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Can Stock Adjustment Model of Canadian Investment Be Meaningful Case for Multicointegration Analysis?

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Abstract. Cointegration which indicates a long-run relationship has become a major issue for models which are based on economic theory. Stock adjustment, being an example for such kind, has been used for stock-flow relationships in economics like investment and capital stock. Related to the stock adjustment models, the multicointegration issue extends the cointegrated long run relationship concept to deeper and more complicated levels. Although inventory stocking models have been studied well in the literature, the general form of stock adjustment model of investment hasen't been brought into the cointegration discussion. This paper, addressing to general form of stock-adjustment model, considers the higher order cointegration and multicointegration issues which arise around nonstationary investment flows and accumulated stocks. Comparing the similar cointegration issues in stock-adjustment models we determine the divergence in the ability of testing methods. We underline the inability of the testing methods for higher order cointegration case. As for the empirical application, estimates of stock a stock-adjustment model of investment for Canadian industry have pointed to 5-8% adjustment speed for the total industry.

Keywords: Stock Adjustment, Investment, Canada, Cointegration, Multicointegration, I(2) Cointegration.

1. Introduction

Granger and Newbold (1974) showed that most economic time series are nonstationary which can lead to spurious estimations unless appropriate estimation techniques are used[1]. The econometrics of nonstationary series has continued to develop with recent studies employing second order integrated econometric models. Issues such as nonstationarity, multicointegration or cointegrated VAR and VECM have been examined in the context of financial models with nonstationary inflation and related nominal price data with second order integrated features (for example see, Berenguer-Rico and Carrion-i-Silvestre, 2010; Johansen, 1995; Johansen *et al.*, 2010; Juselius, 2004; Paruolo, 2000). However, stock adjustment investment models, estimated with different degree nonstationary data series, have not been re-examined in light of these new econometric developments. In this paper, we apply some of these new methods to data for investment by Canadian industry investment.

The aim of this paper is to describe the features of the relevant time series to see whether a stock adjustment investment model lends itself to multicointegration analysis. Then, examining the cointegration structure of the data we discuss restrictions that currently exist in testing multicointegration. Part 2 of the paper presents a typical stock adjustment model for investment. Part 3 investigates the data and stationarity of the data series clarifying which kind of nonstationarity exists for investment-capital data of Canadian industries. Cointegration and multicointegration issues related to the stock adjustment investment model are investigated in part 4 and part 5 presents conclusions.

2. Model

2.1. General stock adjustment model

Stock adjustment theory can be formalized in the most general case for any economical stock and flow variables as Equation (1):

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[1] Harris gives the date of the 1986-March special edition of the *Oxford Bulletin of Economics and Statistics* as the beginning for cointegration studies (Harris, 1995, pp. 1-2).

$$\Delta Q_t = \lambda \cdot (Q_t^* - Q_{t-1}) \quad (1)$$

where

$$\Delta Q_t = Q_t$$

Here Q_t represents an economical stock variable, X_t represents an economical flow variable, Q_t* shows the optimum stock value, λ is an estimated adjustment coefficient and Δ is a difference operator. The deviation from the optimum value multiplied by an adjustment coefficient (here λ) gives the rate of adjustment process towards the optimum value of the stock (see Greenberg, 1964; Kopcke and Brauman, 2001). This approach can be applied for different economic variable such as a price index, the stock and flow of money, and the flow of investment-capital stock. In the case of investment and capital different interpretations have important results for cointegration and multicointegration which forms the main focus of this paper.

2.2. Stock adjustment model of investment

Stock adjustment models have been applied to investment behaviour in a few different ways. A specific form of stock adjustment behaviour has been formulized in Eq. (2). In this form, inventory stock is connected to the two flow variables: production and sales or shipment (Granger and Lee, 1990; Lee, 96). This specification was studied in the well-known early studies of Metzer (1941) and Holt et al. (1960).

$$\Delta Q_t = \alpha + \beta(Q_{t-1} - k y_{t-1}) + \gamma(x_{t-1} - y_{t-1}) \quad (2)$$

where

$\Delta = 1 - L$, a difference operator

In Equation (2) Q_t represents inventory as a stock variable, x_t and y_t represent flow variables such as production and sales. The coefficient k stands for the stock-flow ratio, β and γ parameters represents the speed of adjustment.

