

# Acceptance Sampling – Economical Approach

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**Abstract.** In this paper we shall deal with the AOQL single sampling plans when the remainder of rejected lots is inspected. We shall consider two types of AOQL plans - for inspection by variables and for inspection by variables and attributes (all items from the sample are inspected by variables, remainder of rejected lots is inspected by attributes). These plans we shall compare with the corresponding Dodge-Romig AOQL plans by attributes. The calculation of these plans is considerably difficult, we shall use an original method. From the results of numerical investigations it follows that under the same protection of consumer the AOQL plans for inspection by variables are in many situations more economical than the corresponding Dodge-Romig attribute sampling plans (saving of the inspection cost is 70% in any cases).

**Keywords:** Acceptance Sampling, AOQL Plans, Inspection by Variables.

## 1. Introduction

Under the assumption that each inspected item is classified as either good or defective (acceptance sampling by attributes) in [1] are considered sampling plans which minimize the mean number of items inspected per lot of process average quality

$$I_s = N - (N - n) \cdot L(\bar{p}; n, c) \quad (1)$$

Under the condition

$$\max_{0 < p < 1} AOQ(p) = p_L \quad (2)$$

(AOQL single sampling plans), where  $N$  is the number of items in the lot (the given parameter),  $\bar{p}$  is the process average fraction defective (the given parameter),  $p_L$  is the average outgoing quality limit (the given parameter, denoted AOQL),  $n$  is the number of items in the sample ( $n < N$ ),  $c$  is the acceptance number (the lot is rejected when the number of defective items in the sample is greater than  $c$ ),  $L(p)$  is the operating characteristic (the probability of accepting a submitted lot with fraction defective  $p$ ),  $AOQ(p)$  is average outgoing quality (the mean fraction defective after inspection when the fraction defective before inspection was  $p$ ).

Condition (2) protects the consumer against the acceptance of a bad lot. The AOQL plans for inspection by attributes are in [1] extensively tabulated.

## 2. AOQL Plans by Variables and Attributes

The problem to find AOQL plans *for inspection by variables* has been solved in [2] under the following assumptions:

Measurements of a single quality characteristic  $X$  are independent, identically distributed normal random variables with unknown parameters  $\mu$  and  $\sigma^2$ . For the quality characteristic  $X$  is given either an upper specification limit  $U$  (the item is defective if its measurement exceeds  $U$ ), or a lower specification limit  $L$  (the item is defective if its measurement is smaller than  $L$ ). It is further assumed that the unknown parameter  $\sigma$  is estimated from the sample standard deviation  $s$ .

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The inspection procedure is as follows: Draw a random sample of  $n$  items and compute  $\bar{x}$  and  $s$ . Accept the lot if

$$\frac{U - \bar{x}}{s} \geq k, \text{ or } \frac{\bar{x} - L}{s} \geq k. \quad (3)$$

We have determine the sample size  $n$  and the critical value  $k$ . There are different solutions of this problem. In paper [2] we used for determination  $n$  and  $k$  a similar condition as Dodge and Romig in [1].

Now we shall formulate this problem. Let us consider *AOQL plans for inspection by variables and attributes* – all items from the sample are inspected by variables, but the remainder of rejected lots is inspected only by attributes. Let us denote

$$\begin{aligned} c_s^* & - \text{the cost of inspection of one item by attributes,} \\ c_m^* & - \text{the cost of inspection of one item by variables.} \end{aligned}$$

Inspection cost per lot, assuming that the remainder of rejected lots is inspected by attributes (the inspection by variables and attributes), is  $n \cdot c_m^*$  with probability  $L(p; n, k)$ , and  $n \cdot c_m^* + (N - n) \cdot c_s^*$  with probability  $1 - L(p; n, k)$ . The mean inspection cost per lot of process average quality is therefore

$$C_{ms} = n \cdot c_m^* + (N - n) \cdot c_s^* \cdot [1 - L(\bar{p}; n, k)] \quad (4)$$

Now we shall look for the acceptance plan  $(n, k)$  minimizing the mean inspection cost per lot of process average quality  $C_{ms}$  under the condition (2). The condition (2) is the same one as used for protection the consumer Dodge and Romig in [1]. Let us introduce a function

$$I_{ms} = n \cdot c_m + (N - n) \cdot [1 - L(\bar{p}; n, k)], \quad (5)$$

Where

$$c_m = c_m^* / c_s^*. \quad (6)$$

Since

$$C_{ms} = I_{ms} \cdot c_s^*, \quad (7)$$

Both function  $C_{ms}$  and  $I_{ms}$  have a minimum for the same acceptance plan  $(n, k)$ . Therefore, we shall look for the acceptance plan  $(n, k)$  minimizing (5) instead of (4) under the condition (2).

