DOI: 10.7763/IPEDR. 2014. V75. 29

Estimation the System Reliability using Weibull Distribution

D. R. Dolas [1 +], M.D. Jaybhaye [2] , S. D. Deshmukh[3]

[1] PhD Student , MGM'S JNEC Aurangabad , Aurangabad-431003 India
[2] Associate Professor in Production Engineering Dept, COEP, Pune-411005, India.
[3]Principal MGM'S JNEC Aurangabad - Aurangabad, 431003, India

Abstract. An increasing complexity of systems often leads to an increase of failure mechanisms. A statistical analysis of the lifetime of systems with several failure mechanisms consisting of several sub-components considered The Weibull distribution is commonly used as a lifetime distribution in reliability applications. The two-parameter Weibull distribution can represent a decreasing, constant or increasing failure rate. This paper presented the estimation of system reliability using two parameter Weibull distributions. The parameters are estimated using Weibull probability plot & statistical analysis and the results being presented in charts. Computation is made using 'Windchill quality solution 10.1 Tryout' software.

Keywords: Reliability, Weibull distribution, WPP Method.

1. Introduction

Weibull distribution is named after Walodi Weibull (1887 – 1979). It is very flexible and can through an appropriate choice of parameters and model many types of failure rate behaviors. This distribution can be found with two or three parameters; scale, shape and location parameters. There are a number of methods for estimating the values of these parameters; some are graphical and others are analytical. Graphical methods include Weibull probability plotting and hazard plot. These methods are not very accurate but they are relatively fast. The analytical methods include maximum likelihood method, least square method and method of moments. These methods are considered as more accurate and reliable compared to the graphical method. In this article an attempt is made to estimate the system reliability using two parameter Weibull distributions & Computation is made using 'Windchill quality solution 10.1 Tryout' Software.

The Maximum likelihood and median rank regression methods are used by researchers to estimate parameters of Weibull distribution [1]. Parameter estimation method for the machine tool reliability analysis to overcome the problem of unavailability of a well-defined failure data collection mechanism was given by Lad et al.[2]. It uses the knowledge and experience of maintenance personnel to obtain the parameters of lifetime distribution of the repairable as well as non-repairable components subassemblies. The Weibull distribution is the standard function used by the wind energy community to model the wind speed frequency distribution and compare the methods [3]. Estimation the reliability function using the Maximum likelihood Method & Weighted Least Square Method & simulation procedure using Monte Carlo Method are used and several experiments are implemented to find the best estimators which have smallest mean square error [4]. Monte Carlo simulations are used to investigate the correlation between system complexity and component lifetime distributions. [5].Moments, maximum likelihood and least squares are compared & used the means square error & total deviation, as measurement for the comparison between these methods [6]. Weibull technique is used for the reliability analysis & the Pareto analysis is carried out. The spare parts optimization was also carried out for a few vital components of this wind farm and the results are presented [7]. Gear box assembly analysis and the failure data in various operating conditions was taken from the logbooks of the vehicles. For modeling purposes the Weibull distribution has been chosen & result will be useful to the maintenance engineer to find the unreliability of the gear box assemblies and also for future strategic decision making [8]. Paritosh Bhattacharya presented the analytical methods for estimation of parameters [9].

[+] Corresponding author. Tel.: + 91 9422655271; fax: +0240- 2482893.
E-mail address: *dhananjay_dolas@rediffmail.com*

Reliability analysis of fans using Weibull and lognormal models & analyze the current test design of fans [10]. The comprehensive analysis for complete failure data using Weibull Distribution and the Median rank regression (MRR) for data- fitting method is described and goodness-of-fit using correlation coefficient [11]. Analyses of real data set & compare these estimators in terms of deficiency via Monte Carlo simulation [12]. The moment-based piecewise polynomial model are proposed to estimate the parameters of the reliability & probability distribution of the products [13]. The shape and scale parameters of the distribution are frequently used to design and characterize commercial wind conversion machines, examine three different models [14]. Two parameter Weibull distribution's linear regression model is used for analyzing wind speed pattern variations. [15].

