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Economic Structural Change and Growth with Two Capital Goods

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Abstract. This paper proposes a three-sector growth model with heterogeneous capital. It structurally generalizes the Uzawa growth model by introducing heterogeneous capital into the growth model with homogeneous capital. We show the dynamic properties of the model and simulate the motion of the national economy over time. We also examine effects of changes in preferences and technologies on the economy. The model with heterogeneous capital reveals different properties from those of the model with homogeneous capital. These include the saddle-path stability of the steady state in contrast to the homogeneous capital case. The comparative statics analysis also provides the results which cannot be obtained from the two-sector model with homogeneous capital. For instance, we demonstrate that a rise in the propensity to save will result in the following changes in the steady state: the national output, the wealth, total stocks of the two capital goods, the output levels of the three sectors are all increased; the labor force shifts from the consumption good sector to the capital sectors; the price of consumption good is increased but the price of heavy industrial good is reduced.

Keywords: Heterogeneous Capital, Economic Growth, Economic Structur

1. Introduction

Over the past three decades, the study of the economic growth with economic structure and heterogeneous capital has caused a lot of attention in economic growth theory. Yet, we argue that economics still needs an analytical framework for examining structural change with accumulation of heterogeneous capital. How to model economic growth with heterogeneous physical capital continues to present a challenge for modeling of economic dynamics. It is well known that most of the models in the neoclassical growth theory model are extensions and generalizations of the pioneering works of Solow in 1956. Important extensions to the case of two-sector economies was initiated by Uzawa [1, 2]. Since Uzawa published his seminal work, the Uzawa model resulted in an explosion of research in the 1960s on the two-sector growth model. Economists have made many efforts in generalizing and extending the Uzawa two-sector model by, for instance, introducing more general production functions, money, externalities, knowledge, human capital, and fictions in different markets [3, 4, 5, 6, 7]. Most of these extensions are developed within the framework of one capital and one consumption goods. However, as observed by Farmer and Wendner [8: 773], “In multi-sector growth theory, the two-sector growth model with homogeneous capital dominates. However, economies are usually characterized by heterogeneous capital.” The purpose of this paper is to develop a dynamic three-sector model with structural change à la Uzawa in which structural change is the outcome of rational behavior of consumers and firms. The reminder of the paper is organized as follows. Section 2 defines the growth model with an alternative approach to consumer behavior with saving and consumption. Section 3 examines dynamic properties of the model when the production functions are specified with the Cobb-Douglas form and simulates the model. Section 4 studies effects of changes in some parameters on the economic structure. Section 5 concludes the study.

2. The Model

The economy consists of three distinct sectors with a fixed population. The three production sectors are called heavy industrial, light industrial, and consumption good (and service) sectors. The industrial sectors produce capital goods, which can be used only as production inputs. The light industrial commodity is selected to serve as numeraire. Labor and capital markets are perfectly competitive and labor force and

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capital are fully employed. Let subscripts $h, i, s,$ stand for heavy industrial sector, light industrial sector, and consumption good sector. Let N stand for the fixed labor force and $w(t)$ the wage rate.

First, we describe production side of the economy. We consider that each sector employs two capital and labor as inputs. Let $F_j(t)$, $K_j(t)$, $N_j(t)$, and $k_j(t)$, stand for, respectively, the output of, the heavy industrial capital good, labor force, and light industrial capital good employed by sector j $(j = h, i, s)$. We specify the production functions as

$$F_j(t) = A_j K_j^{\alpha_j}(t) N_j^{\beta_j}(t) k_j^{\gamma_j}(t), \ \ \alpha_j + \beta_j + \gamma_j = 1, \ \ \alpha_j, \beta_j, \gamma_j > 0, \ \ j = i, h, s, \tag{1}$$

where A_j, α_j, β_j, and γ_j to be constant. We use $p_h(t)$ and $p_s(t)$ to represent the prices of heavy industrial good and consumption good. The marginal conditions are

$$r_h + p_h \delta_k = \frac{\alpha_j p_j F_j}{K_j}, \ \ w = \frac{\beta_j p_j F_j}{N_j}, \ \ r_i + \delta_i = \frac{\gamma_j p_j F_j}{k_j}, \tag{2}$$

where $r_j(t)$ is the interest rate of capital good j $(j = h, i)$ and δ_k is the depreciation rate of capital.

