Empirical Portfolio Analysis: M-V vs CAPM

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Abstract. In order to assess the two-asset portfolios from the different approaches: Mean-Variance (M-V) and Capital Asset Pricing Model (CAPM), this article provides an empirical portfolio analysis based on two stocks - David Jones Limited (DJS) and the Commonwealth Bank of Australia (CBA). It critically examined and discussed rationales and assumptions of the two methods, and concluded that although each method has several limitations, M-V approach is better than CAPM when estimating risk and return of 2-stock portfolios due to unavoidable unsystematic risk of stocks and limited explanatory capacity of beta.

Keywords: portfolio, Mean-Variance approach, Capital Asset Pricing Model, unsystematic risk, beta.

1. Introduction

The Mean-Variance approach (Markowitz 1952) and CAPM (Sharpe 1964) are two fundamental theories in finance. The Mean-Variance approach is an important method to evaluate the risk and return of a portfolio (Yoshimoto 1995) [25]. The CAPM (Capital Asset Pricing Model) is widely used in evaluating the performance of managed portfolios (Fama and French 2004) [6]. In order to assess the investment feasibilities of two-asset portfolios from the two different approaches, this article provides an empirical portfolio analysis based on two stocks - David Jones Limited (DJS) and the Commonwealth Bank of Australia (CBA) and critically examined rationales and assumptions under these two methods.

2. Mean-Variance Approach

Mean-variance theory is an important model for investments developed by Markowitz (1952) [16]. It is considered to be the simplest model of investments as it can be directly used in application. In this part, we use M-V approach to calculate the expected return (ER) and standard deviation (SD) of DJS and CBA, and construct a series of portfolios of the two stocks by varying the weights of each at 2.5% intervals.

2.1. Justification of data selection:

Ane' and Lobidi (2004) pointed out that the monthly data shows approximately normally distributed results compared with the daily or weekly data. According to the Central Limit Theorem, if the sample size is larger than 30, normality approximation can be applied in the calculation and when the sample size exceeds 50 the normality approximation can be used confidently (Woolridje 2008) [24]. Bartholdy and Peare (2011) said in general it is believed that when estimating a parameter the more observations the better, however if the time period is too long then one runs into instability problems [1]. Besides, if the estimation period we choose is too long, the long pasted data would affect the accuracy of estimation to predict future. Because of the reasons above, monthly price data over five years period (from June 2006 to July 2011) which we choose as sampling frequency and the estimation period including 61 samples is appropriate to calculate the expected return (ER) and the standard deviation (SD).

2.2. Calculate of ER and SD of each share

According to our calculation, the monthly expected return (mean return) of DJS and CBA is 1.278861% and 0.866616% respectively. When it comes to the risk, we usually refer to the deviations from the mean return. Here we use the standard deviation of return method to estimate the risk of the given two shares. The standard deviation of returns of DJS and CBA is 12.414335%, 6.938949% respectively.

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Covariance of returns of DJS and CBA is 0.004233, and the coefficient of correlation of the two is 0.491402, which means the returns of DJS and that of CBA are positively correlated.

2.3. Calculation of ER and SD of portfolios

We choose to make a weight bias of 2.5%, thus 41 portfolios will be formed and analysed via the mean returns and SDs of the combinations.

$$r_p = x_1\bar{r}_1 + x_2\bar{r}_2 \tag{1}$$

$$\sigma_p^2 = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\sigma_{12} \tag{2}$$

x_1, x_2 = proportion of the portfolio invested in DJS's share and CBA's share, $\bar{r}_1, \bar{r}_2$ = mean return on DJS's share and mean return on CBA's share, σ_1, σ_2 = the SD of DJS and CBA, σ_{12} is the covariance of returns on DJS and CBA.

By using the equations above, we compute all mean returns and SDs of the 41 portfolios and make a diagram hereafter to help to make a comparison on the different results.

