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Research on the problem of the Multi-echelon Inventory Model of the Repairable Spare Parts with Two-indenture

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Abstract—According to the practical backgrounds of the repairable spare parts in the weapon equipment of the PLA, the multi-echelon inventory model with two-indenture was constructed in this paper. In the mathematical model, the minimum value of expected backorders under the constraint of support budget was used as optimizing objective, and the model was analyzed by the simulation of practical instance.

Keywords-Multi-echelon inventory; repairable spare parts; two-indenture

1. Intrduction

Repairable spare parts is an important material basis for weapon equipment maintenance, has and they have a direct impact on keeping the good condition of weapon equipment. With the increasing complexity of the weapon equipment, the quantities and types of the repairable spare parts notably increased, and simultaneously, the support costs of the repairable spare parts remarkably increased. Hence, it is an important problem to ascertain the optimum inventory level of repairable spare parts under budget constraint [1]. In the foreign countries, many researches on the inventory model of the repairable spare parts were completed, and some models of the multi-echelon inventory have been used in equipment maintenance support. However, in our country, literatures in the field are very rare [2-8].

According to current practical background of weapon equipment support in the PLA, the multi-echelon inventory model of the repairable spare parts with two-indenture was constructed in the paper. By considering demands of repairable spare parts follow negative binomial, analyzing the physical flow of repairable spare parts, and utilizing the minimum value of the expected backorders as optimizing objective, we obtain the optimal inventory level of each part under the constrain of support budget. At the end of the paper, the model is analyzed by the simulation of practical instance.

2. The Multi-echelon Inventory Model of The Repairable Spare Parts with Two-indenture

In the model, two-echelon refers to the base and depot, and two-indenture consists of LRU (line-replaceable unit) and SRU (shop-replaceable unit).

2.1The physical flow of the repairable spare parts

When a unit on the weapon equipment gets failed, it will be checked and taken apart from the weapon equipment at the base. If a LRU gets failed and it can be repaired at the base, the LRU will be repaired. Otherwise, it will be sent to the depot for repair, at the same time a resupply demand of the LRU from the depot is placed. If one of the SRU that composed the LRU gets failed, and it can be repaired at the base, it will be repaired. Otherwise, the failed SRU will be sent to the depot for repair, and a resupply demand of the SRU is simultaneously placed. The depot repairs the failed parts received from the base, and the repaired units will be sent back to base, or put on the shelf. The order and transportation time is defined to be the time from sending a request at base until the unit is received from the depot. If the depot (or a base) hasn’t available inventory on hand when received a supply request, a backorder occurred [2].

2.2Basic Assumption

1) Each echelon follows (s,s-1) inventory policy. When a unit on the weapon equipment gets failed, it will be repaired on a one-for-one basis. i.e., the units are not batched for repair.
2) There are the test equipment, personnel, and other resources at bases and depot. Only a fraction of repairs can be performed at the base, while all failures can be repaired at the depot.
3) The base is resupplied from the depot, and lateral supply is not allowed.
4) The ordering cost of the repairable spare parts is considered, but the cost of inventory holding, transportation, and other management is ignored.
5) The ordering and transportation time for each base is ordinary.
6) Each LRU failure dues to only one SRU.
7) The demand rate at each base follows negative binomial distribution.

The inventory balance equation of the multi-echelon system is as follow:

$$s = OH + DI - BO$$

where constant, s represents the inventory position. OH is the number of the repairable spare parts on hand at base, DI is the number of the repairable spare parts which is repaired and transshipped to the base from depot, BO is the number of backorders, which are variable and positive. Due to the order number is 1, the reorder point is s-1 [9-10].

2.3Computation of expected backorders and variance

The mean μ , variance var , and variance-to-mean ratio V of the negative binomial are given as follows:

$$\mu = aq/p, \qquad var = aq/(1-q)^2, \qquad V = 1/p$$

According to the multi-echelon inventory theorem, the mean and variance of the expected backorders can be computed as follows:

$$EBO[S] = \sum_{X=S+1}^{\infty}(X-S)P(x) = \sum_{X=S+1}^{\infty} XP(x) - S\sum_{X=S+1}^{\infty} P(x)$$
$$= \sum_{X=0}^{\infty} XP(x) - S\sum_{X=0}^{\infty} P(x) - \sum_{X=0}^{S} XP(x) + S\sum_{X=0}^{S} P(x)$$