We now describe behavior of households. The approach to household behavior in this study is discussed at length by Zhang (2005). Let $K(t)$ and $k(t)$ stand for the total stocks of heavy and light industrial goods at time. Let us denote $Y(t)$ the current net income of the population. The net income consists of wage incomes and interest payment, i.e., $Y(t) = r_h(t)K(t) + r_i(t)k(t) + w(t)N$. The disposable income at any point of time is then equal to $\hat{Y}(t) = Y(t) + p_h(t)K(t) + k(t)$. The disposable income is used for saving and consumption. We assume that the utility level, $U(t)$, of a typical household is dependent on consumption good, $C(t)$, and savings, $S(t)$. The utility function is specified as follows: $U(t) = C^{\xi_0}(t) S^{\lambda_0}(t), \ \xi_0, \lambda_0 > 0,$ in which the parameters, ξ_0, and λ_0 are respectively called the propensities to consume services and to hold wealth. The budget constrain is given by: $p_s(t)C(t) + S(t) = \hat{Y}(t)$. Households determine two variables, $C(t)$ and $S(t)$ at each moment. Maximizing $U(t)$ subject to the budget constrain yields

$$p_s(t)C(t) = \xi \hat{Y}(t), \ \ S(t) = \lambda \hat{Y}(t), \tag{3}$$

where $\xi \equiv \xi_0 / (\xi_0 + \lambda_0)$ and $\lambda \equiv \lambda_0 / (\xi_0 + \lambda_0)$. Let $a(t)$ stand for the total wealth of the households. We have $a(t) \equiv p_h(t)K(t) + k(t)$. According to the definitions of $s_j(t)$, the wealth accumulation is given by

$$\dot{a}(t) = S(t) - a(t). \tag{4}$$

The equation simply states that the change in the wealth is equal to savings minus dissavings.

Consider now an investor with one unity of money. He can either invest in heavy industrial capital good thereby earning a profit equal to the net own-rate of return $r_h(t)/p_h(t)$ or invest in light industrial capital good thereby earning a profit equal to the net own-rate of return $r_i(t)$. As we assume capital markets to be at competitive equilibrium at any point of time, two options must yield equal returns, i.e.

$$\frac{r_h(t)}{p_h(t)} = r_i(t). \tag{5}$$

Assume that the labor force is always fully employed. We have

$$N_h(t) + N_i(t) + N_s(t) = N, \tag{6}$$

Assume that the two capital goods are always fully employed. We have

$$K_h(t) + K_i(t) + K_s(t) = K(t), \ \ \ k_h(t) + k_i(t) + k_s(t) = k(t). \tag{7}$$

The balance of demand of and supply for the consumption good is represented by

$$C(t) = F_s(t). \tag{8}$$

The change in capital stock is equal to its output minus its depreciation. We have

$$\dot{K}(t) = F_h(t) - \delta_h K(t), \ \ \ \ \dot{k}(t) = F_i(t) - \delta_i k(t). \tag{9}$$

We have thus built the model.

3. Properties of the Dynamic Economic System

This section examines properties of the dynamic system. First, we show that the motion of the economy can be expressed as $3-$ dimensional differential equations. Before stating our analytical results, we introduce two variables $\bar{\Lambda} \equiv k / k_i$ and $\tilde{\Lambda} \equiv K / K_i$. Introduce

$$\theta_h(\bar{\Lambda}, \tilde{\Lambda}) \equiv \frac{\bar{\Lambda}}{\bar{\alpha}_s} - \frac{\tilde{\Lambda}}{\bar{\gamma}_s} + \frac{1}{\bar{\gamma}_s} - \frac{1}{\bar{\alpha}_s}, \ \theta_s(\bar{\Lambda}, \tilde{\Lambda}) \equiv \frac{\tilde{\Lambda}}{\bar{\gamma}_h} - \frac{1}{\bar{\gamma}_h} - \frac{\bar{\Lambda}}{\bar{\alpha}_h} + \frac{1}{\bar{\alpha}_h},$$