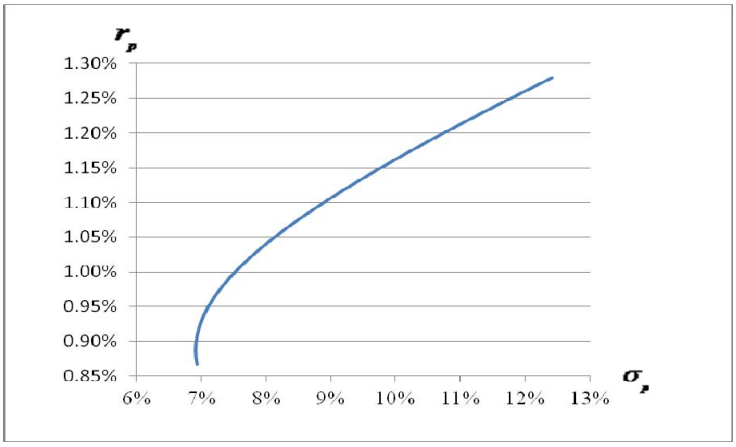

Fig. 1: Return and SD of the Portfolios (drawn by excel).

3. CAPM Approach

The capital asset pricing model (CAPM) was firstly introduced by Sharpe (1964) [23] based on the portfolio theory of Markowitz (1952)[16]. It is a commonly used model in modern finance. In this part, we are going to use the CAPM to calculate β of DJS and CBA, to estimate expected returns of each stock and to construct portfolio beta/return combinations of the two stocks.

3.1. Basic formulas

CAPM is a linear function of market risk premium. According to Frino, Hill and Chen (2009) [9], the CAPM for individual security and portfolio are as follows:

$$E(r_i) = r_f + E(r_m - r_f)\beta_i \tag{3}$$

$$E(r_p) = r_f + E(r_m - r_f)\beta_p \tag{4}$$

For a portfolio which contains only two stock, stock A and Stock B, the expected return of the portfolio can be written as $E(r_p) = W_A E(r_A) + W_B E(r_B)$, where W_A and W_B refer to the weight of stock A and that of stock B respectively; $E(r_A)$ and $E(r_B)$ refer to the expected return of stock A and that of stock B respectively. Thus, according to CAPM, we have:

$E(r_p) = W_A E(r_A) + W_B E(r_B) = W_A\left[r_f + E(r_m - r_f)\beta_A\right] + W_B\left[r_f + E(r_m - r_f)\beta_B\right] = (W_A + W_B)r_f + E(r_m - r_f)\times(W_A\beta_A + W_B\beta_B)$

$= r_f + E(r_m - r_f)\times(W_A\beta_A + W_B\beta_B)$ .Comparing it to the CAPM of a portfolio, we have

$$\beta_p = W_A\beta_A + W_B\beta_B \tag{5}$$

Where r_f = the risk free rate of return, $E(r_i)$ =the expected return on security i, $E(r_m)$ =the expected return on the market, and β_i =the beta of security. r_p refers to the rate of return of the portfolio and β_p refers to the beta of the portfolio.

3.2. Calculation of beta for individual shares

3.2.1 Ex-dividend

According to Goddard (2008), dividends make a difference on the stock share price which cannot be ignored. However, which date should we consider the dividend payment and choose the dividend data? It is recommended that we should consider dividend payment on ex-dividend date. The ex-dividend date is defined as 'the date two business days before the date of record establishing those individuals entitled to a dividend' (Ross, Westerfield and Jordan 2008) [22]. 'On the ex-dividend date the price of the shares typically falls by the value of the dividend payable to the shareholders' (Frino, Hill and Chen 2009)[9]. Therefore, we should consider dividend effect on ex-dividend date.

3.2.2 The choice of sampling interval and horizon

Changes may incur in the managerial strategies and the business scale over 10 years, and this can lead to an un-relevant and inaccurate beta (Bradfield 2003)[3] This leads us to think about shortening the period. Hawawini (1983) illustrated that a security's beta may vary substantially depending upon the different interval lengths [12]. Bartholdy and Peare (2011) figured out that if we move from monthly to daily returns, for example, then it may increase in the amount of noise in the data, thus the efficiency of the estimates reduces [1]. They argue that 5-year of monthly data are recommended when estimating beta based on the CAPM. Moreover, the estimation period and the sampling frequency of CAPM approach should be in accordance with those of the mean-variance approach, thus we can get a more accurate comparison for the purpose of making a decision and recommendation. Hence, a 5-year-period monthly data from 31th July 2006 to 29th July 2011 is selected to estimate beta.