$$\begin{aligned}
VBO[S] &= E[(X-S)^2] - [E(X-S)]^2 \\
&= \sum_{X=S+1}^{\infty}(X-S)^2 P(x) - [E(X-S)]^2 \\
&= \sum_{X=S+1}^{\infty} X^2 P(x) + S^2 \sum_{X=S+1}^{\infty} P(x) - 2S\sum_{X=S+1}^{\infty} P(x) - [E(X-S)]^2 \\
&= \sum_{X=1}^{\infty} X^2 P(x) - \sum_{X=1}^{S} X^2 P(x) + S^2 \sum_{X=1}^{\infty} P(x) - S^2 \sum_{X=1}^{S} P(x) - \\
&\quad 2S\sum_{X=1}^{\infty} XP(x) + 2S\sum_{X=1}^{S} XP(x) - [E(X-S)]^2 \\
&= (aq/(1-q))^2 + aq/(1-q)^2 - 2S\cdot aq/(1-q) + S^2 - \\
&\quad \sum_{X=1}^{S} X^2 P(x) - S^2\sum_{X=1}^{S} P(x) + 2S\sum_{X=1}^{S} XP(x) - [E(X-S)]^2
\end{aligned}$$

According to the multi-echelon inventory theorem, the number of unit in the pipeline of base j, x_j is related to the number of unit is repairing at the depot, x_0. If $x_0 \le s_0$, there are no backorders at the depot. So, the average number of unit in the pipeline to the base is just the average number in resupply.

$$E[X_j \mid X_0] = m_j O \qquad x_o \le s_0 \tag{1}$$

If $x_0 > s_0$, there are $(x_0 - s_0)$ backorders to the depot, and the expected backorders of unit in the pipeline to the base j is as follow:

$$E[X_j \mid X_0] = m_j O + m_j (x_0 - s_0)/m_0 \qquad x_0 > s_0 \tag{2}$$

$$Var[X_j \mid X_0] = \begin{cases} m_j O & x_0 \le s_0 \\ m_j O + (m_j/m_0)(1 - m_j/m_0)(x_0 - s_0) & x_0 > s_0 \end{cases} \tag{3}$$

From (1)、(2)、(3), we can obtain:

$$E[X_j] = m_j O + m_j EBO(s_0)/m_0 \tag{4}$$

$$Var[E[X_j \mid X_0]] = m_j{}^2 VBO(s_0)/m_0{}^2 \tag{5}$$

$$Var[X_j] = m_j O + (m_j/m_0)(1 - m_j/m_0) EBO(s_0) + m_j^j VBO(s_0)/m_0^2 \tag{6}$$

3. The Multi-echelon Inventory Model

In this section, we will formulate the spare parts support system involves two-echelon and two-indenture.

Variables and parameters setting:

O_j : The average order and transportation time for the base

m_{oj} : The mean demand number of LRU every year at base *j*

m_o : The mean demand number of LRU every year at depot

m_{kj} : The mean demand of SRU *k* every year at base *j*

m_k : The mean demand of SRU *k* every year at depot

r_{oj} : The probability that a failure LRU can be repaired at base *j*

r_{kj} : The probability that a failure SRU can be repaired at base *j*

T_{oj} : The mean repair time for LRU at base *j*

T_{kj} : The mean repair time for SRU at base *j*

T_o : The mean repair time for LRU at depot

T_k : The mean repair time for SRU at depot

q_{kj} : The probability that a LRU being repaired at base *j* due to the failure of SRU *k*

q_k : The probability that a LRU being repaired at depot due to the failure of SRU *k*

Apparently,

$$\sum_{k=1}^{K} q_{kj} = \sum_{k=1}^{K} q_k = 1$$

3.1 Derivation of the demand rates

The sequence of the derivation of demand rates is given in Figure 1.

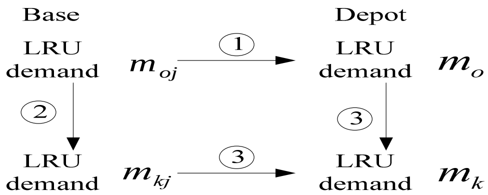

Figure 1. The sequence of the derivation of demand rates

1) The mean annual demand for LRU at the depot is the number of LRU ordered by all bases. That is the number of LRU that can not be repaired at bases.

$$m_o = \sum_{j=1}^{J} m_{oj}(1 - r_{oj}) \tag{7}$$

2) The mean annual demand for SRU *k* at base *j* is the mean annual demand for LRU at base *j* times the probability that a failure LRU is repaired there times the probability that a LRU being repaired at base *j* due to the failure of SRU *k*.

$$m_{kj} = m_{oj} r_{oj} q_{kj} \tag{8}$$

3) The mean annual demand for SRU *k* at depot is the sum of resupply demand at all bases plus the mean annual demand for SRU *k* due to the failure LRU is repairing at depot.

$$m_k = \sum_{j=1}^{J} m_{kj}(1 - r_{kj}) + m_o q_k \tag{9}$$

3.2 Derivation the mean number of pipeline

The sequence of the derivation of the mean number of the pipeline is shown by Figure 2.

