Street as a Stage: A Model of Dynamic Provision of Public Goods under the Threat of Protest

Wen-Kai Wang [1 +] and Kannika Thampanishvong [2]

[1] Department of Finance, National University of Kaohsiung, Taiwan

[2] Thailand Development Research Institute (TDRI), Thailand

Abstract. It has become increasingly common that the politically and economically weak citizens use protest as a channel through which they express their dissatisfaction with the policies engendered by the elites. This paper aims to provide a better understanding on this issue by using differential games to study whether protest could trigger some changes in public good investment under uncertainty. In this paper, we show that protest on its own is not sufficient for inducing the elites to invest more in public goods. The elites choose policy to increase their income and to directly transfer resources from the rest of the society to themselves.

Keywords: Protest, Public Goods, Uncertainty, Differential Games, Class Conflict

1. Introduction

When the group with political power has preference over inefficient policies, this translates into inefficient economic institutions (Acemoglu, 2006). Even though the dictatorship of the elites is inefficient or inappropriate, the elites still have an incentive to preserve the system as it is, while other group in the society will fight to induce changes in the policies. Protest is perceived as a mean through which people, who lack de jure power and are unsatisfied with policies, express their dissatisfaction (Muller and Opp, 1986). Lohmann (1993) argues that, on several occasions, major shifts in policies are preceded by different forms of political actions, which allow people to express their dissatisfaction with the status quo.

Reiss (2007) documents series of case studies which discuss the emergence of protests and organized public demonstrations between the nineteenth century and the end of the twentieth century. A more recent example in the twenty-first century is the wave of protests in South Africa in July 2009 over negligence by the South African government under the leadership of President Jacob Zuma in the area of basic public services. Thus, protests are still commonly used and have indeed become a global phenomenon, although factors that lead people to protest could be diverse. In this paper, we view protest as being a consequence of government's public good policies. Since, from the point of view of the elites, it is wasteful to devote a large amount of economy's resources to investment in public goods, whose benefits are mostly enjoyed by the citizens public good provision, in these countries could be limited (Lizzeri and Persico, 2004). When the politically and economically weak groups are adversely affected by the policies determined by the elites, protest could help pave way for some changes and open up opportunities for them. This paper is devoted to provide a better understanding of the concerns facing the politically and economically weak groups of people.

We use differential games to study the dynamic interactions between these two groups over time. Facing with uncertainty of investment in public good and their preference for direct transfer of resources to themselves, non-elites' engagement in protest alone is not sufficient to trigger policy shift by the elites. Our results show that only when the revolution is looming because the level of public good is too low that the elites start to increase their investment in public goods.

The remainder of the paper is structured as follows. Section 2 is devoted for reviewing the related literature both in economics. Section 3 describes the model environment and we present the method applied to solve the model. Section 4 presents the sensitivity analysis, while Section 5 concludes.

2. Related Literature

[+] Corresponding author. Tel.: +886 7 5916676; fax: +886 7 591 9329.
E-mail address: wkwang@nuk.edu.tw.

In the economic literature, there are very few papers which looked at the microfoundation of protest. Buenrostro et al. (2007) use a game-theoretic approach to analyze the state's response to protest movements and its impact on potential protesters. They use a reputation model by Kreps and Wilson (1982) to explain different state's responses to protest, where they allow the state to be either tough or weak. With more than one protest group, a weak state may choose not to give in to protesters in order to build a reputation for being tough and thus deter other groups from protesting.

Similar to Buenrostro et al. (2007), in this paper, we study the government's response to protest and how the non-elites could use protest to bias policy on public goods in their favour. Unlike in Buenrostro et al. (2007) which allow for heterogeneity across different protest groups, in this paper, we assume that the citizens are homogenous. Moreover, while in Buenrostro et al. (2007), there is incomplete information about each player's type, in our model, there is complete information so we abstract from the signalling of information between protesters and the government.