3.2.3 Market proxy and risk free security

Generally, the All Ordinaries Index is the commonly known market index and widely used in Australia. It includes nearly 97.7% of the market capitalisation of shares and 500 securities in Australia (Frino 2009). However, the All Ords does not take the dividends into consideration when giving the index, which may lead to the biased calculation. In addition, in our case, considering dividends in market index are in accordance with the treatment of individual securities Therefore, we choose the S&P/ASX 200 Accumulation Index which incorporates dividends as the market benchmark. It covers 200 securities and 80% of ASX market capitalisation (Frino 2009), and is widely used market index in Australia [9].

In order to estimate the Beta, which serves as a significant role in the estimation in CAPM, we should determine the risk free rate of return. In the modern economic circumstance, absolute risk free asset does not exist. However, government bond could be assumed as zero risk asset when calculating Beta and pricing capital (Frino 2009). According to Officer and Bishop (2008)[21], retaining a 10-year government bond as a proxy for the risk free rate is both theoretically and practically sustainable. Thus, we use the 10-year government bond as a risk free security in the report. In addition, as we use the monthly data of DJS and CBA, we convert the yearly risk free rate into monthly rate.

3.2.4 Calculation of beta

According to Bodie, Kane and Marcus (2008) [2], the risk premium on individual stock is proportional to the risk premium of the market. Beta of individual stock can be calculated as:

$$\beta = \frac{\text{covariance } (X_t, Y_t)}{\text{variance}(X_t)} \qquad (6)$$

Where $Y_t = (r_{i,t} - r_{f,t})$; $X_t = (r_{m,t} - r_{f,t})$; $r_{i,t}$ = the return on share i, earned over period t; $r_{f,t}$ = the risk-free rate of return, earned over period t; $r_{m,t}$ =the market rate of return, earned over period t. However, here we calculate beta by constructing regression models rather than this approach, as regression analysis can provide significance test of the estimates.

For the CAPM of DJS (R^2 =0.35), β_{DJS} is estimated as 1.60306 with a p-value of 7.09×10^{-7}. For the

CAPM of CBA (R^2 =0.43), β_{CBA} is estimated as 0.993671 with a p-value of 1.46×10^{-8} . Thus, the estimates of beta for DJS and CBA are both significant given a significance level of 1%. Additionally, it can be indicated that DJS is more volatile than the market as 1.603060 is greater than 1, and that CBA is slightly less volatile than the market as 0.993671 is slightly less than 1.

3.3. Estimation of individual stock returns based on CAPM

3.3.1 Current risk free rate and market risk premium

According to our justification in 3.2.3, 10-year government bond is our proxy for risk free security to construct CAPM. According to statistics database of Reserve Bank of Australia (2011), the up-to-date yield of 10-year government bond is 4.48% p.a. in August, 2011.Transfer it into monthly data by dividing 12 ,we can get 0.373333% of the current monthly risk free rate which can be used in our CAPM model.

It is well known that the market risk premium (MRP) is the expected value of the excess market returns. Many researchers have made efforts in estimating the market risk premium of Australia. Among them, Brailsford, Handley and Maheswaran (2008) argue that market risk premium in Australia relative to 10- year government bonds is approximately 6.2 % p.a. over the sample period of 1883–2005, which is the longest periods used to estimate Australian market risk premium so far. This estimate is well-qualified as it drawn a very large sample size to estimate the population value of MRP. Although it did not incorporate information in these 6 years (2006-2011), the estimate may change little if it incorporate these years' data as 6 is very small compared to 123. Therefore, we use 6.2% p. a. as our MRP in the CAPM model, and converted it by dividing 12 to 0.516667% as our monthly market risk premium.