$$\Omega(\bar{\Lambda}, \tilde{\Lambda}) \equiv \delta\left(\bar{\Lambda} + \frac{\alpha_i \tilde{\Lambda}}{\gamma_i}\right)\left(\Lambda - \gamma_i \bar{\Lambda} - \alpha_i \tilde{\Lambda} - \frac{\beta_i N}{N_i}\right)^{-1}, \ \Psi(k, \bar{\Lambda}, \tilde{\Lambda}) \equiv \frac{\tilde{\Lambda}}{N_i^{\beta_i/\alpha_i}}\left(\frac{\Omega}{A_i}\right)^{1/\alpha_i}\left(\frac{k}{\bar{\Lambda}}\right)^{(\alpha_i+\beta_i)/\alpha_i},$$

$$\bar{\alpha}_h \equiv \frac{\beta_h \alpha_i}{\alpha_h \beta_i}, \ \bar{\alpha}_s \equiv \frac{\beta_s \alpha_i}{\alpha_s \beta_i}, \ \bar{\gamma}_h \equiv \frac{\beta_h \gamma_i}{\gamma_h \beta_i}, \ \bar{\gamma}_s \equiv \frac{\beta_s \gamma_i}{\gamma_s \beta_i}, \ \theta \equiv \frac{1}{\bar{\alpha}_s \bar{\gamma}_h} - \frac{1}{\bar{\alpha}_h \bar{\gamma}_s}, \ \delta \equiv 1 - \delta_k.$$

Lemma 1

The dynamics of the economy is described by the following differential equations

$$\dot{k} = \bar{\Psi}_3(k, \bar{\Lambda}, \tilde{\Lambda}) \equiv F_i - \delta_h k, \ \dot{\tilde{\Lambda}} = \bar{\Psi}_1(k, \bar{\Lambda}, \tilde{\Lambda}) \equiv \frac{\Psi_{01}\Psi_{\bar{\Lambda}} + \Psi_{02}\tilde{\Lambda}}{\Psi_{\bar{\Lambda}}\bar{\Lambda} + \tilde{\Lambda}\Psi_{\tilde{\Lambda}}},$$

$$\dot{\bar{\Lambda}} = \bar{\Psi}_2(k, \bar{\Lambda}, \tilde{\Lambda}) \equiv \frac{\bar{\Lambda}\Psi_{02} + \Psi_{\tilde{\Lambda}}\Psi_{01}}{\Psi_{\bar{\Lambda}}\bar{\Lambda} + \tilde{\Lambda}\Psi_{\tilde{\Lambda}}}, \tag{10}$$

where Ψ_{0j} are determined by solving

$$\Psi_{\tilde{\Lambda}}\dot{\tilde{\Lambda}} + \Psi_{\bar{\Lambda}}\dot{\bar{\Lambda}} = \Psi_2 - \bar{\Psi}_3 \Psi_k, \ \frac{\alpha_i k}{\gamma_i \bar{\Lambda}}\dot{\tilde{\Lambda}} - \frac{\alpha_i k \tilde{\Lambda}}{\gamma_i \bar{\Lambda}^2}\dot{\bar{\Lambda}} = \Psi_1 - \frac{a\bar{\Psi}_3}{k},$$

