2011 International Conference on Economics and Finance Research
IPEDR vol.4 (2011) © (2011) IACSIT Press, Singapore

The Performance of Mean-Variance Portfolio Selection and Its Opportunity Cost: The Case of Thai Securities

Budsabawan Maharakkhaka
School of Management
Assumption University
Bangkok, Thailand
e-mail: budsabawan@gmail.com

***Abstract*— Mean-variance efficient portfolio selection is one of the most frequently used criteria in portfolio investment. But since the approach has been subjected to controversy due to its limitation, economists have been trying to validate the reliability of the approach until recently. In this paper, the performance of the mean-variance efficient approximation to maximize expected utility has been evaluated. By assuming that there are three classes of asset in the portfolio, we use monthly returns on SET index, Thai investment grade corporate bond index, and Thai government Treasury bill to compare maximum expected utility of the mean-variance efficient portfolio to the maximum expected utility derived from direct optimization. The findings indicated that picking the portfolio on the basis of the mean-variance criteria does not lead to maximum expected utility. In addition, investors with various utility functions are found to require significant optimization premium to bring up their welfare to the level achieved by holding expected utility maximization portfolio.**

I. Introduction

The most frequently used mean-variance approximation to expected utility maximization introduced by Markowitz (1952) in the early 50's has been the subject of much controversy. It is often claimed that the analysis applies exactly only when returns are jointly normally distributed, or the utility function is quadratic. However, other researches, for example, Levy and Markowitz (1979), Pulley (1981), and Tew, Reid, and Witt (1991) suggested that the portfolio selection based on the mean-variance approach was almost identical to that obtained by expected utility approach for various utility functions and historical distributions of returns.

This present paper follows the works of Pulley (1981) and Kroll, Levy, and Markowitz (1984) in comparing the expected utility of well selected portfolios from the mean-variance efficient frontier with the expected utility of the optimal portfolio for given utility functions. This research examines the portfolio with three classes of assets including stocks, bond and risk free assets in Thailand with one-month Thai government Treasury bill representing the risk free security. The study also follows Simann (1993) and Tew, Reid, and Witt (1991) in evaluating opportunity cost of holding mean-variance efficient. The main objective is to investigate whether the mean-variance efficient portfolio will provide similar level of expected utility as those generated by the maximum expected utility portfolio approximation using the monthly return of Thai securities. If there is a different between portfolios selected from the two approaches, we approximate optimization premium investors require to hold the mean-variance efficient portfolio.

II. Lierature Review

Many approaches have been proposed by economic theorists for decision making under uncertainty and one of the most widely used criterion in investment decision was originally developed by Markowitz (1952). His mean-variance rule states that investors would want to select the portfolios with minimum variance for given expected return and maximum expected return for given variance under the conditions that the utility preference function must be quadratic and the returns have to be jointly normally distributed. However, a number of studies have empirically investigated the ability of mean-variance analysis to maximize the expected utility. Although the conclusions of these studies have been mixed, the general difficulties found so far have been accuracy in measuring the quality of mean-variance efficient solution and how well relying on only mean and variance would work in the real asset allocation problems.

Levy and Markowitz (1979) estimated the expected utility by a function of mean and variance of return of 149 mutual funds and found that ordering portfolios by mean-variance rule was almost identical to the order obtained by using expected utility. Pulley (1981) indicated that the mean-variance formulation provides a very good local approximation to expected utility for more general utility functions using both monthly and semiannual return data. According to the study, investors can confidently rely on mean-variance optimization, with attitudes toward local changes in portfolio value reflected by the local relative-risk-aversion (Pratt, 1964). The paper also suggested that the mutual fund should select portfolios which maximize utility for a wide class of individual investors having different utility function and wealth levels, regardless of the actual form of their utility function. The importance of Pratt-Arrow risk-aversion coefficient can be found in the work of Kallberg and Ziemba (1983) whose theorem was proved that under the assumption of jointly-normally-distributed security returns, maximizing expected utility with different global utility functions will generate identical optimal portfolios if certain computed statistics such as those resembling Pratt-Arrow risk-aversion are identical. Using a normal return distribution for ten securities, Kallberg and Ziemba (1983) demonstrated that the optimal portfolio for seven different utility functions are very similar when global averages of Pratt-Arrow risk aversion measures are similar. While Pulley (1981) has shown that an expected-utility-maximizing investor could do quite well for himself with a mean-variance formulation based on the local relative-relative-risk-aversion coefficient, Kallberg and Ziemba (1983) verify the importance of the Pratt-Arrow risk-aversion measure in global comparisons of expected-utility-maximizing portfolio.