3.3.2 Estimation of Expected returns based on CAPM

As betas of DJS and CBA are calculated, we are ready to estimate the expected return of each stock based on the CAPM. According to 3.3.1, the monthly market risk premium is 0.516667% and monthly risk free rate is 0.373333%. Hence, the expected returns and betas of DJS and CBA based on CAPM can be listed as follows:

Table. 1: Betas and expected returns based on CAPM (calculated by excel)

CAPM-based	DJS	CBA
Beta	**1.60306**	**0.993671**
Expected Return	**1.201581%**	**0.886730%**

3.4. Portfolio Combinations based on CAPM

According to (4) and (5), we can calculate the expected returns and betas of the 41 portfolios (Appendix 6.) of the two stocks by varying the weights of each at 2.5% intervals. Portfolio no.41 gives us the lowest beta, 0.993671, with an expected return of 0.886730%.

The following graph is the picture of the portfolio combinations. In the graph, the extended line, SML has a slope of 0.516667% that is the monthly market risk premium. The intercept of the line is 0.373333% which is the current monthly risk free rate.

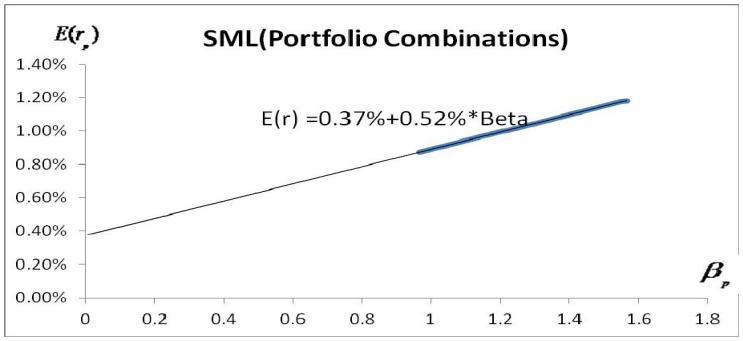

Fig. 2: SML based on CAPM of portfolios of DJS and CBA

4. Analysis and Recommendation

4.1. Rationale and assumptions of M-V method

The M-V approach was derived from the portfolio theory by Markowitz (1952)[16]. Under M-V, a rational investor would choose a portfolio with highest expected return given a certain level of risk, or a portfolio with the lowest risk given a certain level of expected return. The mean of historical returns is calculated to represent the expected return; the standard deviation of these returns is calculated to represent the risk which is consists of systematic risk and unsystematic risk (Ross, Westerfield and Jordan 2008) [22].

There are several assumptions underlying this method. Firstly, 'the process that generates returns in the past is also the process that generates returns in the future' (Frino, Hill and Chen 2009). Secondly, the chosen historical returns represent a random sample of the population returns (Frino, Hill and Chen 2009) [9]. Thirdly, investors are rational and risk-averse, and want to obtain higher return compensation for higher risk (Fame and French 2004) [7]. Fourthly, the M-V model we use in the article is a single period model, in other words, at the beginning of the time period an investor constructs his portfolio according to the chosen mean-variance-criterion and retains the proportion of assets in the portfolio until the end of the period (Korn and Korn 2001)[14]. Lastly, the distribution of returns is assumed to be normal distribution (Markowitz 1952)[16], which is the reason that investors can focus only on mean and variance of the returns. Besides, it has some other implied assumptions, such as investors are price takers, and there is no income tax or transaction fee.

4.2. Critiques of M-V approach

4.2.1 Historical return is questionable to predict future

The mean-variance approach regards sample statistics of historical returns as good predictors of future performance. However, according to Gray (1993)[10], historical returns on investments are no certain guide to future returns. Even if history returns are good indicators of future performance, how to choose historical returns as representatives of population of returns is still a hard problem. Therefore, it is questionable to use historical returns to evaluate performance in order to predict future performance.

4.2.2 Are returns on assets normally distributed?

Mittnik, Paolella and Rachev (2000)[20] point out that financial return data can be leptokurtic, oftentimes skewed. In addition, Hu and Kercheval (2010) indicate that equity returns are not normally distributed. Although normal distribution makes it easier for investors to select portfolios as normal distribution only has two parameters (mean and variance), financial returns are not always normally distributed [13].