in which $\Psi_1 \equiv S - a$, $\Psi_2(k, \bar{\Lambda}, \tilde{\Lambda}) \equiv F_h - \delta_h K$, $a = k(\alpha_i \tilde{\Lambda} / \gamma_i \bar{\Lambda} + 1)$. At any point of time the other variables in the dynamic system can be expressed as unique functions of $k(t)$, $\bar{\Lambda}(t)$ and $\tilde{\Lambda}(t)$ as follows: $k_i = k / \bar{\Lambda} \Rightarrow n_h = \theta_h / \theta \Rightarrow n_s = \theta_s / \theta \Rightarrow N_i = N / (1 + n_h + n_s) \Rightarrow N_h = n_h N_i \Rightarrow N_s = n_s N_i \Rightarrow \Lambda\ (\equiv \hat{Y} / F_i) = \beta_i n_s / \xi \beta_s \Rightarrow F_i = k_i \Omega \Rightarrow K = \Psi \Rightarrow \hat{Y} = \Lambda F_i \Rightarrow K_i = K / \tilde{\Lambda} \Rightarrow K_h = n_h K_i / \bar{\alpha}_h \Rightarrow K_s = n_s K_i / \bar{\alpha}_s \Rightarrow k_h = n_h k_i / \bar{\gamma}_h \Rightarrow k_s = n_s k_i / \bar{\gamma}_s \Rightarrow F_h$ and F_s by (1) $\Rightarrow p_h = \alpha_i k \tilde{\Lambda} / \gamma_i \bar{\Lambda} K \Rightarrow r_i$, r_h and w by (2) $\Rightarrow p_s$ by (2) $\Rightarrow C$ and S by (3).

The proof of the lemma is available from the author. The lemma implies that once we solve the differential equations, then we can determine all the other variables, such as the national output, the labor distribution, capital distribution between among three sectors, the rate of interest, the wage rate. As the expressions are tedious, it is difficult to explicitly interpret economic implications of the conditions for existence of a steady state. In the reminder of this study, we show properties of the dynamic system by simulation. We specify the parameter values as follows

$$N = 5, \ A_h = 1, \ A_i = 0.8, \ A_s = 1.1, \ \alpha_h = 0.3, \ \beta_h = 0.5, \ \alpha_i = 0.25, \ \beta_i = 0.45,$$
$$\alpha_s = 0.3, \ \beta_s = 0.55, \ \lambda_0 = 0.8, \ \xi_0 = 0.2, \ \delta_k = 0.05. \tag{11}$$

The propensity to save out of the disposable income is 80 percent and the propensity to consume is 20 percent. The total productivity factor of the consumption good sector is higher than the total productivity factors of the other sectors. The depreciation rate is 5 percent. We calculate the equilibrium values of the variables as follows

$$F = 6.32, \ F_i = 0.36, \ F_h = 0.78, \ F_s = 6.49, \ p_h = 1.11, \ p_s = 0.78, \ r_i = 0.079,$$
$$w = 0.66, \ r_h = 0.087, \ k = 7.80, \ K = 12.73, \ a = 21.90, \ k_i = 0.85, \ k_h = 1.02, \ k_s = 5.94,$$
$$K_i = 0.64, \ K_h = 1.38, \ K_s = 10.72, \ N_i = 0.25, \ N_h = 0.50, \ N_s = 4.25,$$

in which $F \equiv F_i + p_h F_h + p_s F_s$ is the national output. The most of labor force is employed by the consumption good sector whose share of the national output is also much higher than the share of the other two sectors. Similar to the labor distribution, the share of the two capital goods in the national capital goods employed by the consumption good sector is higher than the share of the other two sectors.

The system has a unique equilibrium point for the given value of the parameters. The question now is whether this equilibrium point is stale. The three eigenvalues are calculated as follows:

$\{-352262, 0.192, -0.030\}$. One eigenvalue is positive. Hence, the dynamic system is unstable. It is important to note that in their two-sector dynamic model with heterogeneous capital, Farmer and Wendner [8] examine the nature of transition paths of the two-capital model and compare the properties of a Cobb-Douglas-Leontief economy with the same economy with homogeneous capital. They demonstrate different dynamic properties between the model with heterogeneous capital and homogeneous capital. The model with heterogeneous capital has the saddle-path stability. The difference is similar to the model of heterogeneous capital proposed here and the two-sector model of homogeneous capital proposed in Zhang [9: Chap. 6]. As the system is unstable, in general it does not necessarily move to the steady state. Due to the initial conditions, it may move far away from the steady state. Nevertheless, as we have found the differential equation controlling the motion of the system, it is straightforward to plot the motion of all the variables over time. Following the computing procedure given in Lemma 1, we now simulate the model to illustrate motion of the system. The initial conditions are specified as: $\tilde{\Lambda}(0) = 21$, $\bar{\Lambda}(0) = 7$, $k(0) = 7$. The initial state is near the steady state. As sown in Figure 1, the system does not converge to its steady state.