Kroll et al. (1984) also report that the best mean-variance efficient portfolio has almost maximum obtainable expected utility and the same is true even when 50% borrowing is allowed. Their work and that of Pulley (1981) both compared expected utility of mean-variance efficient portfolios to the expected utility of the optimal portfolios but Kroll et al. use annual holding period to poses a greater challenge for mean-variance approximation since it is agreed by these researchers that the higher portfolio variance the less likely is a mean variance approximation to do almost as well as actual expected utility maximization.

By introducing the concept of opportunity cost, Simaan (1986) explained that investors holding mean-variance portfolio are expected to receive an optimization premium to equate their expected utility with those of investors holding optimal portfolio. Tew et al. (1991) estimated the opportunity cost of using mean-variance analysis for various jointly distributed investment return distribution. The small opportunity cost found in their study supports the view that mean-variance analysis performs exceptionally well and is most practical approach for most investment scenarios. On the other hand, Simaan (1993) found that when investor's opportunity set includes the riskless asset, the premium to replace the mean-variance investment strategy by its optimal one is very small. But the riskless asset is unavailable, the opportunity cost of mean-variance investment strategies increase with degree of risk aversion indicating the greater difference in expected utility derived from the two solutions since investor's frustration is higher the higher absolute risk aversion is in the non existence of riskless asset.

In 2007, Sharpe justified that all relevant probability distributions do not necessarily have the same form and investor may not consider only quadratic utility function. Thus, mean and variance may not be sufficient statistics to identify the full distribution of returns for the portfolio. In his paper, Sharpe presented alternative approach of mean-variance and expected utility optimization base on the assumption of asset prices that would prevail if there were single representative investor who desired to maximize expected utility. His work suggested that when investor has quadratic utility function, his optimal portfolio will be similar to that derived from mean-variance optimization but when investor's utility function is non-quadratic portfolio may not be. Given diverse investor preferences, there should be diverse portfolio holdings and the expected utility optimization procedure allows one to find preferred asset combination for investors whose utility functions are different.

A. The maximization problem

Portfolio is mean-variance efficient if it maximizes expected rate of return for a given variance and minimize the variance for a given expected return. Let x_i be the proportion of i^{th} asset in the portfolio. Assuming standard constraint set with borrowing allowance 200%, an efficient portfolio $x' = (x_1, x_2, \dots x_n)$ solves the problem

$$\begin{array}{ll} \text{Min} & x'\,\Omega\, x \\ \text{subject to} & -2 \le x_i \le 2 \\ & \Sigma_i\, x_i = 1 \\ \text{and} & \Sigma_i\, x_i r_i = e \\ \text{for all} & e \in [e_{\min}, e_{\max}] \end{array}$$

where Ω = the covariance matrix of security returns
r_i = the expected or mean returns of securities i
e = the mean return of the portfolio.
$e_{\min}$ = the return of the minimum variance portfolio
$e_{\max}$ = the maximum feasible return of any portfolio.

Since it is possible now to calculate the expected utility of the portfolio, for each value of return on asset i, r_i, we can calculate expected utility:

$$\text{EU}\ (\Sigma_i\, x_i r_i)$$

where x_i represents the proportion of i^{th} asset and r_i denotes its corresponding return.

The expected utility of a portfolio is a function of asset return and proportion. Thus, for investors with different utility functions, they seek to maximize the expected utility of their portfolio by

$$\begin{array}{ll} \text{Max} & \text{EU}\ (\Sigma_i\, x_i r_i) \\ \text{subject to} & -2 \le x_i \le 2 \\ \text{and} & \Sigma_i\, x_i = 1 \end{array}$$

In this paper, we will follow Kroll et al. (1984) by denoting the maximum expected utility of the mean-variance efficient portfolio with E*U and the maximum expected utility derive by direct maximization without the constraint to include only efficient portfolio with EU.