2. Methods & Materials

2.1 Weibull Distribution

The two parameter Weibull distribution requires characteristic life (η) and shape factor (β) values. Beta (β) determines the shape of the distribution. If β is greater than 1, the failure rate is increasing. If β is less than 1, the failure rate is decreasing. If β is equal to 1, the failure rate is constant. There are several ways to check whether data follows a Weibull distribution, the best choice is to use a Weibull analysis software product. If such a tool is not available, data can be manually plotted on a Weibull probability plot to determine if it follows a straight line. A straight line on the probability plot indicates that the data is following a Weibull distribution. Weibull shape parameter β also indicates whether the failure rate is constant or increasing or decreasing if β= 1.0, β >1.0, β <1.0 respectively. The cumulative % failures versus operating time data are plotted on Weibull graph [10, 11] Fig. (1, 2,3,4,5 & 6) and the values of the parameters are obtained using the statistical software 'Windchill quality solution 10.1Tryout' software.

2.2 Weibull Probability Plotting

A distribution's probability plotting paper is constructed by linear the cumulative density function (CDF) or unreliability function of the distribution. Once this has occurred, the scales for the x- and y-axis of the distribution's plotting paper can be constructed. The CDF or unreliability function [6] of the two-parameter Weibull distribution is given by equation (1):

$$\mathrm{F(t)} = \mathrm{Q(t)} = 1 - \mathrm{e}^{-\left(\frac{\mathrm{t}}{\mathrm{\eta}}\right)^{\beta}} \tag{1}$$

Where

β & η are scale & shape parameters.

The equation (1) may be written in linear form as follows

$$\mathrm{y} = \mathrm{mx} + \mathrm{c} \tag{2}$$

$$\mathrm{Q(t)} = 1 - e^{-\left(\frac{\mathrm{t}}{\mathrm{\eta}}\right)^{\beta}}$$

$$\ln(1 - \mathrm{Q(t)} = \ln\left(e^{-\left(\frac{t}{\eta}\right)^{\beta}} \right)$$

$$\ln(1 - \mathrm{Q(t)} = -\left(\frac{\mathrm{t}}{\mathrm{\eta}}\right)$$

$$\ln(-\ln(1 - \mathrm{Q(t)})) = \beta ln\left(\frac{\mathrm{t}}{\mathrm{\eta}}\right)$$

$$\ln(-\ln\left(\frac{1}{(1-\mathrm{Q(t)}}\right) = \beta\ln(\mathrm{t}) - \beta\ln(\mathrm{\eta})$$

$$\mathrm{y} = \ln\left(\ln\left(\frac{1}{(1-\mathrm{Q(t)}}\right)\right)$$

$$\mathrm{x} = \ln(\mathrm{t})$$

The CDF equation can now be rewritten as: $\mathrm{y} = \beta\mathrm{x} - \beta\ln(\mathrm{\eta})$

This is now a linear equation, with a slope of β and an intercept of $\beta\ln(\mathrm{\eta})$. Now the x- and y-axes of the Weibull probability plotting paper can be constructed. The x-axis is simply logarithmic, since $\mathrm{x} = \ln(\mathrm{t})$

The y-axis is slightly more complicated, since it must represent

$$y = \ln\left(\ln\left(\frac{1}{(1-Q(t))}\right)\right)$$

Reliability is defined as the probability in which an item or an entity performs its intended function over a period of time understated conditions. The reliability function for the two-parameter Weibull distribution is given as.

$$R(t) = e^{-\left(\frac{t}{\eta}\right)^{\beta}} \quad (3)$$

The Weibull failure rate function is defined as the number of failures per unit time that can be expected to occur for the product. It is given as.

$$\lambda(t) = \frac{\beta}{\eta} \times \left(\frac{t}{\eta}\right)^{\beta-1} \quad (4)$$

The two-parameter Weibull probability density function f(t) is given as

$$PDF = f(t) = \frac{\beta}{\eta} \times \left(\frac{t}{\eta}\right)^{\beta-1} \times e^{-\left(\frac{t}{\eta}\right)^{\beta}} \quad (5)$$

$$CDF = F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}} \quad (6)$$

The mean life or mean time of failure (MTTF or MTBF) is defined as the average time of failure-free operation up to a failure event calculated from a homogeneous lot of equipments under operation. The MTTF or MTBF of the Weibull PDF is given as

$$MTBF = E(T) = \gamma + \eta \times \Gamma\left(\frac{1}{\beta} + 1\right) \quad (7)$$

