

# Mathematical Models for Analyzing the Market Demand of Luxury Goods in China

Yixun Shi<sup>1</sup>

Department of Mathematics, Computer Science and Statistics  
Bloomsburg University of Pennsylvania  
Bloomsburg, PA 17815, USA

**Abstract.** Since China opened up its economy about 30 years ago, the nation has been growing amazingly rapidly in terms of gross domestic product (GDP) as well as in terms of consumption market demand. While almost all sections of China's domestic demand are growing, the growth of the demand for luxury goods is much faster than all others. In this paper, we introduce a few mathematical models for studying the market demand for luxury goods in China. We apply a Markov chain model with changing transition matrices to analyze consumer behaviors in China's luxury goods market, we suggest a linear regression model to explore the interactions among the luxury goods demand and various other factors in economics and social values, and we use a dynamical system model to estimate the long term trend of the market demand for luxury goods in China. The paper is concluded with a number of further comments.

**Keywords:** Luxury Goods, Market Demand in China, Markov Chain, Linear Regression, Dynamical System

## 1. Introduction

Since China opened up its economy about 30 years ago, the nation has been growing amazingly rapidly in terms of gross domestic product (GDP) as well as in terms of consumption market demand. Today, China's economy is the second largest in the world only after that of the United States. During those 30-some years China's economy has gradually changed from a centrally planned system to a more market-oriented one that has a rapidly growing private sector. A major component supporting China's rapid economic growth has been exports growth. However, with the formation of an extremely wealthy group and the growth of an upper-middle class, China has started stepping into a transition from an investment-led and export-driven economy to one that is driven by consumer demand. It is expected that this transition will play out in the next decade, as policies are introduced to enable reforms that encourage increased domestic demand.

While almost all sections of China's domestic demand are growing, the growth of the demand for luxury goods is much faster than all others. As pointed by Park [1], although Japan is still today's largest luxury goods market, China is expected to take its place to become the world's foremost luxury goods market by 2020, with total expected sale amount to be about 100 billion US dollars. A recent article [2] published online by *The Economist* also said: "A new report by CLSA, a broker, forecasts that overall consumption in China (including boring everyday items) will rise by 11% annually over the next five years. That is very fast. But sales of luxury goods will grow more than twice as quickly, reckons CLSA: by 25% a year." Here CLSA stands for Credit Lyonnais Securities Asia, which is a leading independent brokerage and investment group in Asia, founded in 1986 and headquartered in Hong Kong, China.

The purpose of this paper is to introduce a few mathematical models to analyze consumer behaviors in China's luxury goods market, to explore the interactions among the luxury goods demand and various other factors in economics and social values, and to estimate the long term trend of the market demand for luxury goods in China. We will use a Markov chain model to analyze the consumer behavior on where to buy the goods, a linear regression model to explore the interactions among the luxury goods demand and various other factors, and a dynamical system model to estimate the long term trend of the market demand. Our discussions will be carried out in following sections.

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<sup>1</sup> Email: yshi@bloomu.edu

## 2. Consumer Behavior – Where to Buy?

Statistics show (see [2]) an estimate that about 55% of the luxury goods bought by Chinese people are bought outside mainland of China. There are two main reasons for that: high tariffs on luxury goods and counterfeiting being rife in mainland of China. Realizing this consumer behavior, China will soon be slashing import duties on certain luxury goods to encourage wealthy shoppers to buy more within the mainland of China. However, the fighting against the counterfeiting would still have a long way to go.

Under the current status and with those ongoing changes, what would be the trend of the consumer behavior about where to buy? In this section, we are trying to use a Markov chain model with changing transition matrices to estimate this trend.

Let us use the notation of a market status vector  $S = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$  where  $x_1$  stands for the percent of the luxury goods bought by Chinese people within mainland of China and  $x_2$  for the percent of purchase outside the mainland of China. Clearly  $0 \leq x_1, x_2 \leq 1$  and  $x_1 + x_2 = 1$ . To be more precise, let  $S_0$  be the initial status vector, that is, the status vector at the initial time moment,  $S_1$  be the status vector at the end of the first time period, and in general  $S_k$  be the status vector at the end of the  $k$ -th period.

