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Mathematical Modeling of Treatment SIR Model with Respect to Variable Contact Rate

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Abstract. We describe and analyze compartmental models for disease transmission. We begin with models for epidemics, showing how to calculate the basic reproduction number and the final size of the epidemic. We also study models with multiple compartments, including treatment of infective. We also consider models including births and deaths in which there may be an endemic equilibrium. We argue that if we lack detailed knowledge of the biology of the transmission process, parameter estimation should be accompanied by a structural sensitivity analysis, in addition to the standard statistical uncertainty analysis. Here we focus on the estimation of the basic reproductive number from the initial growth rate of an outbreak as this is a setting in which parameter estimation can be surprisingly sensitive to details of the time course of infection.

Keywords: Epidemic modelling, Variable Infectiousness, SIR Model.

1. Introductions

Estimation of epidemiological parameters from disease outbreak data often proceeds by fitting a mathematical model to the data set. The SIR Model currently used is extremely simplistic. Only considers three compartments, namely Susceptible, Infected and Recovered. Two directions of changes, namely from Susceptible to Infected or from Infected to Recovered. Since most vector-borne diseases do not work in such a way, this project aims to modify this SIR model so that it can encompass much more factors that the original SIR model.

The following is the summery of the notation which are used in following models

S= Susceptibles

I= Infectives

R= Removed with Immunity

T= Treatment class

β= Contact Rate

μ= Recovery Rate

B= Natural Birth Rate

α=Natural Death Rate

ρ=Infection Rate for Tratment

δ=Reduce Infection by Treatment

R_c=Relative Removal Rate

R_0= Basic Reproduction Number

N= Total Population

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2. Model formulation

2.1. SIR Model

In 1927 W.O.Kermack and A.G.Mckendric created a model in which they considered a fixed population with only three compertment Susceptable S(t), Infective I(t) and recovered R(t). The compertments used for this model consist of three classes

S(t) is used to represent the number of individuals not yet infected with the disease at time t or those susceptible to the disease.

I(t) denotes the number of individuals who have been infected with the disease and are capable of spreading the disease to those in the susceptible catagory.

R(t) is the compertment used for those individuals who have been infected and then recovered from the disease. Those in these catagory are not able to be infected again or to transmit the infection to others.

$$\frac{ds}{dt} = -\beta SI \text{....(1)}$$

$$\frac{dI}{dt} = \beta SI - \mu I \text{.....(2)}$$

$$\frac{dR}{dt} = \mu I \text{.....(3)}$$

Several assumption were made in the formulation of this equations[1;2]. First and individual in the population must be considered as having a equal probability as agree other individual of contracting the disease with a rate of β which is considered the contact or infection rate of the disease[3]. Therefore, an infected individual makes contact an is able to transmit the disease wth βN others per unit time and the fraction of contacts by an infected with a suscepltable is S/N. The number of new infection in unit time per infective then is βN(S/N) giving the rate of new infections (or those living the susceptable catagory) as βN(S/N)I=βSI. For the 2nd and 3rd equation consider the population leaving the infectives are living this class per unit time to enter the remove class. This process which occure simultaniously are reffer to as the low of muss action. A widely accepted idea that the rate of contract between the two groups in a population is proportional the size of each of the groups cocerned [4;5]. Finally it is assume that the rate of infection and recovery is much faster then the time scales of births & deaths and therefore this factores are ignore in this model.

Now epidemic is spreading if

$$\frac{dI}{dt} = (\beta S - \mu)I > 0$$
$$\Rightarrow \beta S(0) - \mu > 0$$
$$\Rightarrow S(0) > R_c; \qquad \text{Where } R_c = \frac{\mu}{\beta}$$

Here R_c is called relative removal rate and $R_0 = \frac{\beta}{\mu}$ is called basic reproduction number.

Epidemic is control if $S(0) < R_c$

Now we can derive some useful analytical results from this simple model. We 1st assume that initially we have

$S(0) = S_0, I(0) = I_0, R(0) = 0.$

From (1) and (2) we have

$$\frac{dI}{ds} = -1 + \frac{R_c}{S}$$

Therefore we have, $I + S - R_c \ln S = constant$

Using initial conditions we have, $I_0 + S_0 - R_c ln S_0 = constant$

$$I + S - R_c lnS = I_0 + S_0 - R_c lnS_0$$
$$=> I = I_0 + S_0 - S + R_c ln\frac{S}{S_0}$$

Now *I* become in a maximum from ,

$$\frac{dI}{dt} = 0$$
$$=> S = \frac{\mu}{\beta} = R_c$$
$$I_{max} = I_0 + S_0 - R_c + R_c ln\frac{R_c}{S_0} \text{.....(4)}$$

