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The Behaviour of Exchange Rates:

A Wavelet Perspective

Yihui Lan, The University of Western Australia
Accounting and Finance Discipline
Business School, UWA
Crawley WA 6009, Australia
e-mail: yihui.lan@uwa.edu.au

***Abstract*—This paper aims to enhance understanding of the behaviour of both nominal and real exchange rates at different horizons. It focuses on the most prominent exchange rate theory, purchasing power parity (PPP), which is tested using wavelet analysis, an engineering tool that has been increasingly applied to economic and financial data since the late 1990s. This paper applies wavelet analysis to PPP using two approaches. The first approach examines the relationship of the two components of PPP -- the nominal exchange rate and relative price. The second approach tests for a unit root of the real exchange rate. The empirical results from both approaches lend support for PPP for the British pound, Japanese yen, and Swiss franc. The estimated length of the long run for these three currencies varies from 1.5 to 5 years. The results for the Canadian dollar and the euro suggest possible long-range dependence.**

Keywords-Purchasing power parity, wavelet analysis, nominal exchange rates, real exchange rates, long run

I. INTRODUCTION

Exchange rates are an area of intense research activity in international economics and many attempts have been made to understand their behaviour, both in the short and long terms. This paper focuses on the most prominent exchange rate theory, purchasing power parity (PPP) theory, which is the fundamental building block of many exchange-rate models. PPP builds on the law of one price. It states that goods and services should cost the same in all countries when measured in a common currency. During the last three decades, there has been a huge and heated debate on the validity of PPP. Since the breakdown of the Bretton Woods system in 1973, the exchange rates in most industrial countries have experienced substantial volatility. PPP almost reached its demise after the publication of Frenkel (1981) and Meese and Rogoff (1983). However, the last two decades has seen a renewed interest in PPP, with the availability of large datasets and employment of advanced econometric methods. There is now mounting evidence supporting PPP; for surveys see, e.g., Froot and Rogoff (1995), Lan and Ong (2003), Rogoff (1996) and Taylor (2006).

Despite the large and growing literature in favor of PPP, there have been considerable controversies surrounding the theory. The first three are summarized in Clements et al. (2009). First, does PPP indeed hold over the long run? Second, if PPP is valid as a theory pertaining to the long run, then exactly how long is the long run? Third, what exactly are the prices to which PPP refers? Finally and most importantly, empirical research on PPP is hindered by the problem of temporal aggregation bias. The main objective of the paper is to test for PPP using wavelet analysis -- a new (and arguably superior) methodology. Being an engineering tool, wavelet analysis found applications in economics and finance in the late 1990s. While PPP has been tested using a variety of econometric methodologies, including panel data, cointegration, nonlinear modeling, etc, this paper, to my knowledge, is the first attempt of applying the wavelet approach to PPP.

As wavelet analysis allows for the decomposition of a signal into multi-resolution components -- both fine and coarse, its application to the tests for PPP has four important desirable features. First, wavelet filtering can approximate not only a stationary time series, but also local details such as a temporary shock, or a structural change that happens at a particular point in time. Second, wavelet analysis is able to recover the original signal by isolating the signal from the noise component. Third, the problem of temporal aggregation bias can be avoided since the data can be decomposed into orthogonal time horizons (scales), such as day by day, month by month, or year by year. Economists have long recognised that there are many different time periods for decision making, whereas analyses are usually restricted to two time horizons, the short run and the long run. Using wavelet analysis enables the investigation of time series data over a spectrum of well-defined time horizons, rather than some subjectively-chosen short run or long run. Finally, the wavelet approach enables the analysis of the variances of the PPP exchange rate over different time horizons. Specifically, using the new technique the paper will answer the first, second and final questions regarding PPP posed in the previous paragraph.

This paper investigates the behavior of both nominal exchange rates and real exchange rates. In its absolute form, PPP states that the nominal exchange rate should be proportional to the ratio of the domestic to the foreign price level. This is the so called stage-one tests coined by Froot and Rogoff (1995), which are flawed by their failure to take into account possible non-stationarities in the series of interest. As mentioned above, wavelet analysis does not require such a strict time series property. Therefore, we can simply run regressions of nominal exchange rates against relative prices at all scales. We also examine the stationarity of real exchange rates, which underlies stage-three tests (Froot and Rogoff, 1995). The difference between the wavelet analysis of real exchange rates and the traditional approach is that the unit-root test is carried out not just at one scale, but at all available scales of the variable.