The sequence  $\{S_0, S_1, S_2, \dots\}$  forms a Markov chain with changing transition matrices

$$P(k) = \begin{bmatrix} p_{11}(k) & p_{12}(k) \\ p_{21}(k) & p_{22}(k) \end{bmatrix} \quad \text{for } k = 1, 2, 3, \dots \quad (1)$$

such that

$$S_k = P_k S_{k-1}. \quad (2)$$

More precisely, in the transition matrix  $P(k)$ ,  $p_{11}(k)$  stands for the percentage of total purchase within mainland of China in the  $k$ -th period that continues to stay within,  $p_{21}(k)$  for the percentage of total purchase within mainland of China in the  $k$ -th period that switches to outside mainland. Similarly,  $p_{12}(k)$  stands for the percentage of total purchase outside the mainland of China in the  $k$ -th period that switches to within, while  $p_{22}(k)$  for the percentage of total purchase outside the mainland in the  $k$ -th period that continues to stay outside. Values of  $p_{11}(k)$ ,  $p_{21}(k)$ ,  $p_{12}(k)$  and  $p_{22}(k)$  change from time period to time period, and hence they are functions of  $k$ . However, it is always true that

$$0 \leq p_{11}(k), p_{21}(k), p_{12}(k), p_{22}(k) \leq 1 \quad (3)$$

and

$$p_{11}(k) + p_{21}(k) = 1, \quad \text{and} \quad p_{12}(k) + p_{22}(k) = 1 \quad (4)$$

When China starts its efforts on reducing duties on luxury goods and fighting against the counterfeiting, more purchase is likely to happen within mainland of China. Hence a growth of the value of  $p_{12}(k)$  may be observed, showing that purchases start to switch from outside to within. The value of  $p_{11}(k)$  may stay quite stable and high during those early time periods, reflecting the phenomena that purchases within the mainland are encouraged to stay within. After a while, depending on the effectiveness of China's efforts, the values of  $p_{11}(k)$  and  $p_{12}(k)$  may start to decrease, and now the long term where-to-buy trend starts to show. In long run, the transition matrix  $P(k)$  may eventually get closer and closer to the identity matrix  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ , meaning

that a certain percentage of the total purchase will stay within the mainland of China, and the rest of the total purchase will happen outside the mainland of China. The market status vector  $S_k$  reached in those time periods will then approach the equilibrium status for the consumer behavior about where-to-buy. For more detailed discussions on establishing formulas for  $P(k)$ , numerically computing parameters involved in those formulas, and examples showing applications of such Markov chain models, see [3] and [4].

## 3. Interactions Among the Luxury Goods Demand and Other Factors

What makes such a rapid growth of demand for luxury goods in China? Obviously one of the major reasons is the formation of an extremely wealthy group and the growth of the upper-middle class. But there are also a number of other factors that play a role here. The *age* – the average age of Chinese millionaire is only 39, which is 15 years younger than the average elsewhere (see [2]); *gender of luxury goods consumers* – while luxury goods market is normally dominated by women in many other countries, both men and women in China are enthusiastic luxury goods consumers; *social value* – in today’s China one often needs to show off to be taken seriously; *corruption* – use of luxury goods as gifts to lubricate business; and *economic environments* – financial crisis, uncertainty in investments, and depreciation of money drive people with extra money to spend today. If we use  $Q$  to denote the total market demand for luxury goods in China,  $W$  to denote the population size of wealthy and the upper-middle class in China,  $A$  to denote the average age of that population,  $G$  for the rate of Chinese male luxury goods consumers to the female ones,  $S$  for the weight of social value on luxury goods,  $C$  for the extent of corruptions involving luxury goods, and  $E$  for the extent of uncertainty in economic environments, then the total market demand  $Q$  can be expressed as a function of  $W, A, G, S, C, E$  plus a term for the effect of other random reasons. That is,

$$Q = F(W, A, G, S, C, E) + R \quad (5)$$

where  $F(W, A, G, S, C, E)$  is the formula of a selected model and  $R$  stands for the random part caused by other factors.