4.2.3 Risk measurement

Under mean-variance method, it is assumed that all investors are risk averse. In reality, this does not hold. According to Bodie, Kane and Marcus (2008) [2], investors can be divided into three types, which are risk-averse, risk neutral and risk lover. Investors who are risk averse only consider risk-free assets or assets with positive risk premiums. Risk –neutral investors judge risky prospects only by their expected rate of return. Risk lovers are happy to take part in gambles.

In this approach, variance is calculated to represent risk, however, 'a minimization or restriction of variance punishes the desired positive deviations of the portfolio rate of return from its expected value' (Korn and Korn, 2001)[14]. There are therefore other risk measures developed to replace variance, such as value at risk, expected shortfall and lower partial standard deviation, which are also good indicators of vulnerability to extreme negative returns (Bodie, Kane and Marcus 2008)[2].

4.2.4 Single period model

The extreme assumption is that investors care only about mean and variance of one-period portfolio returns (Fama and French 2004) [6]. It is necessary to incorporate the effect of time lapse into the model. As time goes on, the expected returns, variance of returns and covariance of returns may change, so the portfolio preference of the investor changes accordingly. Though researchers, such as Hakansson (1971)[11], have developed multi-period method to solve this problem, our analysis is based on the single period assumption.

4.3. Rationale and assumptions of CAPM method

The capital asset pricing model (CAPM) is the equation of the security market line showing the relation

between expected return and beta (Ross, Westerfield and Jordan 2008)[22]. It depends on three aspects: the risk free rate, the reward for bearing systematic risk and the amount of systematic risk. CAPM is derived from Markowitz's mean-variance framework, so it shares several assumptions with M-V approach, such as the assumption that investors are rational and risk averse, the assumption of normality of returns and the assumption of single period model.

On the other hand, it has its own assumptions compared to M-V approach. According to Bodie, Kane and Marcus (2011)[2], these assumptions are as follows: 'one, investors are price-takers; two, all investors plan for one identical holding period; three, investments are limited to publicly traded financial assets, and investors may borrow or lend any amount at a fixed risk free rate; four, there is no taxes on returns and no transaction costs; five, all investors are rational mean- variance optimizers; six, all investor analyse securities in the same way and share the same economic view of the world'.

4.4. Critiques of CAPM approach

4.4.1 Beta Shifts as the Return interval Changes

Estimation of beta is an important factor in construction of the CAPM. However, beta may change as the return interval changes. For example, 'for the four-year period January 1970 to December 1973, Eastman Kodak had a beta of 1.25 based on daily returns, but a beta of 0.93 based on monthly returns' (Hawawini 1983) [12]. The reason for beta shift is the intertemporal relationships between the daily individual securities returns and daily market returns (Hawawini 1983). Therefore, proper selection of return interval according to investment timeline is a controversial task in constructing the CAPM.

4.4.2 Estimation of market risk premium is a difficult task

Estimating market risk premium is one of the difficult tasks in construction of CAPM. According to Fernandez, Aguirreamalloa and Corres (2011) [8], the average market risk premium used in Australia in 2011 is 5.8% p. a. with a standard deviation of 1.9%. In essence, the market risk premium (MRP) is the expected value of the excess market returns. It is a population statistic, so sampling historical data to estimate MRP is generally problematic unless the sample is sufficiently representative of the population [4]. Merton (1980) introduces method of estimating the risk premium based on the theoretical relationship between expected returns and variance of returns[19]. However, empirical research has failed to prove a significant positive relationship between expected returns and market volatility (Mayfield 2004)[17]. Although researchers such as Mayfield (2004) provide methods for estimating the market risk premium, the estimating the MRP remains a problem in constructing CAPM.