For empirical analysis, we use the currencies in five major developed countries (area) -- Canada, the euro area, Japan, Switzerland and UK, with the US as the base country. The sample period is the post-Bretton-Woods era of 1973 to 2008. Two approaches based on wavelet analysis are employed. In the first approach, we investigate via a regression framework the relationship between the two components of PPP -- the nominal exchange rate and the relative price between two countries at different time scales. Then tests for PPP are carried out based on the regression results. The second approach to PPP only focuses on one time series variable, the real exchange rate. As the rejection of the unit root in the real exchange rate implies support for PPP, we test for the unit root in the real exchange rate over various time horizons. An important conclusion can be drawn regarding the length of long-run with regard to PPP, which is about one to five years. It is also found that the test results from the second approach are in broad consistency with those from the first approach. This paper is organized as follows. Section 2 briefly introduces wavelet analysis and the two most commonly-used techniques -- discrete wavelet transform and maximum overlap discrete wavelet transform. Section 3 gives the detailed procedure for absolute PPP and relative PPP tests, and the unit root test, using wavelet techniques. The data and empirical results are discussed in Section 4. Section 5 summarizes and concludes.

II. Wavelet Analysis

Generally speaking, wavelets are one type of data filters. To describe data, economists sometimes use data filters to extract signals and eliminate noises. A wavelet is a function similar to sine or cosine in that it also fluctuates about zero. As implied by the name, a wavelet is a small wave. Wavelet analysis involves using wavelet functions to decompose and analyse the fluctuations in a time series variable. This section gives a brief overview of wavelet analysis and mainly draws on Gencay et al. (2002).

The fundamental step in the wavelet analysis is wavelet transform, i.e., the decomposition of a time series variable into high frequency or noisy components and low-frequency or trend components. There are many ways sorting the types of wavelet transform. The division based on the wavelet orthogonality gives the use of an orthogonal basis in the discrete wavelet transform (DWT), and a non-orthogonal basis in the maximal overlap discrete wavelet transform (MODWT), as well as the continuous wavelet transform (CWT).

There are two types of wavelet functions used for decomposition: father and mother wavelets. The father wavelet $\phi(t)$ essentially represents the smooth, trend (low-frequency) part of the series, and is thus a low-pass filter. The mother wavelet $\varphi(t)$ represents the detailed (high-frequency) parts and as such is a high-pass filter. Wavelet transform is a refinement of the classical Fourier transform, a filtering technique in the frequency domain. Different from the Fourier transform, which only summarizes information in the data as a function of frequency and does not preserve information in time, wavelet transform utilizes the *dilation* and *translation* of the mother wavelet $\varphi(t)$ to capture features that are unique along two dimensions: frequency and time. For example, DWT requires a total number of observations $N = 2^J$, where J is a positive integer. For a given number of resolution scales j $(j = 1, ..., J)$, the sequence of wavelet functions $\varphi_{j,k}(t)$ are obtained through *scaling* and *translation* of the mother wavelet $\varphi(t)$ as follows:

$$\varphi_{j,k}(t) = \frac{1}{2^{j/2}}\varphi\left(\frac{t-2^j k}{2^j}\right), \quad j = 1, 2, \cdots, J,$$

where *translation* is represented by the parameter k and *dilation* is represented by 2^{j-1}.

The DWT is implemented via a pyramid algorithm (Mallat, 1989). When a high-pass filter (mother wavelet) is applied to a time series x_t, we obtain the wavelet detail for the time scale $j=1$, denoted as $D_j = \sum_k d_{j,k}\varphi_{j,k}(t)$ with $d_{j,k}$ giving wavelet detail coefficients for level j at location k. Applying a low-pass filter (father wavelet) to x_t gives the wavelet approximation corresponding to the lower-frequency component for the time scale $j=1$, denoted as $A_j = \sum_k a_{j,k}\varphi_{j,k}(t)$, where $a_{j,k}$ gives scale coefficients for level j at location k. If A_j is the coarser scale, then $A_{j-1} = A_j + D_j$. Wavelet transforming a time series x_t is realised through a filter cascade in the form of $x_t = A_1 + D_1 = A_2 + D_2 + D_1 = ...$, etc. The end result of is the so-called multi-resolution analysis (MRA) defined as

$$x_t = A_J + D_J + D_{J-1} + \cdots + D_1 \quad (1)$$

Each of the decomposed components is called a crystal, which is obtained from the convolution of the wavelet coefficients and the relevant filter. To be specific, (1) can be written as

$$\begin{aligned} x_t = &\sum_k a_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\varphi_{J,k}(t) + \sum_k d_{J-1,k}\varphi_{J-1,k}(t) \\ &+ \cdots + \sum_k d_{1,k}\varphi_{1,k}(t) \end{aligned} \quad (2)$$

Note that the approximation component at the chosen level J, A_J, is the sum of all higher-level details components, i.e., $A_J = \sum_{j>J} D_j$. In summary, the MRA provides the decomposition of a time series into several layers of orthogonal sequences of scales. Each scale can be analysed individually. In addition, the relationship of each scale with the corresponding scale from different series can be examined.

