2011 International Conference on Economics and Finance Research
IPEDR vol.4 (2011) © (2011) IACSIT Press, Singapore

Fuzzy Logic and Behavioral Finance

A Connection

Renato Aparecido Aguiar
dept. of electrical engineering
Centro Universitário da FEI
São Bernardo do Campo, Brazil
e-mail: preraguiar@fei.edu.br

Roberto Moura Sales
dept. of electrical engineering
Escola Politécnica da USP
São Paulo, Brazil
e-mail: roberto@lac.usp.br

Abstract— **The aim of this paper is to show the connection between the behavioral finance theory and fuzzy sets theory. Such connection, very little explored by researchers, may produce new models for the financial market, aiming to better understand the anomalies of the financial market not explained by modern theory of finance. In this paper is established that the clustering algorithm based on fuzzy sets theory owns, intrinsically, some heuristics (representativeness and anchoring) of the behavioral finance theory.**

Keywords-heuristics; behavioral finance; fuzzy c-means; fuzzy sets

I. INTRODUCTION

The development of tools that help investors' decision making in finance has been for decades object of intense research worldwide. In general, financial decisions aimed at maximizing future returns and in this context, the evolution of the basic premises produced the emergence of conflicting theories, such as the theory of efficient markets [4], which assumes a rational investor and risk averse, and behavioral finance theory, which considers the influence of psychological factors in the decision of an individual. In other words, individuals are not fully rational and their decisions are biased by their preferences and beliefs [6].

Despite differences of opinion, which have produced an intense debate involving these two theories, the behavioral finance have shown to be an appropriate tool for addressing various problems. The theory of behavioral finance suggests models that are based on heuristics such as anchoring, representativeness and availability. According to [9] there is a strong connection between the fuzzy sets theory and the behavioral finance theory, supporting the focus of this work that aims cover, theoretically, the connection between the behavioral finance and fuzzy sets theories.

The paper is organized as follows: In the section II a brief review on fuzzy sets theory and the concepts of fuzzy clustering are presented; an introduction to behavioral finance theory is presented in section III. In the section IV is presented the essential of this paper, which is the connection between fuzzy sets theory and behavioral finance theory; the conclusions are presented in section V.

II. FUZZY CLUSTERING

The fuzzy set theory proposed by Zadeh in 1965 [12] possesses as one of its main characteristics the fact of allowing the treatment of linguistic variables, such as hot, very hot, high, low, advisable, not advisable, highly risky, etc.

The resulting property when considering linguistic variables to characterize objects is that, instead of belonging or not to a certain set, as stated by the classic set theory, these objects will have pertinence indexes associated with different sets. A detailed presentation of the main concepts of the fuzzy theory can be found in [13].

Definition 1: Let the set $X = \{x_1, x_2, ..., x_m\}$, $C_1, C_2, ..., C_n$ subsets of X and real numbers $0 \le \mu_i(x_j) \le 1$, $i = 1,2,...n$, $j = 1,2,..., m$, such that, for every $j = 1,2,..., m$, one has $\sum_{i=1}^{n} \mu_i(x_j) = 1$. Under these conditions, $\mu_i(x_j)$ is denoted membership degree of the element x_j with respect to fuzzy subset C_i . The membership degree may be understood as a measure of the degree of affinity, similarity or compatibility among elements.

Among the techniques for the grouping or classification of elements in subsets of a given set, the Fuzzy c-Means – FCM algorithm has been proved to be an effective tool in those cases in which the features or attributes of the analyzed elements can be represented by a vector of real numbers. In such cases, the FCM algorithm allows identifying clusters of elements from a matrix of dimension $n \times p$, being n the number of elements and p the number of features of these elements [13]. Thus, let $x_1, x_2, ..., x_m$ elements of X and consider the problem of grouping these elements in c subsets. The FCM algorithm determines the subsets c through of the solution of the following problem: given a matrix pattern, the task is to determine c groups, where elements belong to more than one group simultaneously, but with different membership degrees [13]. Each group formed is represented by a center and the distance of each element in regard to each center determines the membership degree of the element in relation to groups. Fig. 1 shows a fuzzy clustering formed by two groups.