For these AOQL plans for inspection by variables and attributes *the new parameter*  $c_m$  was defined – see (6). This parameter must be estimated in each real situation. Usually is

$$c_m > 1. \quad (8)$$

Putting formally  $c_m = 1$  into (5) ( $I_{ms}$  in this case is denoted  $I_m$ ) we obtain

$$I_m = N - (N - n) \cdot L(\bar{p}; n, k), \quad (9)$$

i.e. the mean number of items inspected per lot of process average quality, assuming that both the sample and the remainder of rejected lots is inspected by variables. Consequently *the AOQL plans for inspection by variables* are a special case of *the AOQL plans by variables and attributes* for  $c_m = 1$ . From (9) is evident that for the determination AOQL plans by variables it is not necessary to estimate  $c_m$  ( $c_m = 1$  is not real value of this parameter).

*Summary:* For the given parameters  $p_L$ ,  $N$ ,  $\bar{p}$  and  $c_m$  we must determine the acceptance plan  $(n, k)$  for inspection by variables and attributes, minimizing  $I_{ms}$  under the condition (2).

Solution of this problem is in the paper [2]. Calculation of these plans is considerably difficult. In the first place we shall solve the equation (2), in the second place we shall determine the acceptance plan  $(n, k)$  minimizing  $I_{ms}$  under the condition (2).

For given sample size  $n$  (and given  $N, p_L$ ) we shall look for the critical value  $k$  for which holds (2), i.e. (see [2])

$$M_n(k) - p_L / (1 - \frac{n}{N}) = 0. \quad (10)$$

Under suitable assumptions solution of the equation (10) exists and is unique – see [2]. This solution is considerably difficult (explicit formula for  $k$  does not exist), we must solve (10) two times numerically (in the first step we determine  $x_M$  as a solution of equation  $G'(x) = 0$ , in the second step we determine  $k$  as a solution (10) – see [2]). Numerical solution of equation  $G'(x) = 0$  depends on good first approximation  $x_0$ . In [3] is proved that solution  $x_M$  of this equation is between

$$x_r = \frac{k}{1+A}, \quad x_a = \frac{k + A\sqrt{k^2 - 2(1-A^2)\ln A}}{1-A^2}, \quad (11)$$

Where

$$A = \sqrt{\frac{1}{n} + \frac{k^2}{2(n-1)}}. \quad (12)$$

Therefore we choose for  $x_0$  following point (numerical investigations show that this point is good start value)

$$x_0 = \frac{(100+n)x_r + nx_a}{2n+100}. \quad (13)$$

For the comparison these plans with the corresponding Dodge-Romig LTPD attribute sampling plans from an economical point of view we used parameters  $E$  (inspection by variables) and  $e$  (inspection by variables and attributes), defined by relations

$$E = \frac{I_m}{I_s} \cdot 100, \quad e = \frac{I_{ms}}{I_s} \cdot 100. \quad (14)$$

The AOQL plans for inspection by variables and attributes are more economical than the corresponding Dodge-Romig plans when  $e < 100$ , similarly, if  $c_m$  is statistically estimated and holds  $E \cdot c_m < 100$ , then the AOQL plans for inspection by variables are more economical than the corresponding Dodge-Romig AOQL plans.

*Example.* Let  $N=1000, p_L=0.0025, \bar{p}=0.001$  and  $c_m=1.8$  (the cost of inspection of one item by variables is higher by 80% than the cost of inspection of one item by attributes). We shall look for the AOQL plan for inspection by variables and attributes. Furthermore we shall compare this plan and the corresponding Dodge-Romig AOQL plan for inspection by attributes.

*Solution.* Numerical solution is in paper [3]. Using this solution we obtain the AOQL plan for inspection by variables and attributes  $n=49, k=2.57617$ .