B. Evaluating the approximation

Pulley (1981) evaluate the approximation of the expected utility derived from mean-variance optimization by the ratio of expected utility using mean-variance utility maximization to the expected utility using direct utility maximization. Pulley (1981) index is expressed as

$$I_P = E^*U(\ ) \,/\, EU(\ )$$

According to Kroll et al. (1984), I_P is not invariant to linear transformations of the utility functions. However, one can instead measure the loss incurred from choosing mean-variance portfolio by the index:

$$I_{KLM} = \quad E^*U(\ ) - E_N U(\ ) \,/\, EU(\ ) - E_N U(\ )$$

where is the expected utility of a naïve portfolio in which 1/n is invested in each asset. The expected utility of return from naïve portfolio is written as

$$E_N U(\ ) = EU\, (\Sigma_{i=1}^{N}\ 1/n\ r_i)$$

Pulley (1985) suggested that utility comparison of the portfolios should be made with Certainty Equivalent Index or *CEI*.

$$CEI = CE(E^*U) / CE(EU)$$

where *CE( )* is the certain-dollar amount that yields the same expected utility as the specified risky portfolio. The comment of Reid and Tew (1986) on implementing I_{KLM} stated that the choice of naïve portfolio by Kroll et al. (1984) overlook the preference of very risk-averse informed individual for a high degree of diversification. And after small difference were found between I_{KLM} and CEI, they concluded that when I_{KLM} is calculated with appropriate $E_N U$ value, it is a good measure of relative efficiency and makes Pulley's (1985) suggestion of certainty-equivalents unnecessary. Therefore, the I_{KLM} will be employed for evaluating performance of the mean-variance utility maximization approximation in this present study.

C. The optimization premium

The quality of mean-variance portfolio selection can also be evaluated by an opportunity cost that results from replacing his optimal portfolio with the mean-variance second-best (sub-optimal) alternative portfolio. The opportunity cost, expressed as a rate of return on investment is the premium investor should require over his suboptimal return to bring his welfare to a level achieved by optimal return. An investor requires an optimization premium such that

$$EU(r^* + \theta) = EU(r)$$

Where θ represents optimization premium, r* represents return on mean-variance efficient portfolio, and r represent return on expected utility maximization approach. The optimization premium is invariant under any positive linear transformation of the utility function and depends on the utility function itself.

III. DATA AND METHODOLOGY

A. Data

In this present study, the portfolios include three classes of asset, stock, bond and Treasury bill. We use monthly percentage returns on SET index to represent returns on stock and monthly percentage returns on Thai investment grade corporate bond index for returns on bond. The monthly return on Thai one-month Treasury bill represents the returns on riskless assets. The return data are collected from July 2001 to December 2008 as our sample. There are all together 90 observations for each assets. The statistics of the monthly return are presented in table 1.

TABLE I. SAMPLE STATISTICS FOR RETURN DATA

	SET	CORP BOND	T-BILL
mean	0.0037	0.0039	0.0021
variance	0.0057	0.0000	0.0000
S.D.	0.0754	0.0008	0.0010
Skewness	-1.2853	-0.4282	0.4783
Kurtosis	5.0923	0.0247	-0.8557

B. Selected utility functions

This paper will use three utility functions for empirical analyses each with two different parameter set, these utility function are logarithmic, power and exponential appear in Pratt (1964). The properties of the functions with respect to the absolute and relative risk aversion and parameter values are presented in table 2.

The two logarithmic utility functions represent decreasing absolute risk aversion where, from the two chosen risk aversion coefficients, one exhibits increasing relative risk aversion and another exhibits constant relative risk aversion. Exponential utility functions represent constant absolute risk aversion and the two chosen coefficients of risk aversion show different degree of increasing relative risk aversion. For the last type of utility function, power utility function exhibits decreasing absolute risk aversion and in the study, we choose risk aversion coefficient to reflect increasing and constant relative risk aversion.

TABLE II. UTILITY FUNCTION DESCRIPTION AND PARAMETER SETS

$U(w)$	$R_A(w)$	$R_R(w)$	Parameter value
$\ln(\alpha + w)$,	Decreasing	Increasing if $\alpha > 0$	$\alpha = 0$
		Constant if $\alpha = 0$	$\alpha = -0.5$
		Decreasing if $\alpha < 0$	
$-e^{-\alpha w}$, $\alpha > 0$	Constant	Increasing	$\alpha = 2$
			$\alpha = 5$
$(\alpha + w)^{\beta}$, $0 < \beta < 1$	Decreasing	Increasing if $\alpha > 0$	$\alpha = -0.8, \beta = 0.1$
		Constant if $\alpha = 0$	$\alpha = -0.5, \beta = 0.2$
		Decreasing if $\alpha < 0$	

IV. EMPIRICAL RESULTS

With 200% borrowing allowance, we generate portfolio combination from proportion of minus 2.0 to 2.0 of stock (SET index), bond (Government Bond Index), and Treasury bill. There are all together 110,553 portfolios combination simulated from the process. Each portfolio is calculated for its mean, variance and standard deviation. The results are used as inputs for further maximization problems as described in section 2.