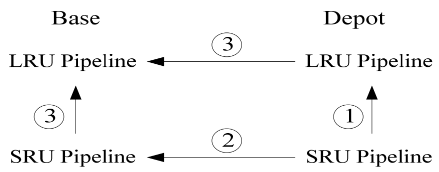

Figure 2. The sequence of the derivation of the mean number of the pipeline

1) The mean and variance for the number of LRU in the pipeline at depot.

The number of LRU being repaired at depot includes two parts: a) the number of LRU being repairing at depot, when there are no delays for SRU. b) the number of LRU delayed in repair, when there are not enough SRU on hand at depot.

Let f_k represent the fraction of depot demand for SRU *k* due to depot LRU repairs, and then

$$f_k = m_o q_k / m_k \qquad k > 0$$

$$E[X_o] = m_o T_o + \sum_{k=1}^{K} f_k EBO(s_k \mid m_k T_k) \tag{10}$$

$$Var[X_o] = m_o T_o + \sum_{k=1}^{K} f_k (1 - f_k) EBO(s_k \mid m_k T_k) + \sum_{k=1}^{K} f_k^2 VBO(s_k \mid m_k T_k) \tag{11}$$

2) The mean and variance for the number of SRU in the pipeline at bases.

The number of SRU *k* in the pipeline at base *j* can be divided into three groups: a) the number of SRU *k* repaired at base *j*. b) the mean number of SRU *k* being resupplied from depot to base *j*. c) the number of SRU *k* delayed because of SRU backorders at base *j*.

Let f_{kj} denote the fraction of all SRU *k* demand at depot that is being resupplied to base *j*.

$$f_{kj} = m_{kj}(1 - r_{kj}) / m_{ki} \qquad k, j > 0 \qquad \sum_{j=1}^{J} f_{kj} = 1 \qquad k > 0$$

$$E[X_{kj}] = m_{kj}[r_{kj} T_{kj} + (1 - r_{kj}) O_j] + f_{kj} EBO(s_k \mid m_k T_k) \quad k, j > 0 \tag{12}$$

$$Var[X_{kj}] = m_{kj}[r_{kj} T_{kj} + (1 - r_{kj}) O_j] + f_{kj}(1 - f_{kj}) EBO(s_k \mid m_k T_k) + f_{kj}^2 EBO(s_k \mid m_k T_k) \quad k, j > 0 \tag{13}$$

3) The mean and variance for the number of LRU in the pipeline at bases.

The number of LRU in the pipeline at base includes four groups: a) the number of LRU in the repair at base. b) the mean number of LRU resupplied from the depot. c) the number of LRU delayed at base due to resuppied delay from the depot. d) the number of LRU in repair at base due to the SRU resuppied delay from the depot.

Let f_{oj} represent the fraction of LRU demand at depot that is being resupplied to base *j*.

$$f_{oj} = m_{oj}(1 - r_{oj}) / m_o \qquad j > 0$$

So we easily see that $\sum_{j=1}^{J} f_{oj} = 1$

$$E[X_{oj}] = m_{oj}[r_{oj} T_{oj} + (1 - r_{oj}) O_j] + f_{oj} EBO(s_o \mid E[X_o], Var[X_o]) + \sum_{k=1}^{K} EBO(s_{kj} \mid E[X_{kj}], Var[X_{kj}]) \quad j > 0 \tag{14}$$

$$
\begin{aligned}
Var[X_{oj}] = {} & m_{oj}[r_{oj}T_{oj} + (1-r_{oj})O_j] + \\
& f_{oj}(1-f_{oj})EBO(s_o \mid E[X_o], Var[X_o]) + \\
& f_{oj}^2 VBO(s_o \mid E[X_o], Var[X_o]) + \\
& \sum_{k=1}^{K} VBO(s_{kj} \mid E[X_{kj}], Var[X_{kj}]) \quad j > 0
\end{aligned}
\tag{15}
$$

4. Simulation and Analysis

Suppose there are 1depot, 3 bases, and all bases with the same parameters. Let C denotes the support budget, and c_i is the cost of LRU, SRU1, SRU2, and SRU3. The parameters are set as follows.

