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Markov Chain Models for Estimating Advertising Effectiveness

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Abstract. Advertising is an important component of marketing efforts. This paper concentrates on the effects of the advertising actions and proposes a class of generalized Markov chain models based on the normal distribution for estimating the advertising effects with changing transition matrices. Numerical procedures for establishing such models and estimating market status are also presented. Further comments are also made in the paper to address the possibilities of using other types of models, real-time on-going modification of models, piecewisely defined models or compositions of models.

Keywords: Advertising, Effectiveness, Markov Chain Models.

1. Introduction

Advertising is one of the important components of marketing efforts. Decisions on how to advertise a particular product in a particular time period and to a particular population are usually based on the estimates of the effects and the costs of advertising actions. In this paper, we concentrate on the effects of the advertising actions and we consider the Markov chain models in estimating the effects.

As an example, consider a television advertising campaign for a certain brand, say brand A, of multivitamin to the population of those who regularly take multivitamin daily. A survey is made at the beginning of the campaign, and then at the end of each month for several months. Each survey shows a market status vector $S = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ where x_1 stands for the percent of the population using brand A and x_2 for the percent of the population not using this brand. Clearly $0 \le x_1, x_2 \le 1$ and $x_1 + x_2 = 1$. To be more precise, let us use S_0 to denote the initial status vector, that is, the status vector at the beginning of the advertising campaign, S_1 to denote the status vector at the end of the first month, and so on so forth.

Suppose the survey at the beginning of the campaign shows that the initial status $S_0 = \begin{bmatrix} 0.05 \\ 0.95 \end{bmatrix}$, that is, at that time 5% of the population use brand A while 95% use other brands. Suppose the succeeding surveys show that in each month, 40% of those using brand A continue to use it and 60% switch to another brand. Meanwhile, 20% of those not using brand A switch to brand A while the other 80% continue to use other brands.

Based these data regarding the advertising effects, we can see that

$$S_1 = \begin{bmatrix} 0.4 & 0.2 \\ 0.6 & 0.8 \end{bmatrix} S_0 = \begin{bmatrix} 0.4 & 0.2 \\ 0.6 & 0.8 \end{bmatrix} \begin{bmatrix} 0.05 \\ 0.95 \end{bmatrix} = \begin{bmatrix} 0.21 \\ 0.79 \end{bmatrix} \tag{1}$$

That is, after one month 21% of the population are using brand A while 79% are with other brands. Similarly we have

$$S_2 = \begin{bmatrix} 0.4 & 0.2 \\ 0.6 & 0.8 \end{bmatrix} S_1 = \begin{bmatrix} 0.4 & 0.2 \\ 0.6 & 0.8 \end{bmatrix} \begin{bmatrix} 0.21 \\ 0.79 \end{bmatrix} = \begin{bmatrix} 0.242 \\ 0.758 \end{bmatrix} \tag{2}$$

So after two months 24.2% are using brand A while 75.8% are not.

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If a simple estimate is made based on the surveys of those few months and we believe that the same advertising effects will happen in every month in long run, then a Markov chain $S_0, S_1, S_2 \dots$ is formed, with the transition matrix

$$P = \begin{bmatrix} 0.4 & 0.2 \\ 0.6 & 0.8 \end{bmatrix} \tag{3}$$

It is easy to see that this is a regular Markov chain with the stationary status vector

$$S = \lim_{k\to\infty} S_k = \begin{bmatrix} 0.25 \\ 0.75 \end{bmatrix} \tag{4}$$

In other words, if this advertising campaign keeps going on, then eventually the percentage of the population using brand A will approach 25%, and the market status will stay that way forever. Note that the stationary status S in equation (4) satisfies

$$PS = S \tag{5}$$

Also note that in this case, the advertising campaign made the best gain in the first month.

It is also well known that for a regular Markov chain like this, the stationary status S does not depend on the initial status vector S_0. That is, for any initial status S_0, under the constant transition matrix $P = \begin{bmatrix} 0.4 & 0.2 \\ 0.6 & 0.8 \end{bmatrix}$, the stationary status will always equal $\begin{bmatrix} 0.25 \\ 0.75 \end{bmatrix}$.

In real life business cases, however, situations are not that simple. The transition matrix P usually changes from time period to time period. Therefore, the above simple model might not work well. In this paper, we consider advertising effects with changing transition matrices, and try to mathematically model the transition matrices and numerically estimate the stationary status.