Many models may be used to establish the formula  $F$ . A frequently used one is the linear regression model

$$Q = c_1W + c_2A + c_3G + c_4S + c_5C + c_6E + R \quad (6)$$

The coefficients  $c_1 \dots c_6$  are to be determined based on available data of  $\{Q, W, A, G, S, C, E\}$ . Since  $W, A, G, S, C, E$  all change when time goes by, the total market demand  $Q$  is also varying with time. Hence the above model can be expressed more precisely as

$$Q(t) = c_1W(t) + c_2A(t) + c_3G(t) + c_4S(t) + c_5C(t) + c_6E(t) + R(t) \quad (7)$$

where  $t$  stands for time. For more details about data collection for establishing a linear regression model, determination of coefficients  $c_1 \dots c_6$ , and evaluation of the effectiveness of a linear regression model, please see [5].

#### 4. Trend of the Market Demand for Luxury Goods in China

As forecasted by CLSA, overall consumption in China (including boring everyday items) will rise by 11% annually over the next five years, and sales of luxury goods will grow by 25% a year. This indicates a trend that the consumption of luxury goods will weigh more and more in China’s overall consumption. A dynamical system model may be used to estimate this trend.

Let us use  $T_1, T_2, \dots, T_k, \dots$  for the total amount of annual overall consumption in China in the first year (current), second year, and so on so forth. Let  $L_1, L_2, \dots, L_k, \dots$  be the annual amount of consumption of luxury goods in corresponding years. If the above growth rates stay unchanged, then we have

$$T_{k+1} = 1.11 T_k, \quad \text{and} \quad L_{k+1} = 1.25 L_k \quad \text{for } k = 1, 2, \dots \quad (8)$$

Consider the rate  $R_k = L_k/T_k$ , that is,  $R_k$  stands for the percentage of total annual overall consumption that is spent on luxury goods. From (8) we see that

$$R_{k+1} = L_{k+1}/T_{k+1} = 1.25L_k/1.11T_k = 1.126R_k \quad (9)$$

The equation (9) formulates a dynamical system that shows that the percentage of overall consumption spent on luxury goods will be increasing at a rate of 12.6%. This is to say that this percentage will be doubled every 6 years! Hypothetically let’s suppose that currently only 5% of total annual overall consumption in

China is spent on luxury goods, that is,  $R_1 = 5\%$ . Then, if the phenomena represented in (9) keeps going on, after 6 years 10% of total annual overall consumption will be in luxury goods market; after 12 years, 20%; after 18 years, 40%; and so on. This would indicate that more and more Chinese are becoming wealthier and wealthier. This could also be an indication that the gap between the wealthy and poor is expanding rapidly. The latter could lead to many social issues. Therefore, it may be reasonable to believe that at a certain point China would make appropriate efforts to slow down the growth rate of the luxury goods market in the nation, and direct the flooding money of the wealthy towards projects that aim to narrow down the wealthy-poor gap.

## 5. Concluding Comments

In this paper we have introduced a few mathematical models to analyze consumer behaviors in China's luxury goods market, to explore the interactions among the luxury goods demand and various other factors in economics and social values, and to estimate the long term trend of the market demand for luxury goods in China. In section 2, a Markov chain model is used to analyze the consumer behavior on where to buy the goods. With available data and appropriately selected formula, the model can be applied to estimate the equilibrium status for the consumer behavior about where-to-buy (within or outside the mainland of China). In section 3, a linear regression model is suggested to explore the interactions among the luxury goods demand and various other factors. Similar models can be applied to study the market of particular luxury goods, such as luxury cars, luxury cloth, popular brand names, etc. In section 4, a dynamical system model is established to estimate the long term trend of the luxury goods market demand. The results suggest that although this market is currently growing extremely rapidly, it is very likely to slow down after a certain point.

Moreover, there is a variety of other models that could be applied in such studies, including exponential models, polynomial models, and so on. Also, more market records would become available when time goes by. Therefore, real-time on-going modification of models may be conducted based on the additional information obtained from the newly received market records. Furthermore, many other factors such as developments in other markets, changes in the economic and political environments, etc, may also have impacts on the luxury goods market. Therefore, more generalized models such as piecewisely defined models or compositions of models may also be needed to address the effects of those factors.

## 6. References

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