O_j =0.02, m_{oj} = [23.2, 23.2, 23.2],

r_{oj} = [0.2, 0.3, 0.5], r_{kj} = [0.1, 0.2, 0.3],

T_{oj} = [0.02, 0.06, 0.04], T_{kj} = [0.01, 0.03, 0.05],

T_o =0.01, T_k = [0.02, 0.03, 0.04],

q_{kj} = [0.1, 0.3, 0.6], q_k = [0.3, 0.5, 0.2],

C=15，c=[5,4,3,2].

The model of repair department is given in Figure 3, and the model of two-indenture is given in Figure 4.

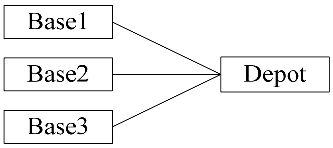

Figure 3. The logic structure diagram

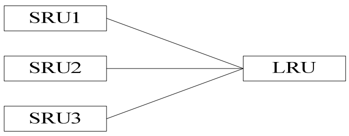

Figure 4. The diagram of two-indenture

The procedure of optimal inventory is offered in Figure 5.

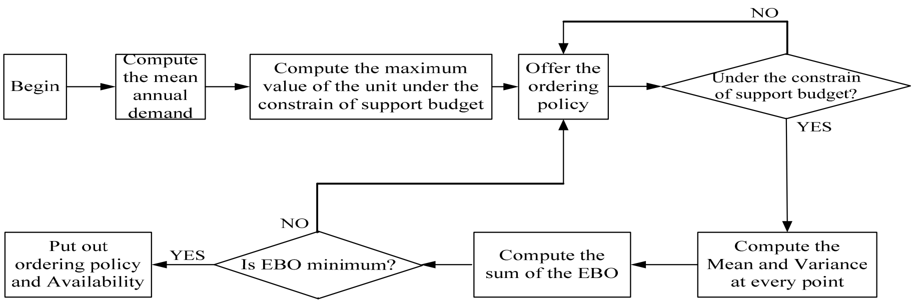

Figure 5. The procedure of the multi-echelon inventory model with two-indenture

According to parameters given above, we implemented the optimal algorithm in Matlab, and computing results are given in TABLE I.

TABLE I THE OPTIMAL INVENTORY POLICY WHEN SUPPORT BUDGET IS 15

	Depot	Base1	Base2	Base3	Sum
LRU	1	0	0	0	1
SRU1	1	0	0	0	1
SRU2	1	0	0	0	1
SRU3	1	0	0	0	1
EBO	0.6098	0.4155	1.3425	1.5615	3.9253

The optimal inventories of expected backorder under different budget are presented in TABLE II.

TABLE II THE OPTIMAL INVENTORY POLICY UNDER THE CONSTRAIN OF THE DIFFERENT SUPPORT BUDGET

Support Budget	Depot				Base1				Base2				Base3				EBO
	LRU	*SRU1*	*SRU2*	*SRU3*	*LRU*	*SRU1*	*SRU2*	*SRU3*	*LRU*	*SRU1*	*SRU2*	*SRU3*	*LRU*	*SRU1*	*SRU2*	*SRU3*	
9	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	4.8887
10	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	4.8887
12	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	4.4615
15	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	3.9253
18	1	1	1	1	0	0	0	0	0	0	0	1	0	0	0	1	3.2290
20	1	1	1	2	0	0	0	0	0	0	0	1	0	0	0	1	3.0274
23	1	1	1	2	0	0	0	0	1	0	0	0	0	0	0	1	2.6325
25	1	1	2	2	0	0	0	1	0	0	0	1	0	0	0	1	2.6031

From the computation of the instance, we can find that the expected backorder is decreasing with the budget increasing, and this trend becomes slow with the budget increasing. The relation curve between the support budget and expected backorders is shown in Figure 6.

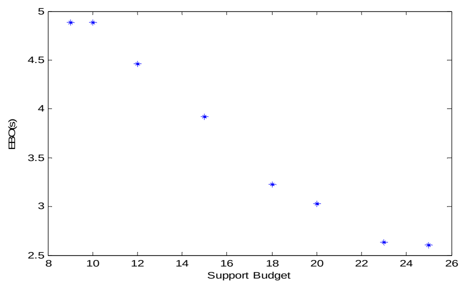

Figure 6. The curve for the support budget versus expected backorders

5. Conclusion

According to the practical background of the repairable spare parts with multi-echelon support in the weapon equipment of the PLA, the minimum value of the expected backorder is used as the optimal objective under constrain of support budget, and the multi-echelon inventory model of the repairable spare parts with tow-indenture is constructed. The model is analyzed by the simulation of an instance, and the computing results show the validity of the model.

6. References

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