2.2. Modified SIR Model

In this case, new susceptible are arriving and those of all classes are leaving. For this type of situation births and deaths must be included in the model [6,7]. The following differential equations are representing this model, assuming a natural death rate α and birth rate is B.

$$\frac{ds}{dt} = B - \beta SI - \alpha S \text{.....(5)}$$

$$\frac{dI}{dt} = \beta SI - \alpha I - \mu I \text{.....(6)}$$
$$\frac{dR}{dt} = \mu I - \alpha R \text{.....(7)}$$

2.3. Treatment SIR Model

One form of treatment that is possible for some diseases is vaccination to protect against infection before the beginning of an epidemic [8; 10]. For example, this approach is commonly used for protection against annual influenza outbreaks. A simple way to model this would be to reduce the total population size by the fraction of the population protected against infection. However, in reality such inoculations are only partly effective, decreasing the rate of infection and also decreasing infectivity if a vaccinated person does become infected [11; 12]. To model this, it would be necessary to divide the population into two groups with different model parameters and to make some assumptions about the mixing between the two groups. We will not explore such more complicated models here.

If there is a treatment for infection once a person has been infected, we model this by supposing that a fraction ρ per unit time of infective is selected for treatment, and that treatment reduces infectivity by a fraction δ. Suppose that the rate of removal from the treated class is μ [13;15]. The SITR model, where T is the treatment class, is given by

$$\frac{ds}{dt} = B - \beta SI - \beta\delta T - \alpha S \text{.....(8)}$$
$$\frac{dI}{dt} = \beta SI + \beta\delta T - \alpha I - \mu I \text{.....(9)}$$
$$\frac{dT}{dt} = \mu I - \rho T - \alpha T \quad \text{.....(10)}$$
$$\frac{dR}{dt} = \rho T - \alpha R \quad \text{.....(11)}$$

3. Results & Discussions

Here we fix the epidemic parameters in equations (1), (2) and (3) as $\mu = 0.2$, $\beta = 0.3$ and t is varied from 0 to 500, in suitable units. Here initially we assume that $S_0 = 0.99999, I_0 = 0.00001, R_0 = 0$. For these values we plot the Fig. 1. Now from Fig. 1, we can see that upto t=150, the rate of susceptible (S) is gradually decreases & at that time the rate of infective (I) also decreases but the rate of recovery (R) is maximum. That means when the rate of susceptible (S) decreases, the rate of infective (I) also decreases & the rate of recover(R) increases.

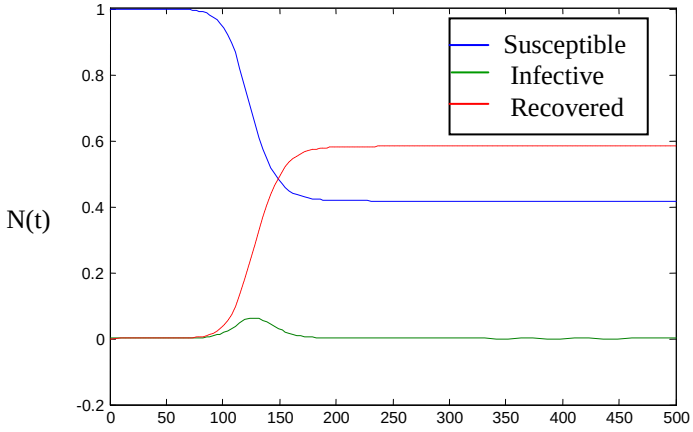

Fig. 1: The rate of susceptible, infective & recovered with respect to time (For $\beta = 0.3$)

Here in Fig. 2 all parameter values are same as used in Fig. 1 only contact rate($\beta = 0.28$) is changed. After changing the contact rate we observe that the infection rate (I) decreases and the recovery rate (R) increases compare to Fig. 1.

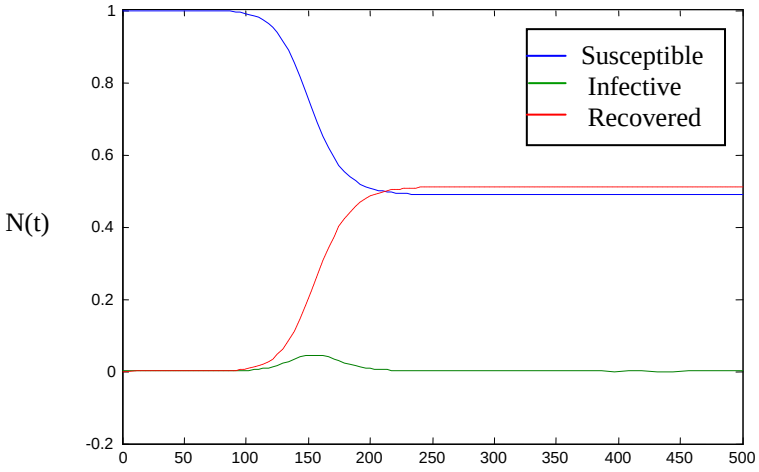

Fig. 2: The rate of susceptible, infective & recovered with respect to time (For $\beta = 0.28$).