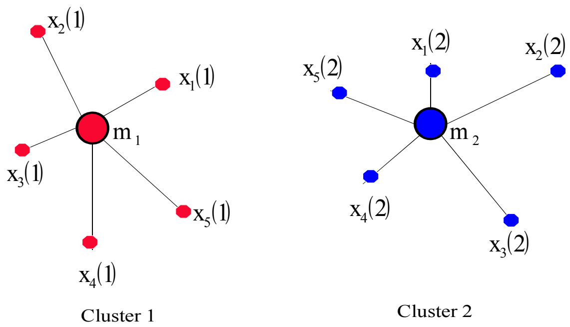

Figure 1 Hypothetical Clustering

In the Fig. 1, the group 1 is represented by the center m_1, the group 2 is represented by the center m_2 and the elements around the centers belong to both groups simultaneously, but with different membership degrees.

As an example, the element $x_1(1)$ has a membership degree higher with regard to cluster 1 than in relation to cluster 2, since this element is closer of the center m_1. Moreover, the membership degree of the element $x_1(1)$ with regard to cluster 1 is greater than the membership degree of the element $x_1(1)$ with regard to cluster 2, since the distance of the element $x_1(1)$ with regard to center m_1 is smaller than the distance of the element $x_1(1)$ with regard to center m_2.

So, for obtain a fuzzy clustering must select a criterion and evaluate it for all possible clustering and select the grouping that optimizes the criterion adopted, [13]. An alternative to solving this problem is to use the Fuzzy c-Means (FCM) algorithm that can be seen in more detail in [3].

Briefly, the FCM algorithm consists of the following steps:

Step 1: Start a matrix of membership degrees, assign randomly degrees of membership for each object in regard to each group, so that $\sum_{i=1}^{c}\mu_{ij}(x_j)=1$;

Step 2: Calculate the centers of each group using (1);

$$c_i = \frac{1}{\sum_{j=1}^{n}\left(\mu_i(x_j)\right)^m}\sum_{j=1}^{n}\left(\mu_i(x_j)\right)^m x_j \quad i=1,\ldots c \qquad (1)$$

Step 3: Recalculate, using (2), the new matrix of membership degrees using the centers obtained in step 2;

$$\mu_i(x_j) = \frac{\left(\frac{1}{\|x_j - c_i\|^2}\right)^{\frac{1}{(m-1)}}}{\sum_{i=1}^{c}\left(\frac{1}{\|x_j - c_i\|^2}\right)^{\frac{1}{(m-1)}}} \quad i=1,\ldots c \quad j=1,2,\ldots,n \qquad (2)$$

Repeat steps 2 and 3 until the value of the objective function, showed in (3) is minimized.

$$J_m = \sum_{i=1}^{2}\sum_{j=1}^{n}\left[\mu_i(x_j)\right]^m \|x_j - c_i\|^2] \qquad (3)$$

In (1), (2) and (3), x_j is the number of objects, c is the number of clusters and m is the fuzziness index [13].

The clusters are formed from the matrix membership degrees randomly chosen, according Step 1, grouping the elements by similarity measure based on the distance of each element relative to each center. In this case, after several iterations, the vectors of the center are adjusted to produce the final group and, consequently, the final matrix of membership degrees.

The use of the FCM algorithm is very useful in pattern recognition for formation of groups whose elements are similar in some sense. So in terms of composition of stock portfolios of the financial market, a method of grouping, such as the FCM algorithm, it becomes a very useful tool in the process of grouping of stocks with similar patterns. Moreover, according to Peters (2003) there is a strong connection, though little explored by researchers, among the heuristics of the behavioral finance theory and the fuzzy sets theory.

III. BEHAVIORAL FINANCE

The modern theory of finance is based on the premise that the investor is rational, risk averse and that operates in a market where stock prices reflect all available information. The research developed in [4], [7] e [11] were consistently the fundamental bases of development of the modern theory of finance.