For each utility function, we choose the portfolio maximizing expected utility then six maximization problems were solved for the expected utility values. As described earlier, this paper will follow Kroll et al. (1984) by denoting the maximum expected utility of the mean-variance efficient portfolio with E*U and the maximum expected utility derive by direct maximization without the constraint to include only efficient portfolio with EU.

Table 3 describes the portfolios selected by the mean-variance efficient approach and expected utility maximization approach as well as their statistics. With a constraint to minimize standard deviation, the M-V efficient portfolio provides lower standard deviation and lower mean return. The portfolio suggests investment proportion of 0, 0.8, and 0.2 of stock, and bond, and T-bill respectively while expected utility maximization approach suggest the proportion of 1, 2, and -2 for the three securities.

Table 4 shows the derived expected utility as well as the values of the index I_{KLM} for various utility functions. Although suggested as not invariant to linear transformations of the utility functions, index I_P is also exhibited in the table.

TABLE III. SELECTED PORTFOLIOS AND STATISTICS

	Proportion of assets			Mean returns	Standard deviations
	Stock	Bond	T-bill		
M-V efficient portfolio (min. S.D.)	0.00	0.80	0.20	0.003575	0.000797
Expected utility max. portfolio	1.00	2.00	-2.00	0.007291	0.075477
Naïve portfolio	0.33	0.33	0.33	0.003257	0.025017

When the value of index I_{KLM} is close to one, the approximation of mean-variance optimization is good and the error in using the mean-variance is small. When the value is close to zero however, the mean-variance criterion does not provide efficient approximation compared to the expected utility maximization criterion given the naïve portfolio as a benchmark. As seen from the table 4, I_{KLM} shows very small value reflecting the fact that the expected utility derived from mean-variance efficient portfolio and expected utility maximization portfolio approximations are different when we use naïve portfolio as criterion.

TABLE IV. EXPECTED UTILITIES AND PERFORMANCE INDEX I_{KLM}

	1	2	3
$U(w)$	**$\ln(\alpha + w)$, $\alpha = 0$**	**$\ln(\alpha + w)$, $\alpha = -.05$**	**$-e^{-\alpha w}$, $\alpha = 2$**
EU	0.00726439	-0.67867079	-0.13337619
E^*U	0.00356819	-0.68602349	-0.1343712
E_NU	0.00325152	-0.68665468	-0.13445662
I_{KLM}	0.07891325	0.07905781	0.07906055
I_p	0.491189	1.01083397	1.00746025
EU-E^*U	0.0036962	0.0073527	0.00099502

	4	5	6
U(w)	**-e-αw, α = 5**	**(α + w)β, α = -.08, β = 0.1**	**(α + w)β, α = -0.5, β = 0.2**
EU	-0.00649674	0.85439365	0.8730747
E*U	-0.00661859	0.85284941	0.87179175
ENU	-0.00662911	0.85271619	0.87168171
IKLM	0.07950137	0.07941259	0.0789997
Ip	1.0187551	0.99819259	0.99853054
EU-E*U	0.00012185	0.00154424	0.00128295

In our study, the indices I_{KLM} are close to 0.079 in every case including logarithmic, exponential, and power utility function. The last column of table 4 show that excess utility provided by expected utility approach over the mean-variance approach for the six cases of utility functions. The difference varies from 0.00012185 and 0.00735270. Although considered to be not invariant to linear transformation, if we consider the I_P index, different conclusions have to be added. In most cases, the index are close to 1 except in the logarithmic utility function with zero α, the case where investors with logarithmic utility function exhibit decreasing absolute risk aversion and constant relative risk aversion.

Table 5 reports the optimization premium investors, with different utility function, holding mean-variance portfolio require to equate their expected utility with those of investors holding optimal portfolio derived from expected utility maximization approach. The premium is expressed as rate of return from investment. The result shows that since there are noticeable different in the expected utility provided by m-v portfolio and expected utility maximization portfolio, investors holding m-v efficient portfolio require significant amount of compensation or optimization premium to bring his welfare to a level achieved by his optimal return derived from expected utility maximization approach.