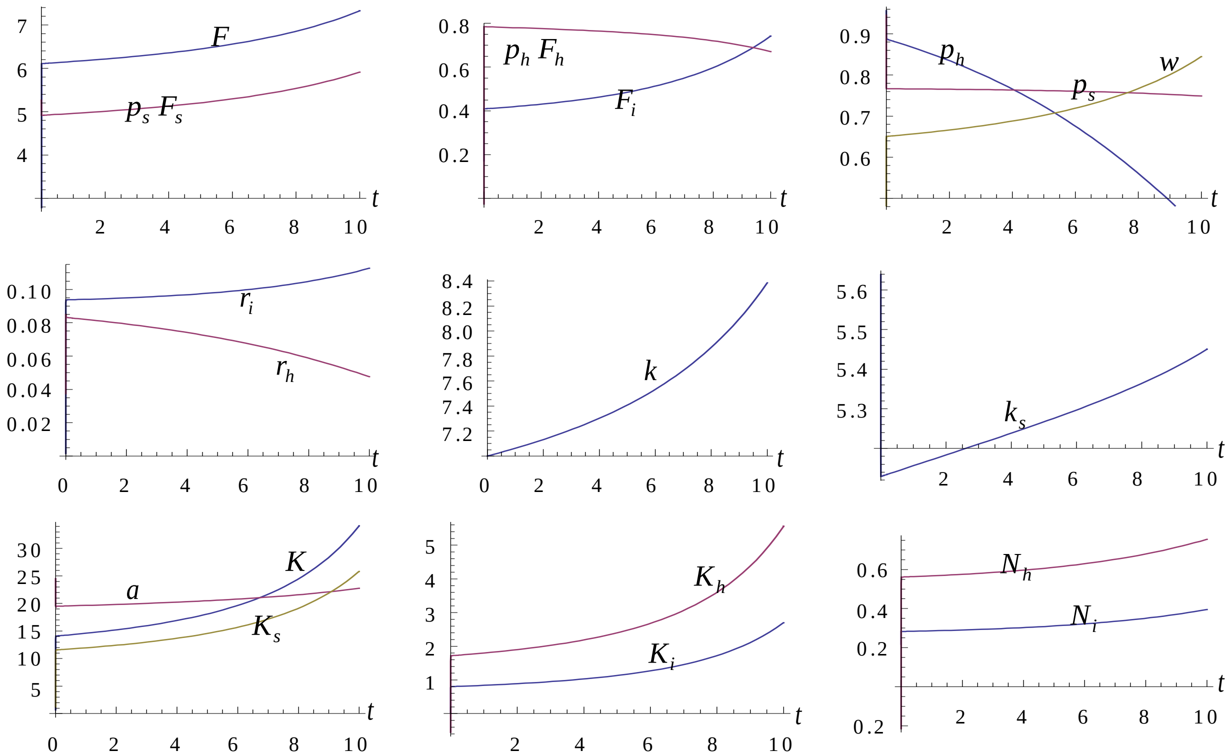

Fig. 1: The motion of the economic system

4. Comparative Statics Analysis

As we have proved that the system has a unique equilibrium point and provided a procedure to solve all the variables, it is straightforward to calculate effects of change in any parameter on the steady state. Yet, as the expressions of the analytical results are too tedious to get explicit conclusions, we still carry out comparative statics analysis by simulation. First, let a variable, $\bar{\Delta} x$, stands for the change rate of the variable x in percentage due to a change in the value of a parameter value. First, we increase the total productivity factor of the light industrial sector in the following way: $A_i : 0.8 \Rightarrow 0.85$. The effects on the equilibrium values of the variables are listed in (12). As the productivity is increased, the output levels of the three sectors are increased. As the technological change occurs in the light industrial sector, it is reasonable to expect that the sector's output is increased more than the other two sectors'. As the light industrial good is chosen as the numeraire, its price is relatively reduced as the prices of the other two products are increased. The labor distribution is not affected by the change in the productivity and the capital levels employed by the three sectors change in the equal rate.