3. The Model

In this section, we present the model which adopts a differential game approach. We consider an economy consisting of elites and non-elites, where the elites are the group with political power. We suppose that the production function of national output, *Y*, requires two types of inputs, investment in public good and labour. Production function is given by

$$P(u,v) = Auv^2, \tag{1}$$

where *u* denotes the investment in the public good by the elites and *v* denote the amount of time the non-elites spend working. It can be seen that the marginal product of *u* is larger than that of *v* if $v < 1$. In this case, the major contribution to the production is the investment in public good. The value of public good, *x*, is assumed to follow a stochastic differential equation:

$$dx(t) = (u(t) - \delta x(t))dt + \sigma x(t)dW(t), \tag{2}$$

where $\delta > 0$ is the depreciation rate and *W(t)* is a standard Wiener process. Suppose that the elites' utility from public good consumption is given by

$$U_e(x) = ax - bx^2,$$

where *a* and *b* are positive. It can be seen that $U_e(x)$ is negative if *x* is sufficiently large, i.e., $x > \frac{a}{b}$. Our discussion is therefore restricted to some specific public goods. The objective functional for the elites is defined by

$$V_e(x) = \max_u \mathrm{E}\left\{\int_t^\infty e^{-rs}\left(ax(s) - bx^2(s) + Au(s)v^2(s) - \frac{u^2(s)}{2}\right)ds \,\middle|\, x(t) = x\right\}, \tag{3}$$

where $\frac{u^2}{2}$ is the individual cost of investment in the public good. Note that the term, $Auv^2 - \frac{u^2}{2}$, denotes the amount of resources the elites directly transfer to themselves.

Given their time endowment and the observed value of public good, the non-elites can choose the amount of time they spend working in the production of output, *P(u, v)*. The opportunity cost of working is given by $\frac{v^2}{2}$, which suggests that the opportunity cost of working borne by the non-elites is increasing in the time the non-elites devote for production and increases at an increasing rate. The interpretation of the opportunity cost of working is as follows. If the non-elites do not express their dissatisfication, the elites may not be concerned about their livelihood. The non-elites' utility is given by

$$U_p(x,v) = c\sqrt{x}v,$$

where *c* is positive. The parameter *c* is negatively related to the minimum consumption of the public good required by the non-elites, i.e., a smaller c implies a higher minimum consumption. It can be seen that the utility of non-elites is positively related to *v*, i.e. the longer the non-elites work, the higher utility they obtain. Moreover, their utility is increasing in the value of public good, i.e., if the non-elites observe that the value of public good becomes smaller, their utility declines. The objective functional for the non-elites is therefore given by

A rich empirical literature has investigated how the collective action problem is solved in practice (Lichbach, 1995; Moore, 1995).

$$V_p(x) = \max_u \mathrm{E}\left\{\int_s^\infty e^{-rs}\left(c\sqrt{x(s)}v(s) - \frac{v^2(s)}{2}\right)ds \,\middle|\, x(t) = x\right\}. \tag{4}$$

To solve the model, the dynamic programming principle leads to the following Hamilton-Jacobi-Bellman (HJB) equations:

$$rV_e(x) = \max_u \left\{ax - bx^2 + Au\left(v^*\right)^2 - \frac{u^2}{2} + (u - \delta x)V_e'(x) + \frac{\sigma^2 x^2}{2}V_e''(x)\right\}, \tag{5}$$

and

$$rV_p(x) = \max_u \left\{c\sqrt{x}v - \frac{v^2}{2} + (u^* - \delta x)V_p'(x) + \frac{\sigma^2 x^2}{2}V_p''(x)\right\}, \tag{6}$$

where u^* denotes the optimal investment of the elites in the public good and v^* denotes the optimal amount of time the non-elites spend working. In this paper, we derive the so called stationary feedback Nash-equilibrium strategies, i.e., u^* and v^* are functions of the state *x*. A necessary condition for v^* is given by

$$v^*(x) = c\sqrt{x}\,. \tag{7}$$

It can be seen that v^* is independent of u^*, which means that the non-elites are not concerned about the the elites' investment in the public good. Instead, their concern is on the value of the public good. Substituting v^* into Equation (5) gives

$$rV_e(x) = \max_u \left\{ax - bx^2 + c^2 Aux - \frac{u^2}{2} + (u - \delta x)V_e'(x) + \frac{\sigma^2 x^2}{2}V_e''(x)\right\}. \tag{8}$$

A necessary condition for u^* is given by

$$u^*(x) = c^2 Ax + V_e'(x)\,.$$

We substitute the form of u^* into Equation (8) and rearrange all terms. We then obtain

$$\frac{\sigma^2 x^2}{2}V_e''(x) + \frac{1}{2}\left(c^2 Ax + V_e'(x)\right)^2 - \delta x V_e'(x) - rV_e(x) + ax - bx^2 = 0\,. \tag{9}$$