In short, the empirical results demonstrated contradiction to the work of Levy and Markowitz (1979), and Kroll et al. (1984) whose works support the mean-variance efficient approximation under various types of utility functions. The results are consistent with Pulley (1981) who use Pulley index (IP) to evaluate performance of mean-variance efficient portfolio.

TABLE V. OPTIMIZATION PREMIUM (Θ)

	$U(w)$	(θ)
1	$\ln(\alpha + w)$, $\alpha = 0$	0.003716600259
2	$\ln(\alpha + w)$, $\alpha = -.05$	0.003716288050
3	$-e^{-\alpha w}$, $\alpha = 2$	0.003716278808
4	$-e^{-\alpha w}$, $\alpha = 5$	0.003716278052
5	$(\alpha + w)^{\beta}$, $\alpha = -.08$, $\beta = 0.1$	0.003716378598
6	$(\alpha + w)^{\beta}$, $\alpha = -0.5$, $\beta = 0.2$	0.003717096677

The findings, however, does contradict to the study of Simann (1993) and Tew et al. (1991) who claimed that the mean-variance analysis performs exceptionally well and investors require only very small opportunity premium to replace his optimal portfolio with the mean-variance second-best alternative.

V. CONCLUSIONS

This paper evaluates the performance of the mean-variance efficient approximation to maximize expected utility, the issue that has been studied by a number of researches although the results of those works have been mixed. Assuming there are three classes of asset in the portfolio, we use monthly returns on SET index, Thai investment grade corporate bond index, and Thai government Treasury bill to represent return on stock, bond, and riskless assets respectively we compare maximum expected utility of the mean-variance efficient portfolio to the maximum expected utility derived from direct optimization.

For three types of utility functions with different degree of absolute and relative risk aversion it is found that picking the portfolio on the basis of the mean-variance criteria does not lead to the maximizing expected utility. The performance of the mean-variance approximation shown in this paper is not much different from the selecting naïve portfolio where investors easily put equal proportion of investment on each asset in their portfolio. Moreover, investors with various utility functions are found to require significant optimization premium to bring up their welfare to the level achieved by holding expected utility maximization portfolio. However, the findings in this study may be subjected to some limitations and it is suggested that further research may be conducted, for example, in the case where many type of assets or larger sample are included in the portfolio.

REFERENCES

[1] J. Kallberg and W. Ziemba, "Comparison of alternative utility functions in portfolio selection problems," Management Science, vol. 29(1), 1983.

[2] Y. Kroll, H. Levy, and H. Markowitz, "Mean-Variance versus direct utility maximization," The Journal of Finance, vol. 39(1), Mar. 1984, pp. 47-61.

[3] H. Levy and H. Markowitz, "Approximating Expected utility by a function of mean and variance," The American Economic Review, vol. 69(3), Jun. 1979, pp. 308-317.

[4] H. Markowitz, "Portfolio selection," The Journal of Economic and Finance, vol. 7(1), Mar. 1952, pp. 77-91.

[5] J. Pratt, "Risk aversion in the small and in the large," Econometrica, vol. 32(1/2), Jan.-Apr. 1964, pp. 122-136.

[6] L. Pulley, "A general mean-variance approximation to expected utility for short holding periods," The Journal of Finance and Quantitative Analysis, vol. 16(3), Sep. 1981, pp. 361-373.

[7] L. Pulley, "Mean-variance approximation to expected logarithmic utility," Operational Research, vol. 31(4), 1983, pp. 685-696.

[8] L. Pulley, "Mean-Variance versus direct utility maximization: a comment," The Journal of Finance, vol. 40(2), Jun. 1985, pp. 601-602.

[9] D. Reid and B. Tew, "Mean-Variance versus direct utility maximization: a comment," The Journal of Finance, vol. 41(5), Dec. 1986, pp.

[10] W. Sharpe, "Expected utility asset allocation," Financial Analysis Journal, vol. 63(5), 2007, pp. 18-30.

[11] Y. Simaan, "What is the opportunity cost of mean-variance investment strategies?," Management Science, vol. 39(5), May 1993, pp. 578-587.

[12] B. Tew, D. Reid, and C. Witt, "The opportunity cost of a mean-variance efficient choice," The Financial Review, vol. 26(1), Feb 1991, pp. 31-43.

[13] W. Young and R. Trent, "Geometric mean approximations of individual security and portfolio performance," The Journal of Financial and Quantitative Analysis, vol. 4(2), 1969, pp. 179-199.