4.4.3 Systematic risk v. s. unsystematic risk

A basic assumption of CAPM is that the risk to determine the expected return of a security is only the systematic risk. This is because all other risk (i.e. the unsystematic risk) can be avoided by diversification (Sharpe 1964)[23]. Nevertheless, considering our case that the portfolio which consists of only two stocks cannot be treated as a well-diversified portfolio, we cannot ignore the existence of unsystematic risk in this case. Therefore, the CAPM may not a satisfactory model when consideration of unsystematic risk is necessary.

4.4.4 Lending or borrowing any amount at risk free rate is unrealistic

One assumption under CAPM is that an investor can lend or borrow any amount at risk free rate. In reality, this is not the case. Although it is possible to buy at any amount of risk-free securities such as treasury bills, it is not possible to borrow at this rate and to borrow as any amount as possible. This is because borrowing rate should be higher than lending rate; otherwise banks would not exist. In addition, borrowing behaviour accompanies with default risk, so it is not allowed to borrow as much as possible.

4.4.5 Is beta sufficient to explain expected return?

Even if beta can represent total risk, is beta sufficient to explain expected return? Fama and French (1993) found that there is no apparent relationship between beta and average return [7]. Even if beta is related to the expected return, the goodness of fit – (in 3.2.4) in our case indicates that there are more than 50% of variation in dependent variable has not been explained by the regression model. In other words, factors other

than beta are omitted. Merton (1973)[18] extended the CAPM and invented the inter-temporal CAPM, or ICAPM. Fama and French (1993) introduced Fama–French three-factor model. Although these researchers have extended and developed CAPM, the explanatory capacity of beta is still a controversial issue[7].

4.5. Portfolio recommendation

The CAPM approach still has some shortcomings while being applied in our case. Firstly, CAPM omits the unsystematic risk. Lintner (1965) stated that under a given expected return, diversification can help minimize unsystematic risk [15]. In most portfolio decisions, investors construct a portfolio based on a variety number of shares, which, according to Lintner's point of view, will reduce the unsystematic risk by stock diversity. However, in our case, we construct portfolios consisted of merely two stocks, which makes little contribution to reducing the unsystematic risk, which cannot be ignored when analysing portfolios. Thus, using the CAPM approach, our estimates may deviate from the real values. Furthermore, as interpreted above in 4.4, other drawbacks of CAPM, including the limited explanatory capacity of beta, the shift of beta with the return interval, difficulty in estimating the market risk premium, impossibility of lending and borrowing rate from risk free rate, have negative effects on the accuracy of our estimates. Taking all the facts into consideration, portfolio recommendation based on CAPM approach is not our preference.

In terms of Mean-Variance approach, we found that the portfolio which consists of 5% of DJS and 95% of CBA is the minimum variance portfolio among all the portfolios. It has the lowest risk of 6.918177% with an expected return of 0.887228%. Based on statement above, we can safely come to the conclusion that we should choose the MV approach rather than CAPM to calculate the expected risk and return, because CAPM does not account enough for unsystematic risk, and beta has limited explanatory ability in the regression model (CAPM). Our recommendation is to choose a portfolio consisting 5% of DJS share and 95% of CBA because this portfolio has the minimum risk level among all the portfolios we constructed.

5. Conclusion and Further discussion

The article provides an analysis of portfolios combined by two stocks- David Jones Limited (DJS) and The Commonwealth Bank of Australia (CBA) over a 5-year period via two different approaches, Mean-Variance approach and Capital Asset Pricing Model (CAPM).

We critically examined assumptions and rationales of M-V and CAPM, and point out limitations for both methods. For M-V approach, assumption of normality is a major limitation, as market returns often do not behave normally. In addition, it has single period assumption, risk-averse assumption, etc. As for CAPM, beta does not stay the same if we swift estimate intervals. Secondly, estimating market risk premium is a difficult task. Most importantly, beta only represents systematic risk and does not sufficiently explain expected return. Based on our empirical portfolio analysis, we recommend that a portfolio constructed by 5% of DJS shares and 95% of CBA shares is the best choice for the investor who seeks minimum risk. It is concluded that the M-V approach is better than CAPM to estimate the risk and return of 2-stock portfolios due to the un-diversifiable unsystematic risk and limited explanatory capacity of beta.

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