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Application of the Bayesian Decision-making

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Abstract—There are large numbers of factors of uncertainty in the presence of market environment, so that the decision-mak- ing of decision-makers have a degree of risk. It is a workable solution that increases the amount of information to increase the reliability of the decision-making. In this article, through analysis of information in general risk-based decision-making role, using the means of Bayesian decision-making, priori analysis and posteriori analysis, information are given to determine the value and improve the reliability of the risk of decision-making effective way.

Keywords—Bayesian decision-making; Full value of information; Value added information

1. Introduction

Access to information is the decision-making. As the market environment, there are many uncertainties and decision-makers state their own constraints and inadequate statistical information. Decision-makers are often unable to grasp all the information relevant to decision making, making the results of subjective decision-making between the objective facts or target deviation. Therefore, under incomplete information to decision makers for decision-making will inevitably bring a degree of risk. Effective control of information is a powerful tool for risk reduction, more adequate access to information, the uncertainty and the risk of decision-making environment are smaller. But no matter by what means the collection of information, we need the input, to spend the necessary human, material and financial resources and time, which is to pay for the cost of access to information, become an information cost. Mastered a large number of high-quality information, it can improve the reliability of decision-making, which reflects the value of the information.

Bayesian decision-making refers to the use of additional information, according to the Bayesian probability formula to estimate the posterior probability, and on the basis of evaluation and selection of a decision making method. Using Bayesian decision-making, information and can be combined with supplementary information to analyze and judge, there by increasing the reliability of decision-making. Meanwhile, the use of the method, the value of information can also be collected and whether additional information to make new scientific judgments.

2. The Risk-based Decision-making Criteria

Risk-based decision-making is in the estimation of the probability distribution of the state space based on the decisionmaking, it is risk-based decision-making objectives, risk identification and risk measurement, based on the risk evaluation method for a variety of selection and combination of scientific and to develop specific programs of risk decision-making process. The risk-based decision-making includes the probability of the use of objective probability and subjective probability. Objective probability is the probability of a general sense, can be derived from the frequency estimation, usually by natural history information projections or in accordance with the results of randomized trials calculated. Subjective probability is based on their knowledge, experience, and an event made the likelihood of subjective judgments.

Each risk-based decision-making problems consists of three elements: Natural state $\Theta = (\theta_1, \theta_2, \cdots, \theta_n)$ (Natural state of the state space form)and the probability distribution of the state $p = (p_1, p_2, \cdots, p_n)(p_j \ge 0, \sum_{j=1}^{n} p_j)$ space (p_j is the probability θ_j); Decision-makers to take action $A = (a_1, a_2, \cdots, a_m)$ (Constitute the action space); Policymakers hope to achieve (With the payoff matrix

$Q=(q_{ij})_{m\times n}$ (q_{ij} denotes under the condition that θ_j, action program adopted a_i revenue value obtained) or reve- nue function described). For example, to develop a new product in the market demand can not accurately predict the circumstances, be sure to manufacture or production, production and other issues is how much a risk-based decision making. The state space is the market sales, such as is in demand, sales in general, sales poor, these conditions are not decision-makers control the decision-makers to make a decision, the result is uncertain with a certain degree of risk. Therefore, the risk-based decision making and accurately estimate the full value of information, scientific and rational collection to increase investment to get the ever-changing market information, to grasp the occurrence of various natural conditions can make the choice of decision-making more scientific and reliable solution, thus increasing economic efficiency.

The probability distribution of the state space, decision making can be calculated and compared to detailed analysis, on this basis, according to a the criteria set to select and judge. Risk-based decision-making expectations criteria: the criterion is risk-based decision-making in the most widely pan, a criteria. It is based on the expected benefits of the program based on the size, to select the appropriate option. Specific decisions can be calculated as earnings expectations:

$$E(Q(a_i))=\sum_{j=1}^{n} q_{ij}\,p_j\,(i=1,2,\cdots,m),$$

where $E(Q(a_i))$ program of income is the expected value of i, q_{ij} is j state in the event of program benefits value of j, p_j state is the probability j. Earnings expectations higher, indicating that, on average, the program gains greater. So the programs can be expected in the larger scheme proceeds as the best solution.