Investment behaviour can be given in general form as in equation (3). In this equation investment constitutes the change in the level of capital is defined by the difference between desired and actual level of capital adjusted by coefficient λ (Kopcke and Brauman, 2001). The adjustment rate (λ) here can be interpreted as the speed of adjustment or adjustment ratio which can be related to the type and structure of the industry (Greenberg, 1964). Desired/optimum level of capital can also be defined according to different theoretical approaches (for example see, Kopcke and Brauman, 2001, pp.10-17), but this general form is suitable for current purposes.

$$\Delta K_t = \lambda(K_t^* - K_{t-1}) \quad (3)$$

or

$$I_{gross} - \delta = \lambda(K_t^* - K_{t-1})$$
$$I_{gross} = \lambda K_t^* + (\delta - \lambda)K_{t-1} \quad (4)$$

K_t*: Desired/optimum capital stock

K_t : Capital stock

I_t: Investment flow

δ : depreciation

t: time

λ: adjustment coefficient

Equation (3) includes the inventory change implicitly. It is not possible to measure inventory explicitly. Nevertheless equation (3) can represent the adjustment process by just the capital stock itself, and the change in the capital stock and desired stock level variables. However, an unexpected fall in the sales would reflect an (involuntary) rise in inventory. This consequence, though, is a minor issue for our study compared to its important effect on cointegration analysis of the model. These effects are made clearer in the next part.

3. The Stock Adjustment Model of Investment and Cointegration

Mulitcointegration issues arise in relations of stocks and flows where the change in a stock variable can be determined by the relationship of flow variable(s) with the stock and/or among themselves. In the investment relationship, defining the stock control process by two different same order flow deviations makes it possible to define elaborate cointegration, and open the way to a “deeper form of cointegration” (Lee, 1996, p.633).

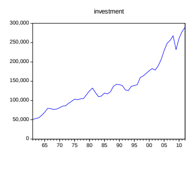
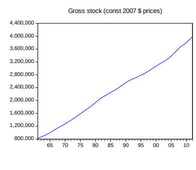
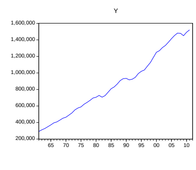
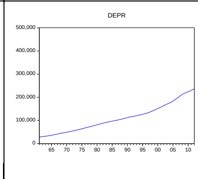
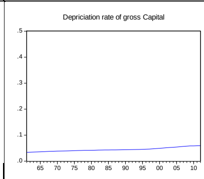
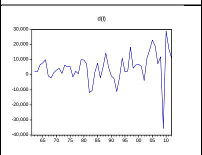
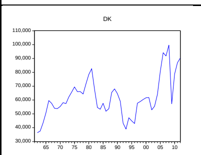
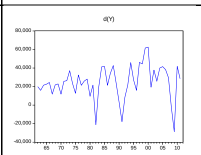
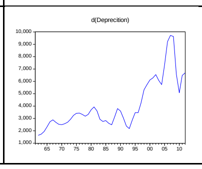
Equation (2) has been applied in important studies as one of the first economic forms for multicointegration (see, Engsted *et al.*, 1997; Granger, 1986; Granger and Lee, 1990; Lee, 1992; Lee, 96). An important feature of equation (2) is that the adjustment relationship in the equation can be represented by the same order nonstationary variables. Whereas, in the form of equations (3) or (4) the capital stock is determined simply by investment, when the investment series show the first order integration I(1) property then it can be expected that the stock variable will be second order integrated. Therefore, determining cointegration in the stock adjustment investment models of equations (3) or (4) is fundamentally different from the analyses that can be applied to equation (2) and it is important to understand and clarify the difference.

The importance has already been stated of determining the integration levels of nonstationary series and the possibility of confusing the first and zero order integration relationship with the second and first order integration relationship. Therefore, nonstationarity levels of the series have been scrutinized in depth.

4. Data Series and Stationarity Analysis

Industrial data of Canada used in this research was obtained from the Statistics Canada database. Industrial investment, capital and capital depreciation data (Cansim Table 0031-0003) include yearly data between 1961- 2012. These data are given in 2007 constant dollar prices. Output of the economy is given as gross domestic product (GDP) data. GDP data were available for 1961 to 2011 (Cansim Table 380-0017) in 2002 constant prices and have been transformed into constant 2007 dollar by using GDP the deflator rate of GDP2007/GDP2002 .

Table I: Statistical Features of the Series

Variables	I ($2007xmillion)	K ($2007xmillion)	Y ($2007xmillion)[a]	δ (depreciation ; $2007xmil)	δ/K (ratio)
Mean	139500.3	2306866.	865173.2	109325.6	0.044700
Median(=**δ**)	125249.0	2328358.	830561.0	100689.5	0.043244
Maximum	289731.3	3982660.	1521431.	236661.1	0.059423
Minimum	51038.80	825469.4	291527.6	28001.50	0.033922
Std. Dev.	62664.09	923500.9	375646.9	59419.57	0.007087
S.Dev/Med					0.16388400
Observation	52	52	51	52	52
Data graph of level series in different scales					
Data graph of differenced series in different scales					

a: Gross domestic product series are given in 2002 dollar by Cansim and transformed to 2007 constant dollars units by using the GDP2002/GDP2007 ratio.