3. Case Study

Estimation of reliability, failure rate failure density (PDF) cumulative density function &unreliability of diesel engine for compressor application of twenty same Make & Model the time to failure of cooling system the failure data as given below

1276,720, 1135, 1854, 1687, 2570, 2440, 2547, 1100, 2117, 1876, 1633, 2646, 1556, 2470, 1250, 1895, 2607,896 & 401

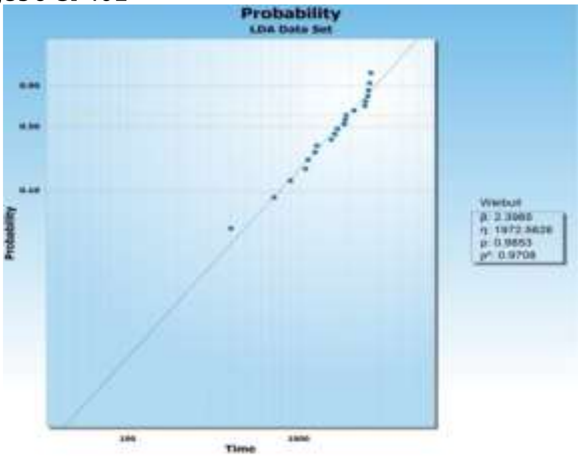


Fig.1: Weibull probability plot

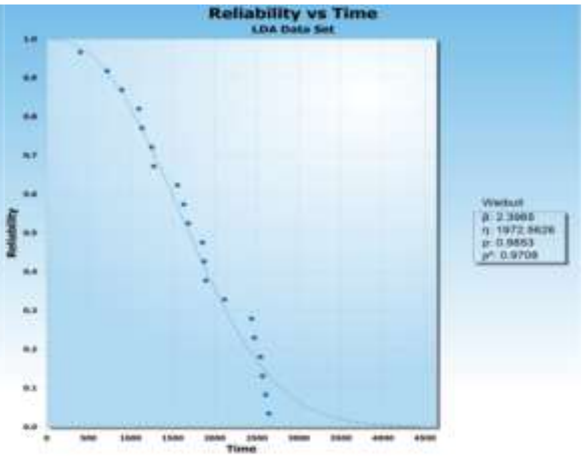


Fig.2: Reliability *Vs* Time plot

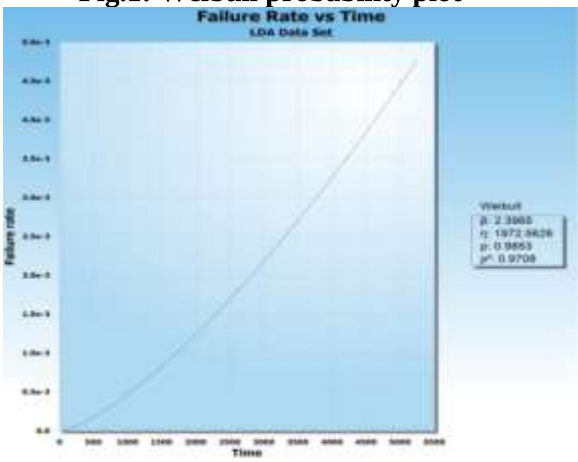


Fig.3: Failure rate *Vs* Time plot

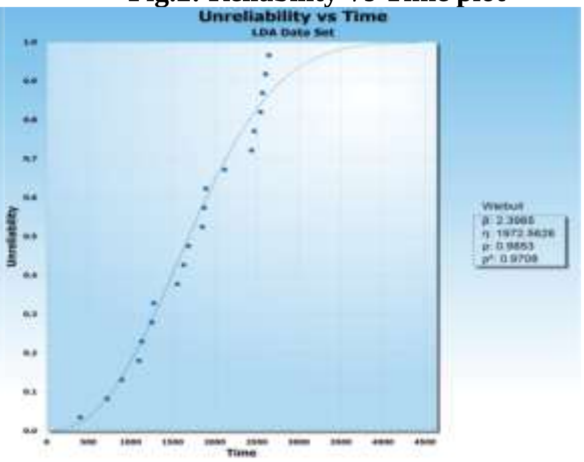
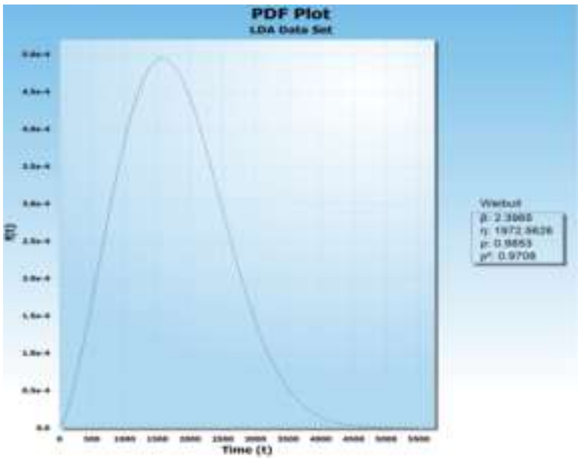