3. Application of Bayesian Decision

Let the prior probability of θ_j certain state (Prior pro-bability of the various states are given)is $p(\theta_j)$, additional information obtained through the investigation for the e_k, if θ_j is given, e_k conditional probability for the $p(e_k/\theta_j)$.

Information is given under the conditions of e_k, θ_j of the conditional probability that the posterior probability (the probability of the Supplementary information amended) $p(\theta_j/e_k)$, can be used Bayesian formula:

$$p(\theta_j/e_k)=\frac{p(\theta_j)\cdot p(e_k/\theta_j)}{\sum_{j=1}^{n} p(\theta_j)\cdot p(e_k/\theta_j)}$$

Decision analysis is divided into a priori and a posteriori analysis. Priori analysis is making use of prior probability, and then test analysis. Posterior probability is used as the selection and determines the appropriate basis for the program. In general, two types of conclusions are inconsistent. Posteriori analysis not only the use of prior information, but also use additional information, therefore, in general if additional information is accurate, then the posterior analysis of the conclusions more reliable.

Case Study: Water Company to be a construction period of the river closure decisions. There are two options: Option 1 is the construction in August; Option 2 is the construction in October. Assuming other conditions are met, of closure the only factor is the weather and hydrological conditions .10 weather and hydrological conditions in January can guarantee a successful closure. And in August the weather and hydrological conditions, there are two possible, if the weather is good, there is no flooding upstream, 8 by the end of a successful closure can be makes the whole project ahead of schedule, which can increase their profits than the Oct. 10 million Yuan construction; if the weather is bad, there upstream flooding, closure fails, the construction of an additional 700 more than in October Mllion loss. Based on past experience, good weather in August the possibility is 0.7, the possibility of bad weather is 0.3. To help decision-making, the company intends to purchase weather forecast weather information. Past data shows that the weather forecast accuracy weather is 0.9, the forecast bad weather accuracy is 0.8. Please select the Water Company the right scientific decision-making programs.

Firstly, use a priori analysis:

Three programs in different states of the payoff matrix table: (Unit: million)

Weather conditions	Good weather θ_1	Bad weather θ_2
Profit program 1	1000	-700
Profit program2	0	0
Option 1 of the prior probability	0.7	0.3
Option 2of the prior probability	1	0

The expected values of the three programs are:

$$E(Q(a_1)) = 0.7\times 1000 + 0.3\times(-700) = 490 \text{(Million)}$$

$$E(Q(a_2)) = 1\times 0 + 0\times 0 = 0 \text{(Million)}$$

$$E(Q(a_1)) \geq E(Q(a_2))$$

In accordance with the expected value criterion a_1 is the best solution.

Value of perfect information and value added information for decision analysis using Bayesian formula of the two commonly used indicators. Full value of information refers to when making decisions about an issue, for all the possible state of nature can provide completely accurate information, by the perfect information before and after the action taken by the benefits of different values to represent the difference. Because different states have different income difference between the values, so we used the difference between the expected profit values to comprehensively reflect the full value of the information. The formula

$$EVPI = \sum_{j=1}^{n} p(\theta_j)[\max_i Q(a_i,\theta_j) - Q(a_*,\theta_j)]$$

$EVPI$ (Expected Value of Perfect Information)is the expected value of perfect information, $\max_i Q(a_i,\theta_j)$ show like the program maximum benefit under the value of θ_j, $Q(a_*,\theta_j)$ said the best solution to the prior analysis under the state revenue value θ_j.

$$EVPI = p(\theta_1)\times[\max_i Q(a_i,\theta_1) - Q(a_1,\theta_1)] + p(\theta_2)\times[\max_i Q(a_i,\theta_2) - Q(a_1,\theta_2) = 210 \text{(million)}$$

Expected value of perfect information at this time is relatively high; indicating that additional supplemental information by, it is possible to further improve the effectiveness of decision-making.

After further inspection the following analysis.