We can see statistical features of the data series and their graphics of level and differenced level in Table 1. Although depreciation is not one of the variables in the general model it does takes place as parameter in equation (4) specification; therefore, it has been included to the scrutiny to be able to assess the stability of the depreciation rate. All series seem trended but as suggested in literature, it is not easy to recognise the difference between first and second order nonstationary series. However, it is easier to see the difference from the differenced series. In Table I, investment and domestic product series are closer to stationary form, whereas, the capital stock series continues to be nonstationary. Moreover, the results of the formal tests given in Table II are consistent with this preliminary analysis.

Table II: Augmented Dickey-Fuller and Phillip-Perron unit Root Tests

Variables	Lag[1]	ADF- with constant	Lag[1]	ADF – no constant or trend	Bandwith[2]	PP – no constant or trend
K	1	1.177	1	-1.200	4	-0.999
Y	1	0.502	1	-2.101	0	-1.540
I	0	1.237	0	-0.707	5	-0.301
DK	0	-2.408	0	-2.719	3	-2.753
DY	0	-4.776***	0	-4.816***	5	-4.655***
DI	0	-6.661***	0	-6.913***	8	-7.231***
D^2K	0	-7.360***	0	-7.282***	10	-7.896***

*Significant at 10% level; **Significant at 5% level; ***Significant at 1% level; MacKinnon (1996) one-sided p-values.
1: Lag lengths are decided according to Schwarz criterion.
2: Bandwith decision that is used in Bartlett kernel spectral estimation is made according to Newey-West method.

In Table II formal unit series tests are applied for the various levels of the series used in the capital stock-adjustment model of investment. The ADF (adjusted Dickey Fuller) tests were applied by using the methodology given by:

The Dickey-Fuller 't-statistics' for the significance of ρ is based on the estimated model:

$$\Delta x_t = \alpha + \beta t + \pi x_{t-1} + \varepsilon_t \tag{5}$$

Alternatively, in the case of autocorrelation in the observed series, estimate the augmented Dickey-Fuller model:

$$\Delta x_t = \alpha + \beta t + \pi x_{t-1} + \sum_{i=1}^{k} \gamma_i \Delta x_{t-i} + \varepsilon_t \tag{6}$$

The null hypothesis is that $x_t = x_{t-1} + \varepsilon_t$ where $\varepsilon_t \sim NID(0,\sigma^2)$.

Under the null πˆ will be negatively biased in a limited sample, thus only a one sided test is necessary for determining $H_0 : \pi = 0 [x_t \sim I(1)]$ against $H_a : \pi < 0 [x_t \sim I(0)]$. This model is less restricted, because it allows a deterministic trend as $x_t = \alpha t + \beta t^2 + \pi x_{t-1} + \varepsilon_t$.The critical values are tabulated in Fuller (1976), p. 373, Table 8.5.2 lower part.

The results in Table II show that all the series have unit roots at the original level. When DF tests are applied to the differenced level, investment (I) and product (Y) series become stationary. However, the capital stock (K) series with -2.42 and -2.72 t-values still show nonstationarity and only second differenced level is it stationary with more than 1% significance level (Table II). To be sure the first level correlogram analyzed. The one term peak in PAC (Partial auto correlation) function value and gradually decreasing escalation structure in the AC (autocorrelation) function values show clear auto correlation in its lagged terms. This supports the results of the formal unit root tests. Finally, the second degree differenced level DF test value with over -7 show clear stationarity, and therefore support the I(2) structure for the capital stock series.

This result would be expected since the investment series is a contributing flow to capital stock and forms the main part of the capital stock. It is reasonable to expect that the first order integrated investment series constitutes the second order integrated capital stock series.

5. Cointegration and Multicointegration for the Capital Adjustment Model of Canadian Industries and Estimation Results

The two different way of modeling to investment and capital stock relationship given by Eq. (2) and equations (3) and (4) are theoretically related. The cointegration analyses of them, however, substantially differentiate between them because of econometric prerequisites for data stationarity and integrity levels of the cointegration methods.

The existence of a cointegration relationship among the series of stock flow model of equation (2) is consistent with the literature. To search and detect cointegration among the same order integrated level series using standard methods such as Engle-Granger two step cointegration analysis, or the Johansen test or ARDL bound test[2] is relatively straightforward. It is also agreed that cointegration between flow series is a sign between the cointegration between flow series and their stock levels. This literally means a cointegration between first order integrated I(1) flows and its second order integrated I(2) accumulated series. This multi-dimensional cointegration relationship is defined as multicointegration[3] in the literature.