2. Models for Changing Transition Matrices

Let $P(k) = \begin{bmatrix} p_{11}(k) & p_{12}(k) \\ p_{21}(k) & p_{22}(k) \end{bmatrix}$ be the transition matrix for the k-th time period. That is, $p_{11}(k)$ stands for the percentage of those using brand A in the k-the period that continue to use it, $p_{21}(k)$ for the percentage that switch to other brands. Similarly, $p_{12}(k)$ stands for the percentage of those not using brand A in the k-the period that switch to use it, while $p_{22}(k)$ for the percentage that continue to use other brands. Values of $p_{11}(k)$, $p_{21}(k)$, $p_{12}(k)$ and $p_{22}(k)$ change from time period to time period, and hence they are functions of k. However, it is always true that

$$0 \le p_{11}(k), p_{21}(k), p_{12}(k), p_{22}(k) \le 1 \tag{6}$$

and

$$p_{11}(k) + p_{21}(k) = 1, \quad \text{and} \quad p_{12}(k) + p_{22}(k) = 1 \tag{7}$$

Often, short term advertising campaigns or early stages of long term campaigns are likely to aim at gaining new customers. This is particularly true when a new brand is being introduced to the market, or when an established brand is being introduced to a newly opened market. In these early time periods, a growth of the value of $p_{12}(k)$ may be observed, reflecting a positive advertising effects. The value of $p_{11}(k)$ may stay quite stable and high, even close to 1, during early time periods because those who start to try a new product may very much likely to stay with it for some time before they decide whether to stick to it or not. After a while, however, the values of $p_{11}(k)$ and $p_{12}(k)$ may start to decease, and now the long term on-going advertising efforts are needed, perhaps mainly aiming at keeping the existing customers. In long run, the transition matrix $P(k)$ may eventually get closer and closer to the identity matrix $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, meaning that those who have used brand A for a long time may very likely to stay with it, and those with other brands stay with other brands. The market status vector S_k reached in those time periods will then become the stationary status for the brand A.

Therefore, the value of $p_{11}(k)$ may stay close to 1 for a while, then starts to decrease, and eventually comes back up and stays close to 1. The values of $p_{12}(k)$ would grow up for a while in the early stages, then gradually decreases and eventually stays close to 0. The values of $p_{21}(k)$ and $p_{22}(k)$ are determined by $p_{11}(k)$ and $p_{12}(k)$, respectively, due to the equation (7).

Based on above assumptions, many mathematical functions may be used to establish models for $p_{11}(k)$ and $p_{12}(k)$. Below we propose two such models.

1. The upside-down normal distribution model $M_{11}(k) = 1 - \frac{\alpha_1}{\sqrt{2\pi}\sigma_1} e^{-\frac{1}{2}(\frac{k-\mu_1}{\sigma_1})^2}$ for $p_{11}(k)$. Three parameters α_1, μ_1, and σ_1 are involved in the formula of this model. The model is formed by subtracting from 1 a portion of the density function of the normal distribution $N(\mu_1, \sigma_1)$. The curve of $M_{11}(k)$, for example, when $\alpha_1 = 10$, $\mu_1 = 20$, and $\sigma_1 = 10$, is shown in Figure 2.1 below. Note that the value of α_1 determines how low the value of $M_{11}(k)$ can get, μ_1 determines when it will get to that low value, and σ_1 determines how long it will take for the value of $M_{11}(k)$ to approach 1.

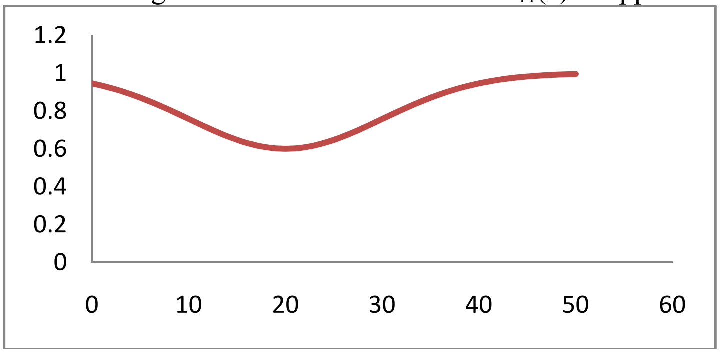

Figure 2.1

2. The normal distribution model $M_{12}(k) = \frac{\alpha_2}{\sqrt{2\pi}\sigma_2} e^{-\frac{1}{2}(\frac{k-\mu_2}{\sigma_2})^2}$ for $p_{12}(k)$. This model is formed by using a portion of the density function of the normal distribution $N(\mu_2, \sigma_2)$. Again, three parameters α_2, μ_2, and σ_2 are involved. In Figure 2.2 below, we display the curve of $M_{12}(k)$ when $\alpha_2 = 5$, $\mu_2 = 18$, and $\sigma_2 = 8$.