Fig.4: Unreliability *Vs* Time plot



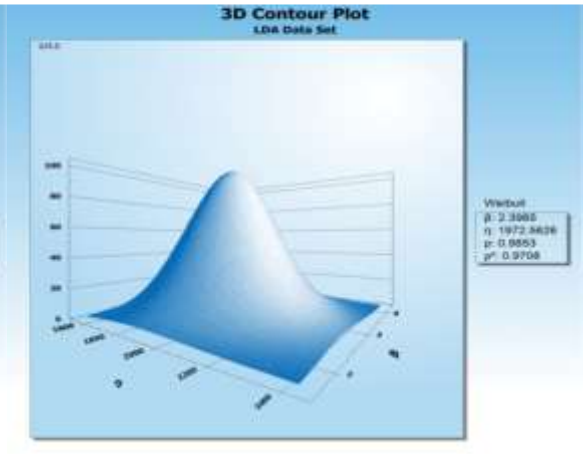


Fig.5: Probability density function plot **Fig.6: 3D Contour plot**

Using the software tool 'Windchill quality solution 10.1 Tryout' the Weibull parameters are:

β= 2.3965 η = 1972.5626 ρ = 0.9853 $\rho2$ =0.9708

Estimation of Mean time between failure (MTBF) or Expected time to system failure E (T)

$\text{MTBF} = \text{E(T)} = \gamma + \eta \times \Gamma\left(\frac{1}{\beta} + 1\right)$ = 0+ 1972.56 × Γ (1/2.3965 +1) = 1972.56× Γ (1.417)

= 1748.48hours

Reliability = $\text{R(t)} = e^{-\left(\frac{t}{\eta}\right)^{\beta}} = e^{-\left(\frac{1748.48}{1972.56}\right)^{2.3965}}$ = 0.47

Cumulative density function = $\text{CDF} = \text{F(t)} = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}}$ = 1- 0.47 = 0.53

Failure rate= $\lambda(t) = \frac{\beta}{\eta} \times \left(\frac{t}{\eta}\right)^{\beta-1} = \frac{2.3965}{1972.56} \times \left(\frac{1748.48}{1972.56}\right)^{2.3965-1}$ = 10.26×10^{-4}

Probability density function = PDF= R (t) × λ (t) = 0.47 ×10.26 × 10-4 = 4.825× 10-4

4. Results & Discussion

Weibull distribution parameters are estimated using 'Windchill quality solution 10.1Tryout' software tool very easily and statistical computation & charts are presented in fig (1, 2,3,4,5, and 6) the Fig 1. Presented the Weibull probability plot with parameters are estimated & failure pattern of diesel engine. Fig 2 presents the reliability of diesel engine using failure data. Fig. 3 shows the failure rate but failure rate is not constant due to repairable system it may increase, constant or decreasing. Fig 4 shows unreliability *vs* time plot of diesel engine. Fig 5 shows probability density function on basis of time to failure of diesel engine & Fig. 6 shows the 3D contour plot .In present case study electric & electronics system & components are not considered which may be analyzed based on similar basis with different Weibull parameters.