Despite its widespread use in many disciplines, the DWT has two major drawbacks: (i) a time series under investigation is required to have a dyadic length (i.e., the sample size must be divisible by 2^J); and (ii) the DWT is not shift invariant, which means that the DWT of a signal and of a time-shifted version of the same signal are not simply shifted versions of each other. In empirical work, the maximal overlap discrete wavelet transform (MODWT) is usually preferred. The MODWT, as its name suggests, computes all possible shifted time intervals and thus the orthogonality of the transform is lost. However, attractions of the MODWT include (i) it provides better resolution than DWT at coarser scales (lower frequencies); and (ii) The MODWT wavelet variance estimator is asymptotically more efficient than the same estimator based on the DWT.

III. TESTING FOR PPP

This section sets out the two approaches to testing for PPP. One is based on the relationship between the two components of PPP -- the nominal exchange rate and relative price. The other is based on the time series properties of the real exchange rate.

There are two versions of PPP: absolute and relative PPP. The former refers to that the exchange rate equals relative price levels between two countries at any time, while the latter states that the change in the exchange rate over any period equals the difference between the changes in the price levels (i.e. inflation differential) in the home and foreign country. The standard PPP model in logarithmic form is

$$s_t = \alpha + \beta r_t + \varepsilon_t , \qquad (3)$$

where s_t is the logarithm of the nominal exchange rate, $r_t = \ln\left(P_t / P_t^*\right)$ is the logarithmic relative price in which P_t and P_t^* are domestic and foreign price levels for an identical basket of goods and services, respectively, α and β are parameters, and ε_t is a random disturbance term. Equation (3) postulates that the exchange rate is proportional to the relative price. Absolute PPP holding requires the coefficients $\alpha = 0$ and $\beta = 1$, while relative PPP requires only $\beta = 1$.

Traditional tests of PPP based on equation (3) are claimed to lead to spurious results due to the possible non-stationarity of the two variables as well as the error term (see, e.g., Froot and Rogoff, 1995). The advantage of wavelet analysis lies in its ability to capture short-lived transitory components of data in short time interval, as well as trend and patterns in longer time intervals, making it ideal for analyzing non-stationary data. As emphasized by Schleicher (2002), since the lowest frequency base scale, A_J, includes any non-stationary components, the data need not be detrended or differenced.

In our first approach of wavelet analysis of PPP, we do not estimate equation (3) using the original time series as in earlier studies of PPP. Instead, we first carry out wavelet decompositions of s_t and r_t using the MODWT:

$$s_t = A_J^s + D_J^s + D_{J-1}^s + \cdots + D_1^s ,$$
$$r_t = A_J^r + D_J^r + D_{J-1}^r + \cdots + D_1^r .$$

Upon obtaining scale coefficients $a_{J,k}$ and detail coefficients $d_{j,k}$ defined before equation (1), we can run J+1 individual regressions based on J-level wavelet decompositions:

$$\begin{aligned} d_j^s &= \alpha_j + \beta_j d_j^r + \varepsilon_j, \qquad j = 1, \ldots, J \\ a_J^s &= \alpha'_J + \beta'_J a_J^r + \varepsilon'_J . \end{aligned} \qquad (4)$$

The above equation is essentially equation (3) with the variables replaced with their respective j^{th} time-scale coefficients. It is to be noted that for MODWT the total number of observations for each level (scale) is equal to the total number of sample periods of the original series. The estimates of α_j and β_j are expected to uncover the relationship between the two series over each time scale. Two F tests for coefficients can be performed for the regression corresponding to each scale: (i) $\alpha_j = 0$ and $\beta_j = 1$ under absolute PPP; and (ii) $\beta_j = 1$ under relative PPP.