In real economic life, generally difficult to obtain complete information. Through various means, such as sampling, consulting and other supplemental information received does not exceed the total amount of information generally, and its accuracy is not as complete information. Therefore, their value is usually less than full value of information. Therefore, their value is often less than the value of perfect information. Supplementary information e_k value of VAI is calculated as:

$$VAI(e_k) = \text{priori } EVPI - \text{posterior } EVPI(e_k)$$

Priori $EVPI$ probability based on the state of computing the value of the expected value of perfect information, posterior $EVPI(e_k)$ is to understand the additional information e_k, the use of information according to the revised calculation of the posterior probability of the expected value of perfect information. $VAI(e_k)$ values are closely linked with the e_k, in order to comprehensively reflect the added value of information, but also need to calculate the expected value of additional information

$EVAI$ (Expected Value of Additional Information):

$$EVAI = E[VAI(e_k)] = \sum_{k=0}^{T} VAI(e_k)\cdot p(e_k)$$

$P(e_k)$ is the probability of e_k:

$$p(e_k) = \sum_{j=1}^{n} p(\theta_j)\cdot p(e_k/\theta_j), \sum_{k=0}^{T} p(e_k) = 1.$$

$EVAI$ collection of additional information to determine whether it is beneficial to the basic standards. generally gather additional information before, it should $EVAI$ the cost of collecting additional information to compare, collect additional information only when the cost is less than $EVAI$, on average, to collect additional information have any value.

In this case, set e_1 good weather forecast issued by the supplementary information for e_2, given the bad weather forecast for the b additional information $p(e_1/\theta_1)=0.9$,

$p(e_2/\theta_2)=0.8$. The following Bayesian formula to amend the probability of various states, to arrive at the decision-making programs more reliable:

$$p(\theta_1/e_1)=\frac{p(\theta_1 e_1)}{p(e_1)}=0.9131$$

$$p(\theta_2/e_1)=1-p(\theta_1/e_1)=0.0869$$

$$p(\theta_1/e_2)=\frac{p(\theta_1 e_2)}{p(e_2)}=0.2258$$

$$p(\theta_2/e_2)=1-p(\theta_1/e_2)=0.7742$$

Posterior probability calculated using $p(\theta_j/e_1)$ posteriori analysis can be carried out:

$$E(Q(a_1))=1000\times 0.9131+(-700)\times 0.0869=304.8 \text{ (Million)}$$

$$E(Q(a_2))=0\times 0.9131+0\times 0.0869=0 \text{ (Million)}$$

So $E(Q(a_1))\geq E(Q(a_2))$, in accordance with the expected value criterion a_1 is the best solution.

Posterior probability calculated using $p(\theta_j/e_2)$

analysis can be carried out:

$$E(Q(a_1))=1000\times 0.2258+(-700)\times 0.7742=-316.14 \text{ (Million)}$$

$$E(Q(a_2))=0\times 0.2258+0\times 0.7742=0 \text{ (Million)}$$

So $E(Q(a_2))\geq E(Q(a_1))$, in accordance with the expected value criterion a_2 is the best solution.

Good weather forecast issued e_1 the value of the posterior complete information:

$$EVPI(e_1)=0.9131\times[\max_i Q(a_i,\theta_1)-Q(a_1,\theta_1)+0.0869\times[\max_i Q(a_i,\theta_2)-Q(a_1,\theta_2)=60.83 \text{ (Million)}$$

Bad weather forecast issued e_2 the value of the posterior complete information:

$$EVPI(e_2)=0.2258\times[\max_i Q(a_i,\theta_1)-Q(a_2,\theta_1)+0.7742\times[\max_i Q(a_i,\theta_2)-Q(a_2,\theta_2)=225.8 \text{ (Million)}$$

The expected value of additional information $EVAI$:

$$VAI(e_1)=210-60.83=149.17 \text{ (Million)}$$

$$VAI(e_2)=-15.8 \text{ (Million)}$$

$$EVAI=\sum_{k=1}^{2}VAI(e_k)p(e_k)=98.0293 \text{ (Million)}$$

From the above analysis of the appropriate conclusions can be drawn: In order to improve the effectiveness of decision-making, the Water Corporation should purchase weather forecasting to more accurately grasp the status of Meteorology and Hydrology. By $EVAI=98.0293$ Mllion Yuan, so the asking price as long as the information below 98.0293 Million purchase of information is beneficial on average. If the weather is good weather forecast, the construction should be in August, if the weather forecast bad weather, construction should be in October.

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