We make a sophisticated guess and assume that the solution of Equation (9) takes the form of

$$V_e(x) = B_2 x^2 + B_1 x + B_0\,.$$

Substituting the above solution form into Equation (9) gives

$$\left[\sigma^2 B_2 + \frac{1}{2}(c^2 A + 2B_2)^2 - 2\delta B_2 - rB_2 - b\right]x^2 + \left[(c^2 A + 2B_2)B_1 - \delta B_1 - rB_1 + a\right]x + \left(\frac{B_1^2}{2} - rB_0\right) = 0.$$

The above equation must hold for all positive *x*. Therefore,

$$B_2^{\pm} = \frac{-(\alpha + 2c^2 A) \pm \sqrt{(\alpha + 2c^2 A)^2 - (4c^4 A^2 - 8b)}}{4},$$

$$B_1 = \frac{a}{r + \delta - (c^2 A + 2B_2)},$$

$$B_0 = \frac{B_1^2}{2r},$$

where $\alpha = \sigma^2 - 2\delta - r$. As long as B_2 is determined, B_1 and B_0 are both determined. Note that $B_2^{\pm}$ are well defined if $\sqrt{(\alpha + 2c^2 A) - (4c^4 A^2 - 8b)} \geq 0$. In practice, the depreciation rate and discount rate are much lower than 1, which leads to a negative α . On the other side, the elites stop benefiting from the public good when $x > \frac{b}{a}$. A larger *b* implies that the elites are more unlikely to raise the value of the public good. This case is more interesting than the one for a smaller *b*. To determine the value of B_2, we apply the finite

horizon approximation introduced in Ewald and Wang (2011) and conclude that either $0 < B_2^- < B_2^+$ or $B_2^- < 0 < B_2^+$ and $B_1 > 0$ leads to $B_2 = B_2^-$. We omit the proof here.

4. Sensitivity Analysis

In this section, we analyze the optimal investment of the elites in the public good and study how it is affected by the parameters in the model. Note that

$$u^*(x) = c^2 Ax + 2B_2 x + B_1 = \left(c^2 A + 2B_2\right)x + B_1 .$$

If $B_2 = B_2^- > 0$, then $b \le \frac{c^4 A^2}{2}$. This case is less interesting in the real world since, in the perspective of the elites, the public good is very much profitable. This leads the optimal investment of the elites in the public good to increase in *x*. Nevertheless, provided that this phenomenon is not caused by the protests by the non-elites, we therefore choose to omit it here.

We now assume that $b > \frac{c^4 A^2}{2}$, which leads to $B_2 = B_2^- < 0$. Furthermore, $c^2 A + 2B_2 < 0$, which implies that the optimal investment of the elites in the public good is decreasing in *x*. The interpretation is as follows. The amount of time the non-elites spend protesting is negatively related to the value of the public good. Moreover, if *x* is sufficiently large, the production is increasing significantly in *x*. Therefore, when the value of the public good is sufficiently high, the elites reduce the investment in the public good to obtain higher direct transfer of the resources.

We now study how some parameters affect the optimal investment of the elites in the public good. We begin with the parameter *c*, which is negatively related to the non-elites' minimum consumption of the public good. We have the following proposition:

Proposition 5.1. $u^*(x)$ is decreasing more significantly in *x* if *c* is smaller.

Proposition 5.1 indicates that if the non-elites require higher minimum consumption of the public good, i.e., the non-elites spend more time protesting, then the elites reduce the investment in the public good significantly when a higher value of the public good is measured. This result is surprising. A possible interpretation is as follows. The elites have less incentives to raise the value of the public good since they prefer the direct transfer of the resources to the consumption of the public good. If the non-elites require higher minimum public good consumption, i.e., they spend more time protesting, the elites have to invest more in the public good to raise the value of it to placate the angry non-elites. Then the elites have to sacrifice higher direct transfer of the resources, which is not in their will. Therefore, the elites reduce the investment in the public dramatically to receive more transfer of the resources if a higher value of the public good is measured. It can also be shown that a smaller *c* implies a smaller optimal investment of the elites in the public good, i.e., B_1 is increasing in *c*. The interpretation is that a higher minimum consumption of the public good of the non-elites implies that the elites not only reduce the investment significantly in *x*, but also invest less in the public good.

We now move on to studying how the uncertainty parameter affects the optimal investment of the elites in the public good. We have the following proposition:

Proposition 5.2. $c^2 A + 2B_2$ is decreasing in σ.