On the other hand, according to behavioral theory, individuals make decisions biased by heuristics, with a rationale that deviates from statistical rules, in contrast, in the context of the economy, with the modern theory of finance. Cognitive psychology, which studies the mechanism of thought, is the basis of this approach and shows that individuals value too recent experience and are overconfident in their own abilities, providing thus the emergence of distortions in their thinking [10.

The heuristics in behavioral finance can be divided into:

i) Heuristics of representativeness: refers to a kind of mental shortcut in which there is a tendency to assume that something belongs to a particular group, based on the similarity with a member of that category. Many probabilistic questions with which people are concerned are: what is the probability that the object A belongs to Class B?

What is the probability that event A originates from process B? To answer those questions people use the representativeness heuristic, in which probabilities are assessed by the degree to which A is representative of B. In a classic example of literature, some individuals must answer what the occupation of a person chosen random from a group of ten people, knowing that eight people in the group are truck drivers and two are brokers. In the first case the ten people are also dressed and, after choosing one person from among the ten most participants, based on the known probability, judged that this person would be a truck driver. In the second case, was added an element of ambiguity, where ten people were dressed differently and was chosen a person, wearing suits, sunglasses and carrying a folder. In this case, most participants identified this person as a broker, although the likelihood of this person be a truck driver to overcome the likelihood, known a priori, to be a broker. In this example, the man wearing suit, goggles and carrying a folder has more similarity to the set of brokers and less similarity to the set of truck drivers. The individuals formed an association based on similarity, without conducting an analysis of the structure of probabilities, responding that the person selected was a broker [8]. In the context of decisions on the economy, the individual under the influence of the representativeness heuristic has a strong tendency to overvalue recent information. As in the previous example, there is a new ambiguous information that reduces the accuracy of the analysis, thereby producing a biased decision [2]. Furthermore, the existence of heuristics in decision making tends to produce over-reaction, meaning that past losers tend to be winners in the future and vice versa [5].

ii) Heuristic Anchoring: refers to a kind of mental shortcut in which verifies the use of a standard as a starting point, adjusting decisions on the basis of this initial anchor. In many situations individuals make estimates supported by an initial value, which is adjusted to produce the final answer. The initial value may be suggested by the formulation of the problem or may be the result of a calculation part. An experiment conducted by Tversky and Kahneman [6] shows the influence of anchoring in the decision of an individual. In this experiment, two student groups must estimate the value of an expression in five seconds:

$8\times7\times6\times5\times4\times3\times2\times1$ for the group 1;

$1\times2\times3\times4\times5\times6\times7\times8$ for the group 2.

Although the correct answer to the two sequences to be 40.320, the average estimate obtained for the sequence 2 was 512, while the average estimate obtained by the group 1 (descending sequence) was 2.250. This occurs because in the descending sequence the first steps of multiplication (from left to right) produce a number greater than the ascending sequence. Thus, when individuals are faced with complex situations, make decisions supported by information available. In the stock market, where the amount of information is very extensive and dispersed, individuals tend to use mental shortcuts, or heuristic judgments in decision-making, transforming a complex trial in a simple task compared. Moreover, people often are based on facts or terms of reference for decision making. In this case, it is said that the decision is based on the anchoring heuristic.

The anchoring heuristic is associated with conservative decisions, causing people to resist sudden changes in their decisions when faced with new information [8].

In terms of investment in stocks, market prices are usually a reference in the decision of an investor, since in the stock market, the information is extensive and scattered. The existence of such heuristics in decision making tends to produce sub-reaction, in which past winners tend to be future winners and losers in the past tend to be losers in the future [5].

iii) the availability heuristic: it is a kind of mental shortcut that produces a tendency to judge the likelihood of an event according to the ease with which examples of that event come to mind. In this case, the decision will depend on the number of examples which are available in the consciousness of the individual in a given time. In an elementary proof of this effect, Tversky and Kahneman [6] presented an experiment in which different lists were presented for different groups of people, who must answer what list contains more names of men than women's names. In some lists the men are more famous, and in other lists women are more famous.