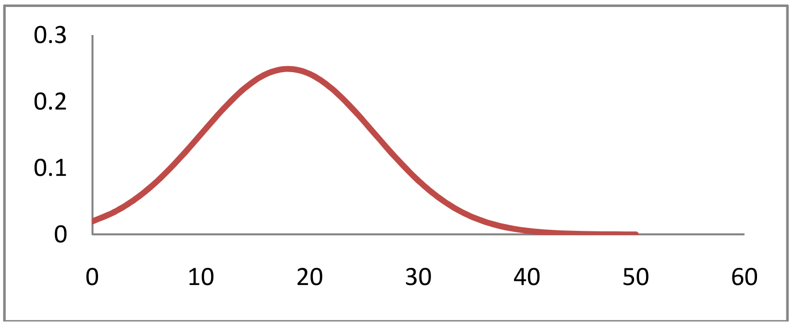

Figure 2.2

Since models $M_{11}(k)$ and $M_{12}(k)$ each involves three parameters, in order to numerically estimate these parameters, surveys need to be conducted for at least three time periods. For example, suppose a survey is made at the beginning of the first time period and then at the end of each time period for the first n periods (with $n \geq 3$) and the values of $p_{11}(k)$ and $p_{12}(k)$, for $k = 1, 2, \ldots n$, are found from these surveys, then by numerically solving the nonlinear system

$$p_{11}(k) = 1 - \frac{\alpha_1}{\sqrt{2\pi}\sigma_1} e^{-\frac{1}{2}(\frac{k-\mu_1}{\sigma_1})^2} \quad \text{for } k = 1, 2, \ldots n \tag{8}$$

we can determine the parameters α_1, μ_1, c_1 in $M_{11}(k)$, and thus establish a model for $p_{11}(k)$. Similarly, by solving the nonlinear system

$$p_{12}(k) = \frac{\alpha_2}{\sqrt{2\pi}\sigma_2} e^{-\frac{1}{2}(\frac{k-\mu_2}{\sigma_2})^2} \quad \text{for } k = 1, 2, \ldots n \tag{9}$$

we can establish the model $M_{12}(k)$ for $p_{12}(k)$. Many numerical methods are available for solving nonlinear systems (8) and (9). See, for example, [2] and [4].

3. Numerical Estimates of Advertising Effects and Stationary Status

In this section, let us use an example to describe the numerical procedure for estimating the advertising effects and stationary market status. Consider this television advertising campaign for the brand A of multivitamin. A survey is made at the beginning of the campaign, and then at the end of each month for four months. Suppose the first survey shows that the initial status $S_0 = \begin{bmatrix} 0.01 \\ 0.99 \end{bmatrix}$, that is, at that time 1% of the population use brand A while 99% use other brands. Suppose the succeeding surveys show that the transition matrix for these four months are:

$$P(1) = \begin{bmatrix} 0.998 & 0.231 \\ 0.002 & 0.769 \end{bmatrix}, \qquad P(2) = \begin{bmatrix} 0.994 & 0.289 \\ 0.006 & 0.711 \end{bmatrix},$$

$$P(3) = \begin{bmatrix} 0.984 & 0.350 \\ 0.016 & 0.650 \end{bmatrix}, \qquad P(4) = \begin{bmatrix} 0.970 & 0.399 \\ 0.030 & 0.601 \end{bmatrix}.$$

Based on these surveys, we have that $p_{11}(1) = 0.994$, $p_{11}(2) = 0.984$, $p_{11}(3) = 0.967$, and $p_{11}(4) = 0.926$. By solving the nonlinear system (8) with $n = 4$, we get $\alpha_1 = 4$, $\mu_1 = 10$, and $\sigma_1 = 3$. Therefore the model for $p_{11}(k)$ in this case is $M_{11}(k) = 1 - \frac{4}{\sqrt{2\pi}\cdot 3} e^{-\frac{1}{2}(\frac{k-10}{3})^2}$. Similarly, the surveys show that $p_{12}(1) = 0.289$, $p_{12}(2) = 0.350$, $p_{12}(3) = 0.399$, $p_{12}(4) = 0.442$. Solving the nonlinear system (9) with these values we see that $\alpha_2 = 6$, $\mu_2 = 6$, and $\sigma_2 = 5$. Thus the model for $p_{12}(k)$ will be $M_{12}(k) = \frac{6}{\sqrt{2\pi}\cdot 5} e^{-\frac{1}{2}(\frac{k-6}{5})^2}$. With these two models we may estimate the monthly transition matrix which represents the advertising effects in each month, using the formula

$$P(k) \approx \begin{bmatrix} M_{11}(k) & M_{12}(k) \\ 1 - M_{11}(k) & 1 - M_{12}(k) \end{bmatrix} \tag{10}$$

The following Table 3.1 shows the estimates of entries in the transition matrices in the 5-th, 10-th, 15-th, and 20-th months. Note that $P(k)$ approached $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ when k gets bigger and bigger.