$$\bar{\Delta} F = 10.13, \ \bar{\Delta} F_i = 10.13, \ \bar{\Delta} F_h = 2.79, \ \bar{\Delta} F_s = 2.30, \ \bar{\Delta} p_h = 7.13, \ \bar{\Delta} p_s = 7.65,$$

$$\bar{\Delta} r_i = 0,\ \bar{\Delta} r_h = 7.13,\ \bar{\Delta} k = 10.13,\ \bar{\Delta} K = 2.79,\ \bar{\Delta} w = \bar{\Delta} a = \bar{\Delta} k_i = \bar{\Delta} k_h = \bar{\Delta} k_s = 10.13,$$
$$\bar{\Delta} K_i = \bar{\Delta} K_h = \bar{\Delta} K_s = 2.79,\ \bar{\Delta} N_i = \bar{\Delta} N_h = \bar{\Delta} N_s = 0. \tag{12}$$

We now increase the propensity to save in the following way: $\lambda_0 : 0.8 \Rightarrow 0.82.$ The results are listed in (13). In comparison to the one-sector growth model, as our model has a refined economic structure, we can examine possible differences in effects on different sectors. As in the one-sector growth model, a rise in the propensity to save increases the national output. The wealth, total stocks of the two capital goods, the output levels of the three sectors are all increased. As the propensity to save is increased, the labor force shifts from the consumption good sector to the capital sectors and the price of consumption good is increased. The price of heavy industrial good is reduced. This partly results from the specified values, $\alpha_h\ (= 0.3)\ > \alpha_i\ (= 0.25)$. Although the capital stocks employed by the three sectors are increased, the increase rate of the consumption good sector is the lowest. This partly explains the rise of the good's price.

$$\bar{\Delta} F = 2.49,\ \bar{\Delta} F_i = 4.72,\ \bar{\Delta} F_h = 4.28,\ \bar{\Delta} F_s = 1.60,\ \bar{\Delta} p_h = -0.03,\ \bar{\Delta} p_s = 0.43,\ \bar{\Delta} r_i = -3.14,$$
$$\bar{\Delta} w = 2.41,\ \bar{\Delta} r_h = -3.17,\ \bar{\Delta} k = 4.66,\ \bar{\Delta} K = 4.47,\ \bar{\Delta} a = 4.52,\ \bar{\Delta} k_i = 6.77,\ \bar{\Delta} k_h = 6.55,$$
$$\bar{\Delta} k_s = 4.03,\ \bar{\Delta} K_i = 6.80,\ \bar{\Delta} K_h = 6.56,\ \bar{\Delta} K_s = 4.06,\ \bar{\Delta} N_i = 2.26,\ \bar{\Delta} N_h = 2.05,\ \bar{\Delta} N_s = -0.37. \tag{13}$$

5. Concluding Remarks

This paper proposed a two-capital-goods model with multiple capital goods with an alternative approach to household behavior. The neoclassical economic growth has a unique unstable equilibrium point when the parameters specified with certain values. We also analyzed changes in the parameters upon the system. We have limited our study to a simplified spatial structure of the economic system. There are numerous extensions of the Solow-Uzawa models. We may introduce more realistic representations of household behavior with endogenous time and multiple kinds of consumption goods. We now point out a few straightforward extensions. For instance, we may consider the economy as a small country, which implies that economy has negligible impact on the interest rate in globally open market. This assumption has been accepted in the literature of international economics. Another extension is to assume that the economy is a small and open city in the national economy. In this case, both the utility level and the rate of interest should be exogenously fixed at the national levels.

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