However, adjustment coefficients which are required in eq. (2) does not help us. We need to estimate the equations of the form (3) or (4). Moreover the cointegration analysis we need to conduct is different here. We have investment series which is I(1), capital stock series which we detected as I(2) and national product series which is also I(1). Having different order integrated data series, the Johansen method is not suitable.. Engle-Granger 2 step analysis is a general method which can refer to the most general definition of the cointegration.. The ARDL bound can test for cointegration of different order integrated series using the unrestricted ECM (error correction model), which can refer all long-short run relations and/or causal relations, has been shown to have some drawbacks[4]. On the other hand the unconventional test statistics of this method are produced for stationary and first order integrated series combinations.

We can progress by applying the Granger 2-step method. Despite its drawbacks mentioned above, the Engle-Granger 2-step method is the only formally legitimate test method for second order integrated series is currently available. As the first step, we estimate equation (7) to check the existence of the long run relationship that we have in our adjustment model:

$$I_t = \alpha + \beta_1\, trend + \beta_2 K_{t-1} + \beta_3 Y_t + \varepsilon_t \qquad (7)$$

The estimation results of the cointegration vector given in Eq. (7) are given below:

$I_t = -170034 - 16293 trend + 0.175 K_{t-1} + 0.379 Y_t + \varepsilon_t$

(-5.14) (-5.58) (4.14) (10.46)

$R^2 = 0.97$

DW=0.56

F-stat=439.08

Here the 0.56 value can be used for the first step CRDW test[5]. Rejecting null hypothesis we conclude the DW statistic here is different from the zero. In the second step we can formally test the stationarity of the residuals of cointegration equation (CE). The results of the ADF test are given on Table III.

Table III: Augmented Dickey-Fuller and Philip-Perron unit Root Tests for CE Residuals

Variables	Lag[1]	ADF- with constant	Lag[1]	ADF – no constant or trend	Bandwith[2]	PP – no constant or trend
ε_t	0	-2.946**	0	-2.995***	2	-3.133***

*Significant at 10% level; **Significant at 5% level; ***Significant at 1% level; MacKinnon (1996) one-sided p-values. [1]:lag lengths are decided according to Schwarz criterion. [2]:Bandwith decision that is used in Bartlett kernel spectral estimation is made according to the Newey-West method.

The ADF and PP test results given in Table III rejects the null hypothesis of having unit root in the residuals of the estimated cointegration vector. This result shows the sign of cointegration. When we estimate the models of form (3) and (4), the results are as below:

$\Delta K_t = 52485 - 0.041 K_{t-1} + 0.114 Y_t + \varepsilon_t$

(10.82) (-3.07) (3.62)

$R^2 = 0.36$

[2] The superiority of ARDL bound test to Johansen method which takes nonstationarity as given is another discussion in this specific subject.

[3] The definition of *multicointegration* is not clear in literature. Some studies use the term of *polynomial cointegration* distinctively for cointegration between second and at least first order integrated series; others use the terms identically.

[4] Two of these drawbacks are the conduct of the first step errors into the second step and the possibility of the construction of the cointegration vector in two different ways.

[5] CRDW (Cointegration regression Durbin-Watson) test is assumed as not conclusive itself.

$F\text{-}stat=13.29$

$DW=0.56$

As explained in the first section this model estimates net investment. The estimation can be applied to gross investment, because investment data are measured as gross investment. The estimation of equation (4):

$I_t=12592-0.050K_{t-1}+0.270Y_t+\varepsilon_t$

(2.09) (-3.02) (6.91)

$R^2=0.94$

$F\text{-}stat=320$

$DW=0.32$

The estimations of stock adjustment models of equations (3) and (4) do not estimate constant and trend significant at the same time. On the other hand, the estimations with either of constant or trend give similar results, and estimation results are given above. The t-statistics which are all significant at the level of 1% or 5% are given in parentheses. The estimations are not free from problems; for example, R^2 with 0.36 value in estimations of (3) and DW statistic with 0.32 value in (4) are said comparatively low. However, the estimated stock adjustment speed values are comparable having 8.4% (adding 4.3% depreciation value determined in Table 1) and 5% respectively.

6. Conclusion

The investigation of the stock adjustment investment model for Canadian industries addresses important issues in both economic and econometric application and points to the necessity of further developments in higher order cointegration and its testing. The scrutiny of Canadian industrial investment data shows signs of cointegration. This result needs to be discussed theoretically in terms of cointegration behaviors of the nonstationary flow series and their accumulations in stocks. The economic result of the estimations can be summarized as a slow adjustment speed for Canadian total industry at a level between 5-8% approximately.

7. References

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