Our second approach to PPP is to test the unit root hypothesis of the real exchange rate. As PPP holding corresponds to the stationarity of real exchange rate, rejecting the unit root lends support to PPP. The real exchange rate for a country at time t, denoted as q_t, is defined as $q_t = \ln\left[P_t / \left(S_t P_t^*\right)\right] = s_t - r_t$, which can be interpreted as the inflation-adjusted exchange rate. The procedure is as follows. After applying the MODWT to the real exchange rate q_t, we test each level of the details as well as the chosen-level approximation for the unit root null using two conventional unit root tests, the ADF test and Phillips and Perron (PP) test. One advantage of the PP test over the ADF tests is that the PP tests are robust to serial correlation and/or heteroskedasticity in the error term.

IV. EMPIRICAL RESULTS

Our sample includes Canada, euro area, Japan, Switzerland and United Kingdom, with the US as the base country. The monthly data for nominal exchange rates and price levels over the post-Bretton-Woods period are obtained from the International Financial Statistics CD-ROM published by the International Monetary Fund.

Prior to wavelet analysis of a time series, a number of decisions have to be made: Which family of wavelet filters is to be used, what type of wavelet transform to apply, and how boundary conditions at the end of the series are handled. Different wavelet family make different trade-offs between capturing local details and generating smoothness of the approximation component. As the least asymmetric (LA) families developed by Daubechies (1992) are argued to allow the most accurate alignment in time between wavelet coefficients at various scales and the original series (see, e.g., Gallegati and Gallegati, 2007, Percival and Walden, 2000), we use a LA wavelet function of the width 8 (LA8, or Sym4 in a different notation).

The ordinary-least-squares regression and test results for level-7 decompositions are presented in Table 1. The regression results for the level-J detail or approximation coefficients include (i) Two parameter coefficient estimates and their corresponding p values in parenthesis. (ii) Two F-tests on coefficient restrictions and the p values in parenthesis. The test for $\alpha_j = 0$ and $\beta_j = 1$ is for testing absolute PPP and that for $\beta_j = 1$ relative PPP. (iii) The ADF unit root test statistics for the dependent variable, independent variable, and the regression residual for each level. (iv). R^2 of each OLS regression, which gives the goodness-of-fit of the regression.

For a level-J decomposition, $J+1$ regressions are required to be examined, i.e., J regressions for J details components and 1 regression for the approximation component. For the purpose of illustration, take the example of a level-3 results for Canada in Table 1. As a level-3 wavelet decomposition of a variable x_t takes the form of $x_t = A_3 + D_3 + D_2 + D_1$, we need to look at rows 1 to 8 of the final three columns corresponding to levels $j = 3, 2, 1$ details as well as rows 9-16 of the third last column corresponding to the approximation component (level $J = 3$). In total, $J+1=4$ sets of regression and tests results have to be investigated.

First we take a detailed look at the results for Canada in Table 1. The ADF statistics in rows 5 to 7 are all less than the 1 percent critical value of –3.46, so that the unit root null is rejected for both the dependent and independent variables and the error term of the regression at each detail. This shows that all results for the details regressions in Panel A are valid. The estimates of the intercept in row 1 are all insignificant, as suggested by their large p-values. However, the slope coefficient for all regressions all take values greater than one in absolute terms, so that when we include the coefficient test of $\beta_j = 1$ in the two F-tests, both are rejected; see the close-to-zero p-values of the F-tests in rows 3 and 4. Therefore according to the results from the details regression, neither absolute nor relative PPP is supported for the Canadian dollar, even though the goodness-of-fit of the regression is reasonably good, with the R^2 value ranging from 14 to 27 percent (row 8). Turning now to the approximation regressions. The last column of rows 13 to 15 shows that the level-1 approximation does not pass the ADF test, so that we should disregard this set of results. One prominent feature of the results in Panel B is that the t-tests for the significance of estimates and the F-tests for coefficient restrictions all have p-values of almost zero; see rows 9 to 12. Therefore, even though there is some significant relationship between the exchange rate and relative price, but the relationship does not conform to what PPP theory predicts. In summary, the results in Table 1 do not provide support for PPP.

The results for the remaining countries can be interpreted in a similar way. Table 2 gives a summary of the results for the five countries. The evidence from three out of five countries, Japan, Switzerland and UK, is positive for both versions of the PPP with the length of the long run estimated to be 1.5 to 5 years. PPP is not supported for the Canadian dollar, given the maximum decomposition level used. Regarding the euro, PPP holds in the approximation regressions, but not in the details regressions, suggesting that the two variables under investigation is likely to have long-range dependence.