As a result of this experiment, in each of the lists individuals erroneously judged that the class (sex) that had the with more numerous famous personalities. That is, the list had more famous personalities was the more numerous. In this demonstration, the salience, represented by the degree of fame for every personality, affected the recoverability of instances and, consequently, the decision of individuals.

IV. CONNECTION BETWEEN FUZZY LOGIC AND BEHAVIORAL FINANCE

There is a strong connection between the behavioral finance theory and the theory of fuzzy sets. More specifically, the fuzzy c- means (FCM) algorithm owns, intrinsically, some heuristics of the behavioral finance theory.

Concerning the representativeness heuristic, it is present in the fuzzy clustering algorithm in the separation and grouping of objects. The groupings are made based on the similarity of each object with regard to each group. Surprisingly, the representativeness heuristic is mainly based on the similarity between objects as described in THE section II.

As an example, Fig. 2 shows the distribution of 100 random data in a two-dimensional plan, each one with two characteristics.

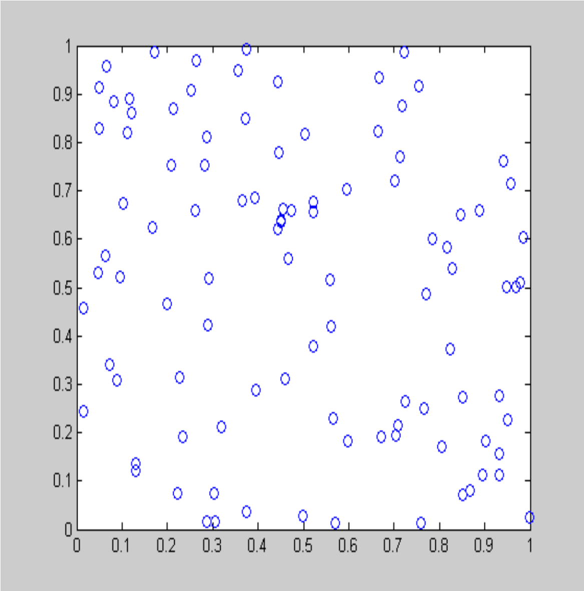

Figure 2Data Matrix

Applying the FCM algorithm in this data set results in the clustering consisting of two groups is obtained as shown in the Fig. 3.

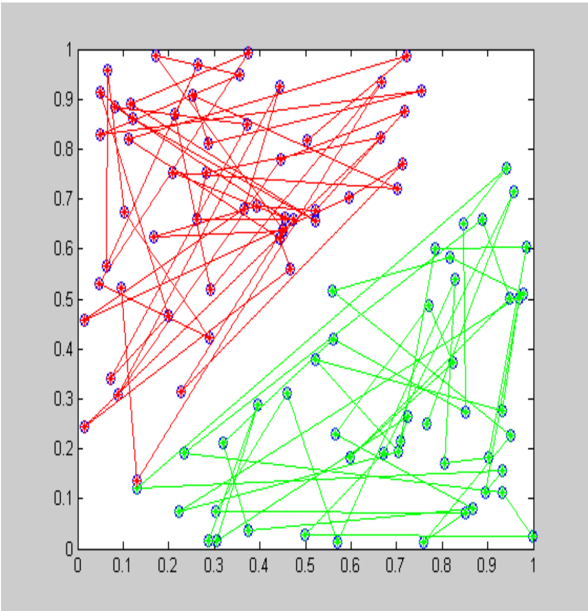

Figure 3 Clustering Fuzzy

Clearly, the elements more similar are grouped and less similar split. This grouping occurs because of the representativeness heuristic contained, intrinsically, in the fuzzy c- means algorithm that, based on the similarity between the objects and each group assigns a membership degree to each element with regard to each cluster.