Here in Fig. 3 also all the parameter values are same as used in Fig. 1 only contact rate ($\beta = 0.25$) is changed by 0.03. After changing the contact rate we observe that the infection rate (I) is almost nil and the recovery rate (R) is maximum compare to Fig. 1 and Fig. 2. So we can conclude that when contact rate (β) decreases, the infection rate (I), susceptible rate(S) are decreases but recovery rate increases (R).

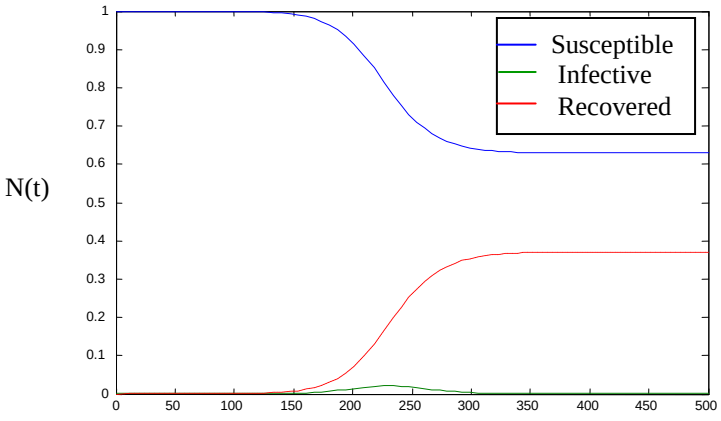

Fig. 3: The rate of susceptible, infective & recovered with respect to time (For $\beta = 0.25$).

Let us now fix the epidemic parameters in equations (1), (4) and (3) as $\mu = 0.1$, $\beta = 0.25$ and t is varied from 0 to 520, in suitable units. Here initially we assume that $S_0 = 0.99999, I_0 = 0.00001, R_0 = 0$. For these values we plot the Fig. 4. Now from Fig. 4,we can see that at time t=50, the infection rate (I) is highest but at that time the susceptible (S) and recovery rate (R) is minimum.After that time, the infection rate (I) decreases, the susceptible rate (S) is finish and the recovery rate(R) increases.

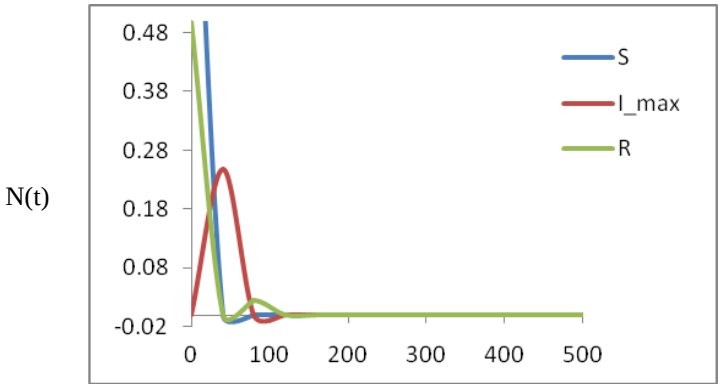

Fig. 4: The rate of susceptible, infective & recovered with respect to time.

After fixing the epidemic parameters in equations (4), (5) and (6), as $\mu = 0.07$, $\beta = 0.3, B = 0.2, \alpha = 0.11$ and t is varied from 0 to 500, in suitable units. Here initially we assume that $S_0 = 0.95, I_0 = 0.04, R_0 = 0$.01For these values we plot the Fig. 5. Now from Fig. 5, we can see that at time t=45 the rate of susceptible is gradually decreases but at that time the rate of infective and recovery rate increases . After certain period of time when t=47, all are in stable position i.e.