The corresponding AOQL plan for inspection by attributes we find in [1]. For given parameters  $N, p_L$  and  $\bar{p}$  we have  $n_2=130, c=0$ . For the comparison these two plans from an economical point of view we use parameter  $e$  (see (14)).

Since  $e = 52.2008\%$  (for calculation see [3]), using the AOQL plan for inspection by variables and attributes (49, 2.57617) it can be expected approximately **48% saving of the inspection cost** in comparison with the corresponding Dodge-Romig plan (130, 0).

Further we compare the operating characteristics of these plans. For example we have  $L_1(\bar{p}) = L_1(0.001) = 0.959306$ , i.e. the producer's risk for the AOQL plan for inspection by variables and attributes is  $\alpha = 1 - L_1(\bar{p}) = 0.04$ . The producer's risk for the corresponding Dodge-Romig plan is  $\alpha = 1 - L_2(\bar{p}) =$

$1 - 0.87 = 0.13$  (for calculation see [3]). Graphic comparison of the operating characteristics of these plans is in Figure 1.

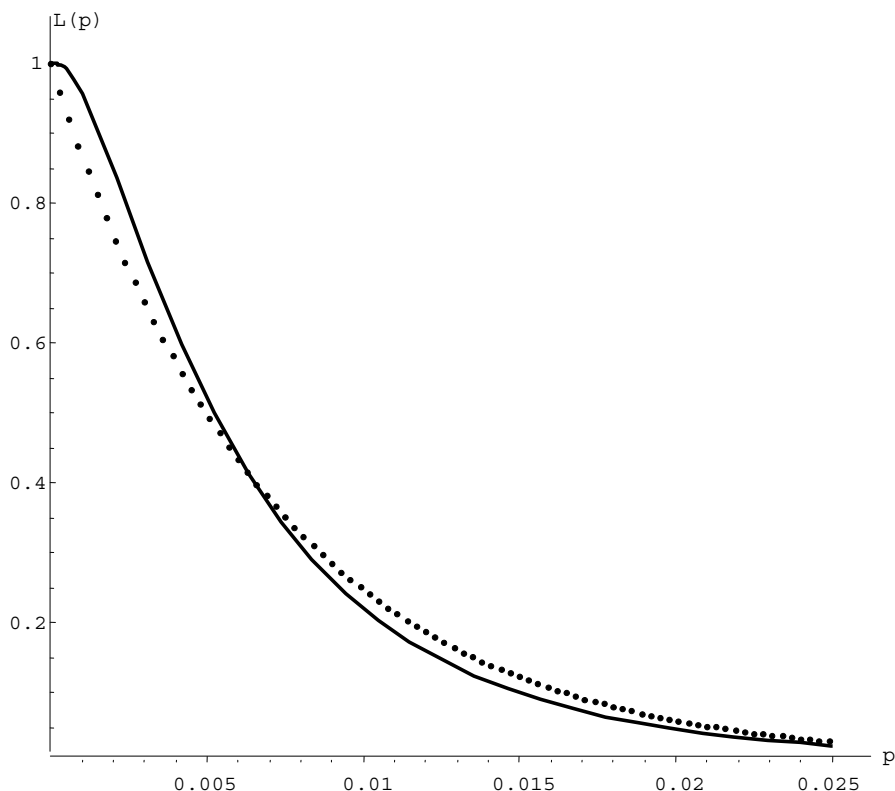


Fig. 1: OC curves for the AOQL sampling plans for inspection by variables and attributes (49, 2.57617) for inspection by attributes (130, 0).

### 3. Conclusions

From results of previous example it follows that the AOQL plan for inspection by variables and attributes is more economical than the corresponding Dodge-Romig AOQL attribute sampling plan (48% saving of the inspection cost). Furthermore the OC curve for the AOQL plan by variables and attributes is better than corresponding OC curve for the AOQL plan by attributes - see Figure 1 (for example the producer's risk for the AOQL plan by variables and attributes  $\alpha = 0.04$  is less than for the corresponding Dodge-Romig plan  $\alpha = 0.13$ ).

These conclusions hold generally. From the results of numerical investigations it follows (see [4]) that under the same protection of consumer the AOQL plans for inspection by variables are in many situations **more economical** than the corresponding Dodge-Romig attribute sampling plans (saving of the inspection cost is 70% in any cases).

### 4. References

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