The anchoring heuristic is also present, intrinsically, in the fuzzy c- means algorithm. As already defined in the section II, a decision based on this heuristic is adjusted for an anchor, an initial value used for produce the final answer. Similarly, as described in the section II, the first step of the algorithm starts with a matrix of membership degrees that will be adjusted, in each iteration of the algorithm, to produce the final answer or the final grouping.

Since that the fuzzy c- means algorithm is based on fuzzy sets theory, there is a strong connection, or a great similarity between the fuzzy sets theory and the theory of behavioral finance. In the model developed by Aguiar and Sales, called Behavioral Fuzzy Model [1], stocks of the financial market are classified by fuzzy c-means algorithm and are defined two groups (winner and loser) and each group is represented by a center. The grouping of the stocks is based on the similarity of each stock with regard to center of each group, forming a winning stock portfolio and a portfolio loser. Aguiar and Sales find that the Behavioral Fuzzy Model is biased by representativeness and anchoring heuristics in decision making [1], exploring, in this way, anomalies (overreaction or underreaction) present in the stock market.

V. CONCLUSIONS

Many models have been developed to assist and improve the performance of investors in decision-making. Some models, based on the theory of modern finance suppose that the investor is rational and risk averse, on the other hand, behavioral finance theory claim that the investor is biased by heuristics, such as representativeness, anchoring and availability in the decision-making of an investor in the stock market.

This paper aims to show the strong connection between fuzzy sets theory and behavioral finance theory. The comparison between these two theories to show that the fuzzy c- means (FCM) algorithm, which has its constructive base in the theory of fuzzy sets owns, intrinsically, the heuristic of representativeness and anchoring heuristic derived from the theory of behavioral finance.

Thus, a model for the financial market that has as constructive base the theory of fuzzy sets, leads an investor to make a biased decision by heuristics of representativeness and anchoring. The presence of these heuristics in decision making of an investor can produce some anomalies in the stock market, known as overreaction and underreaction.

REFERENCES

[1] R. A. Aguiar, R. M. Sales, L. A. Sousa. “A Behavioral Fuzzy Model for Analysis of Over-and Under-reaction reaction in the Brazilian Stock Market”. Revista de Administração de Empresas - RAE, v. 48, n.3, 2008.

[2] E. Amir,; Y. Ganzach. “Overreaction and Underreaction in Analysts' Forecasts”. Journal of Economic Behavior & Organization, v. 37, p. 333-347, 1998.

[3] J. C. Bezdek. “Pattern Recognition with Fuzzy Objective Function Algorithm”. Plenum Press, New York and London, 1981.

[4] E. Fama. “Efficient Capital Markets: A Review of Theory and Empirical Work”. Journal of Finance, v. 25, p. 383-417, 1970.

[5] E. Fama. “Markets Efficiency, Long-Term Returns, and Behavioral Finance”. Journal of Finance Economics, v. 49, p. 283-306, 1998..

[6] A. Tversky, D. Kahneman. “Judgement under Uncertainty: Heuristics and biases”. Science, v. 185, n. 4157, pp. 1124-1131, 1974.

[7] H. .M. Markowitz. “Portfolio Selection”. The Journal of Finance, Chicago: American Finance Association, v. VII, n. 1, p. 77-91, 1952.

[8] E. Peters. “Chaos and Order in the Capital Markets”, 2nd edition. New York: Wiley, 1996.

[9] E. Peters. “Simple and Complex Market Inefficiencies: Integrating Efficient Markets, Behavioral Finance, and Complexity”. The Journal of Behavioral Finance, v. April, n. 4, 2003.

[10] J. R. Ritter. “Behavioral Finance. Pacific-Basin”. Finance Journal, v. 11, p. 429-437, 2003.

[11] W. F. Sharpe. “Capital Asset Prices: a theory of market equilibrium under conditions of risk”. Journal of Finance, Chicago: American Finance Association, p. 425-442, set. 1964.

[12] L. A. Zadeh. “Fuzzy sets”. Information and Control, vol. 8, p.338-353, 1965.

[13] H. J. Zimmermann. “Fuzzy Set Theory and Its Application”. Kluwer Academic, Boston, 1996.