The interpretation for Proposition 5.2 is that a higher uncertainty leads the elites to have less incentives to invest in the public good. The elites could invest more in the public good in the situation of higher uncertainty, but the uncertainty may soon push the value of the public good at a lower level. Therefore, when the elites observe a higher value of the public good, i.e., the amount of time the non-elites spend protesting is lower, the elites reduce their investment in the public good significantly to receive more direct transfer of the resources. In addition, it can be shown that B_1 is decreasing in σ. Therefore, a higher level of uncertainty gives the elites more incentives to reduce the investment in the public good more significantly and less incentives to invest more in the public good.

The depreciation rate δ is the next concern. We first present the following proposition:

Proposition 5.3. $c^2 A + 2B_2$ is increasing in δ.

One may expect that a higher depreciation rate may lead the elites to have less incentives to invest in the public good, but we found the contrary. A possible interpretation is that it takes time to raise the value of the public good at a higher level. Therefore, even though the elites observe a higher value of the public good, they reduce the investment in the public good only slightly. Moreover, in contrast to the effect of the uncertainty on the public good, the effect of the depreciation rate on the public good is slow. Even though the value of the public good declines due to a higher value of the two parameters, the elites do not act the same in the two cases. Note that weather B_1 is increasing or decreasing in δ depends on the parameters chosen. Nevertheless, Proposition 5.3 implies that the elites' optimal investment in the public good is higher given a sufficiently large x if the depreciation rate is larger. A large A , for instance, leads to higher production and the elites tend to invest less in the public good to obtain more direct transfer of the resources if the depreciation rate is high and x is small. The elites will fight against a higher depreciation rate only if they have some spare resources. On the other hand, if A is small, the elites do not have sufficient amount of resources, which leads them to have more incentives to invest more in the public good if a higher depreciation rate is observed..

Last but not least, we present the result of the sensitivity analysis on the parameter b. It can be shown that c^2A+2B_2 is decreasing in b. The interpretation is that the elites can benefit from the public good only if its value lies in a small interval. Therefore, the elites have less incentives to raise the value of the public good and it causes the elites to reduce the investment in the public good significantly if a higher value of the public good is observed. Since c^2A+2B_2 is decreasing in b, B_1 is also decreasing in b. Hence, a larger b implies that the elites invest less in the public good. The interpretation is that a larger b implies that the elites' consumption of the public good is smaller, which gives the elites less incentives to raise the value of the public good.

5. Conclusion

In this paper, we study how the politically and the economically weak non-elites could influence the policy choices of the elites, particularly the investment in public good, by taking part in the protest. Our results show that, if a higher value of the public good is observed, a higher uncertainty leads the elites to reduce the investment in the public good significantly. On the other hand, the investment in the public good declines slightly with respect to the value of the public good if the depreciation rate is high. It is interesting to highlight that a larger minimum consumption of public good does not always lead to more investment in the public good even though the non-elites protest longer, which is contrary to the priori belief.

6. References

[1] D. Acemoglu. A Simple Model of Inefficient Institutions. Scandinavian Journal of Economics, 2006, 108(4), pp. 515-546.

[2] L. Buenrostro, A. Dhillon and M. Wodders. Protest and Reputation. International Journal of Game Theory, 2007, 35, pp. 353-377.

[3] J.P. Conley and A. Temimi. Endogenous Enfranchisement When Groups' Preferences Conflict. Journal of Political Economy, 2001, 109(1), pp. 79-102.

[4] C.-O. Ewald and W.-K. Wang. Analytical Solutions for Infinite Horizon Stochastic Optimal Control Problems via Finite Horizon Approximation: A Practical Guide. Mathematical Social Sciences, 2011, 61(3), pp. 146-151.

[5] D. Kreps and R. Wilson. Reputation and imperfect information. Journal of Economic Theory, 1982, 27, pp. 253-279.

[6] M. Lipsky. Protest as a Resource. American Political Science Review, 1968, 62, pp. 1144-1158.

[7] A. Lizzeri and N. Persico. Why did the elites extend the suffrage? Democracy and the scope of government, with an application to Britain "age of reform". The Quarterly Journal of Economics, 2004, 119, pp. 707-765.

[8] S. Lohmann. A Signaling Model of Informative and Manipulative Political Action. American Political Science Review, 1993, 87(2), pp. 319-333.

[9] E. Muller and K. Opp. Rational choice and rebellious collective action. American Political Science Review, 1986, 80, pp. 471-487.