5. Conclusion

The conducted research regarding the Weibull parameter estimation using 'Windchill quality solution 10.1Tryout' software and estimation of the reliability of diesel engine, the parameter estimation is very fast as compared to the analytical methods. As per literature survey this method is not accurate but relatively fast. The empirical approximation of functions was taken and it showed that the Weibull distribution parameters are β = 2.3965 and η = 1972.56 approximates well the reliability of the diesel engine, and that the expected time of failure-free function E (T) = 1748.48 h. Starting from the established fact that diesel engine has an increasing rate of failure, and that failure causes may be different in nature from overloading the engine and fatigue of material, to wear and corrosion, it was necessary to determine the individual failure rates of diesel engine subsystems (parts) and their contribution to overall reliability and failure rate.

Acknowledgements

The authors are deeply grateful for help of General Manager Mr. Yuvraj Lavhale and employees of Trinity Mahalasa Durga sales & services, Aurangabad during field studies. Also authors acknowledge the guidance of Dr.S.K.Basu, Emeritus Professor, College of Engineering, Pune.

References

[1] Yunn-Kuang Chu and Jau-Chuan Ke , Computation approaches for Parameter Estimation of Weibull Distribution, *Mathematical and Computational Applications*, 2012 Vol. 17, No. 1, pp. 39-47,

[2] Bhupesh Kumar Lad and Makarand S. Kulkarni A parameter estimation method for machine tool reliability analysis using expert judgement**,** *Int. J. Data Analysis Techniques and Strategies,* 2010 Vol. 2, No. 2, pp-155-169..

[3] Salahaddin A. Ahmed, Comparative study of four methods for estimating Weibull parameters for Halabja, Iraq, *International Journal of Physical Sciences*, 2013 Vol. 8(5), pp. 186-192.

[4] Dhwyia S. Hassun, Nathier A, Ibrahim, Assel N. Hussein Comparing Different Estimators of Parameters and Reliability for Mixed Weibull by Simulation, *American Journal Of Scientific And Industrial Research*, 2012, 3(6), pp 406-429

[5] Peng Wang & Tongdan Jin Complex Systems Reliability Estimation Considering Uncertain Component Lifetime Distributions, *IEEE* 2009, PP-395-399

[6] Ahmad Mahir Razali, Ali A. Salih and Asaad A. Mahdi **,** Estimation Accuracy of Weibull Distribution Parameters**,** *Journal of Applied Sciences Research* 5(7): 2009, pp 790-795.

[7] G.M. Joselin Herbert , S. Iniyan , Ranko Goic , Performance, reliability and failure analysis of wind farm in a developing Country, *Elsevier Ltd Renewable Energy* 2010, 35 , pp- 2739-2751.

[8] Sunil Dutta, Dinesh Kumar and Pradeep Kumar , Reliability analysis of defence vehicles gear box assembly under preventive maintenance , *Indian Journal of Science and Technology* ,2010, Vol. 3 No. 3 , PP 328-331

[9] Paritosh Bhattacharya, A Study on Weibull Distribution for Estimating the Parameters, *Journal applied quantitative methods* , 2010, Vol.5 No.2 , pp-234-241

[10] G. R. Pasha, M. Shuaib Khan, Ahmed Hesham Pasha, Reliability analysis of Fans **,** *Journal of Research (Science),* Bahauddin Zakariya University, Multan, Pakistan,. 2007, Vol. 18, No. 1, pp. 19-33

[11] G.R Pasha1, M. Shuaib Khan2, Ahmed Hesham Pasha, Empirical Analysis of The Weibull Distribution for Failure Data, *Journal of Statistics* 2006 Vol: 13, No.1 pp-33-45

[12] Yeliz Mert Kantar Birdal Senoglu estimating the location and scale parameters of the weibull distribution: an application from engineering, International conference applied simulation & modeling 2007 pp-90-93

[13] Chen, S. K. and Ho, Tin Kin and Mao, B. H. Component reliability estimations without field data, *Hong Kong Institution of Engineers, Transactions,* 2007 14(3). pp. 10-17.

[14] F.C.Odo and G.U.Akubue, Comparative Assessment of Three Models for Estimating Weibull Parameters for Wind Energy Applications in a Nigerian Location , *International Journal of Energy Science* 2012 Vol.2 No.1, PP.22-25

[15] Shrinivas Gautamrao , Jamdade, Parikshit Gautam Jamdade, Analysis of Wind Speed Data for Four Locations in Ireland based on Weibull Distribution's Linear Regression Model , *International Journal Of Renewable Energy Research* ,Vol.2, No.3, 2012, pp 451-456.