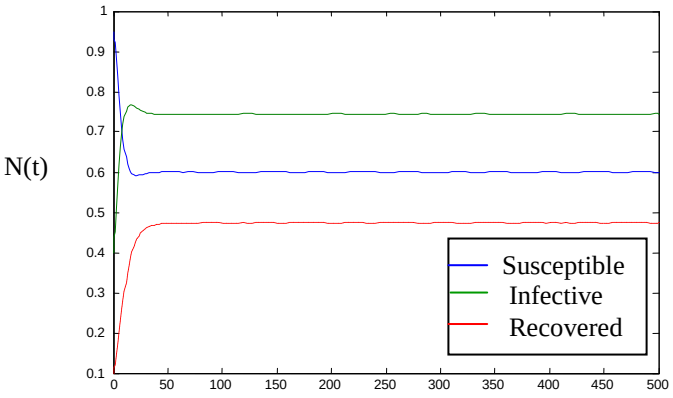

Fig. 5: The rate of susceptible, infective & recovered with respect to time (For $\beta = 0.3$).

Here in Fig. 6 all parameter values are same as used in Fig. 5 only contact rate($\beta = 0.28$) is changed. After changing the contact rate we observe that the infection rate (I) decreases and the recovery rate (R) increases compare to Fig. 5. After certain period of time when t=47, here also all are in stable position.

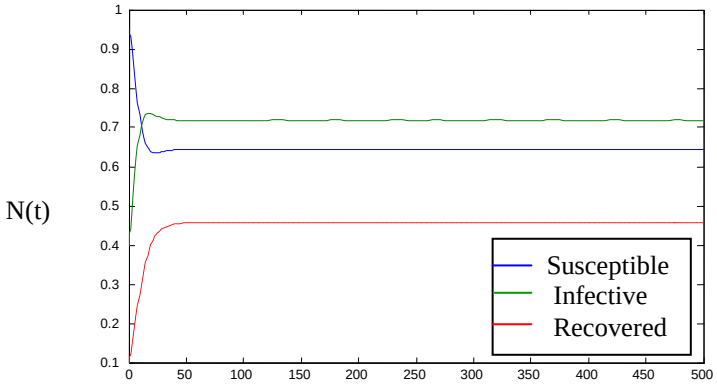

Fig. 6: The rate of susceptible, infective & recovered with respect to time (For $\beta = 0.28$).

Here in Fig. 7 all parameter values are same as used in Fig. 5 only contact rate($\beta = 0.25$) is changed by 0.3. After changing the contact rate we observe that the infection rate (I) decreases and the recovery rate (R) increases compare to Fig. 5 and Fig. 6. In case of Fig. 5 and Fig. 6 we observe that the susceptible rate (S) intersect with the infective rate(R) at N(t)=0.7 but in Fig. 7 they do not intersect and after t=45, they are parallel to each other.

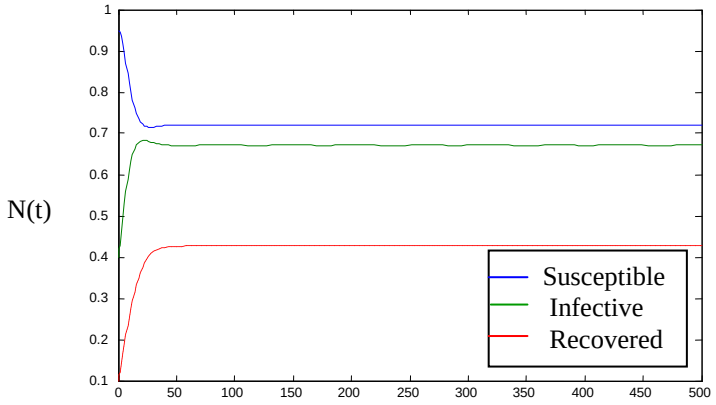

Fig. 7: The rate of susceptible, infective & recovered with respect to time (For $\beta = 0.25$).

Let us now fix the epidemic parameters in equations (8), (9) and (10)as $\mu = 0.07$, $\beta = 0.3, \alpha = 0.21, \rho = 0.32, B = 0.3, \delta = 0.3$ and t is varied from 0 to 500, in suitable units. Here initially we assume that $S_0 = 0.88, I_0 = 0.8, T_0 = 0.3, R_0 = 0.1$. For these values we plot the Fig. 8.Now fromFig. 8,we can see that upto t=25, the rate of susceptible (S) and infective (I) increases but at that time the rate of treatment (T) and recovery (R) rate decreases . In Fig. 8 we observe that, infection (I) starts from N(t)=0.3 and it becomes stable at N(t)=0.4 and we can conclude that compared with all above figures infection rate can not increase beyond N(t)=0.4.