Table 3.1

k	$p_{11}(k)$	$p_{21}(k)$	$p_{12}(k)$	$p_{22}(k)$
5	0.867	0.133	0.469	0.531
10	0.468	0.532	0.348	0.652
15	0.867	0.133	0.095	0.905
20	0.998	0.002	0.009	0.991

With the established model for the transition matrix $P(k)$ and the value of the initial status $S_0 = \begin{bmatrix} 0.01 \\ 0.99 \end{bmatrix}$, we may easily estimate the market status S_k for all the succeeding months as well as the stationary market status. For example, $S_1 = P(1)S_0 = \begin{bmatrix} 0.24 \\ 0.76 \end{bmatrix}$, $S_2 = P(2)S_1 = \begin{bmatrix} 0.46 \\ 0.54 \end{bmatrix}$, and so on so forth. The Table 3.2 below shows the estimates of S_k for k from 1 to 30. For convenience, in the table the status vector S_k is displayed as a row vector instead of a column vector. From the table we can see that for this product the stationary market status will be about $\begin{bmatrix} 0.43 \\ 0.57 \end{bmatrix}$. That is, with the on-going advertising efforts, the percentage of the population using brand A will approach 43%, and the market status will stay that way in long run.

Table 3.2

k	S_k	k	S_k	k	S_k
1	[0.24, 0.76]	11	[0.40, 0.60]	21	[0.42, 0.58]
2	[0.46, 0.54]	12	[0.37, 0.63]	22	[0.43, 0.57]
3	[0.64, 0.36]	13	[0.36, 0.64]	23	[0.43, 0.57]
4	[0.76, 0.24]	14	[0.36, 0.64]	24	[0.43, 0.57]
5	[0.81, 0.19]	15	[0.37, 0.63]	25	[0.43, 0.57]
6	[0.79, 0.21]	16	[0.38, 0.62]	26	[0.43, 0.57]
7	[0.72, 0.28]	17	[0.39, 0.61]	27	[0.43, 0.57]
8	[0.62, 0.38]	18	[0.40, 0.60]	28	[0.43, 0.57]
9	[0.52, 0.48]	19	[0.41, 0.59]	29	[0.43, 0.57]
10	[0.45, 0.55]	20	[0.42, 0.58]	30	[0.43, 0.57]

It is also interesting to mention that, like in the regular Markov chain with constant transition matrix, the stationary status in this model does not depend on the initial status vector. For example, if $S_0 = \begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix}$ instead of $\begin{bmatrix} 0.01 \\ 0.99 \end{bmatrix}$, the stationary market status will still be $\begin{bmatrix} 0.43 \\ 0.57 \end{bmatrix}$; and this remains true for any value of initial status vector.

4. Further Comments

The example used in Section 3 is a simulated case intended to describe the numerical produce to establish models for changing transition matrices and thus to estimate the advertising effects and stationary status. In real business applications, survey results need to be evaluated with care and models should be selected carefully. There is a variety of models that could be applied in the estimating procedure, including exponential models, polynomial models, and so on. Moreover, more survey results may become available when time goes by. Therefore, real-time on-going modification of models may be conducted based on the additional information obtained from the newly received survey results. Furthermore, other factors such as advertising efforts of competitors, changes in the economic environments, growth of the targeted population, etc, may also have impacts on the advertising effects. Therefore, more generalized models such as piecewisely defined models or compositions of models may also be needed to address the effects of those factors.

5. References

[1] W. F. Arens, *Contemporary Advertising,* 10th Ed., McGraw-Hill, New York, 2006.

[2] R. Fletcher, *Practical Methods of Optimization,* 2nd Ed., John Wiley & Sons, New York, 1987.

[3] G.R. Noubary and Y. Shi, Difference equation models for estimating athletic records, *The Journal of Computational and Applied Mathematics*, 91, pp 107 -- 122, 1998.

[4] Y. Shi, A globalization procedure for solving nonlinear systems of equations, *Numerical Algorithms*, 12, pp 272 -- 286, 1996.

[5] Y. Shi, Hot money vs. cool money – A probability model for the trend of investment cash flow, *Proceedings of 2011 International Conference on Financial Management and Economics*, pp 226—230, 2011.