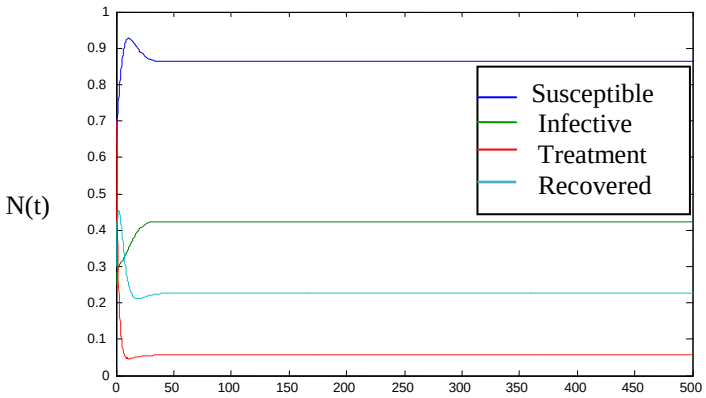

Fig 8: The rate of susceptible, infective, treatment & recovered with respect to time (For $\beta = 0.3$).

Here in Fig. 9 also all the parameter values are same as used in Fig. 8 only contact rate ($\beta = 0.28$) is changed by 0.02. After changing the contact rate we observe that the susceptible (S) rate lightly increase but infection rate (I) decrease and the recovery (R) and treatment (T) rate are also decreases.

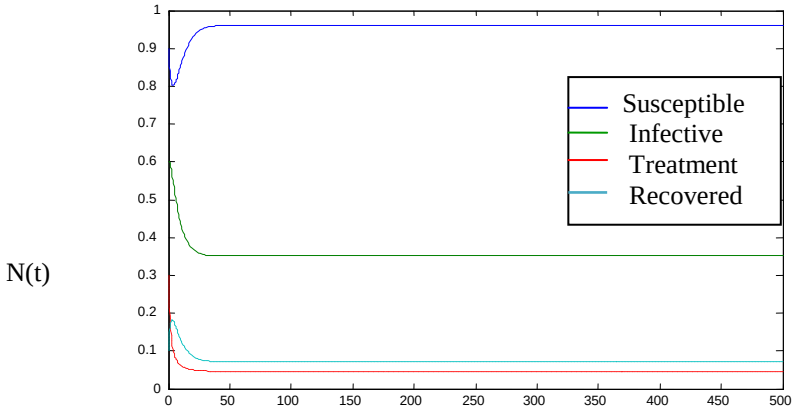

Fig. 9: The rate of susceptible, infective, treatment & recovered with respect to time (Fo\r$\beta = 0.28$).

Here in Fig. 10 also all the parameter values are same as used in Fig. 8 only contact rate ($\beta = 0.25$) is changed by 0.03. After changing the contact rate we observe that the infection rate (I), recovery rate (R), treatment rate (T) but susceptible rate (S) increases compare to Fig. 8 and Fig. 9.

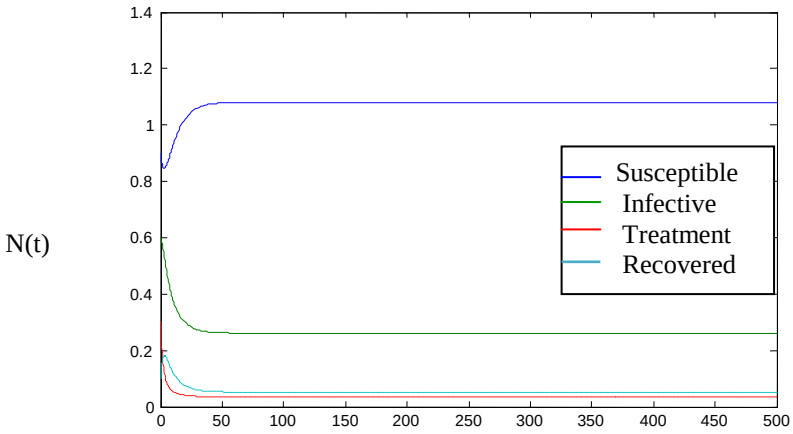

Fig. 10: The rate of susceptible, infective, treatment & recovered with respect to time (For $\beta = 0.25$).

4. Conclusions

This paper presents an overview of the mathematical epidemic models of population disease. Here the SIR model provides a basic framework for the investigation of the epidemic spread. Here we extend the SIR model by modifying the conditions of the infection [9]. In the studied model, a susceptible agent becomes infectious after accumulating exposure larger than its resistance level. The fact that for each of the systems studied here there is a unique solution implies there is some important underlying structure [14].A more detailed analysis of this phenomenon may yield much more general results.Used to determine the rate of change of a function of Infection and recovery obtained via differentiation based on data acquired.

5. References

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