The above results pertain to the two components of the real exchange rate, the nominal exchange rate and the relative price. Now we investigate just the real exchange rate to shed light on whether PPP holds. After decomposing q_t using the LA8 (namely, Sym4) filters, we apply the ADF and PP tests for the wavelet coefficients at each level. Table 3 presents the test statistics. It can be seen from Panel A that the unit root hypothesis is rejected for all currencies from levels one to five details. Such favorable results to PPP are insensitive to different lag lengths in the ADF regression. It is to be noted that one drawback of the ADF test is that it uses a parametric autoregression to model the ARMA structure of the error term. The PP test, on the other hand, corrects for any serial correlation and heteroskedasticity of the errors, and is thus preferred. The PP test results, contained in Panel B, show that the unit root is rejected for all five currencies from level one to three approximations. This suggests that stationarity is present at higher levels (lower frequencies). For the levels four to seven details components of the real exchange rate, the unit root is rejected at the 1 percent level, except for Canada UK at the level five details. This shows that PPP is strongly supported for the four currencies from the horizon of 16 months and longer, while for the Canadian dollar, the length of the long-run is more than five years.

V. SUMMARY AND CONCLUSION

Wavelet analysis has found numerous applications in the economics and finance since the late 1990s. Wavelet filters, unlike conventional filters, have a number of attractions in that (i) they are ideal for analyzing non-stationary data; (ii) the high-frequency noise and low-frequency signal are easy to be isolated; (iii) the horizon under investigation is precise and well-defined; and (iv) the time (location) of an event such as a structural break is easy to be detected, etc. This paper applied wavelet analysis to PPP from two perspectives. One approach examined the relationship of the two components of PPP -- the nominal exchange rate and relative price. The other tested for a unit root in the real exchange rate, as PPP holding corresponds to the real exchange rate following a stationary process. Through wavelet decompositions, we either investigated the relationships between the two variables at different time scales, or carried out tests regarding the required hypothesis testing. Both sets of the empirical results lead to decisive support for PPP for three currencies, Japanese yen, the Swiss franc and the UK pound sterling. The results for the other two currencies from the first approach are not conclusive, although the evidence from the second approach is positive. The euro, in addition, suffers from the problem of a short sample period. It seems that there is possible long-range dependence in the behavior of these two currencies.

With the growing support for PPP, the more recent focus of research has moved to the issue of what constitutes the length of the long run, that is, the period over which exchange rates fully adjust to shocks to relative prices. The general consensus reported in the literature is that the long run is estimated at about 3-5 years. However, the classical approach to PPP is prone to the criticism that the differencing horizon is subjectively chosen. The wavelet method, on the other hand, allows a systematic time scale-by-scale analysis, and thus offers a more powerful tool for analyzing a single time series, and the relationship between two series at different and precise horizons. The estimated length of the long run in this paper is about 1.5 to 5 years.

Several limitations of this study remain. Like in many other PPP studies, the validity of PPP in this paper largely depends on the countries under investigation. The sample period and frequency of data used may also play a role. It is widely agreed that PPP is more easily accepted for developed than for developing countries, especially for the post-Bretton-Woods period. Whether PPP holds, and what approach leads to a better estimate of PPP long run, are still the object of debate. Nonetheless, the application of wavelet analysis provides a fresh look into PPP and an alternative re-assessment of its validity over well-defined horizons.

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TABLE I. REGRESSIONS OF EXCHANGE RATES ON RELATIVE PRICE FOR INDIVIDUAL TIME SCALES: CANADA

	Regression results and tests	Level 7 (128-256 mths)		Level 6 (64-128 mths)		Level 5 (32-64 mths)		Level 4 (16-32 mths)		Level 3 (8-16 mths)		Level 2 (4-8 mths)		Level 1 (2-4 mths)	
								A. Details							
1.	α_i	11.87	(55)	3.20	(58)	2.57	(35)	0.45	(71)	0.30	(64)	0.02	(95)	-0.06	(71)
2.	β_i	-6.75	(0)	-6.27	(0)	-7.10	(0)	-7.22	(0)	-5.35	(0)	-4.02	(0)	-3.77	(0)
3.	$F(\alpha_i=0,\beta_i=1)$	55.23	(0)	97.07	(0)	75.23	(0)	104.70	(0)	64.41	(0)	61.10	(0)	56.81	(0)
4.	$F(\beta_i=1)$	93.42	(0)	190.61	(0)	147.14	(0)	207.76	(0)	128.22	(0)	122.17	(0)	113.48	(0)
5.	ADF(d_i^s)	-33.21		-27.47		-28.17		-29.94		-33.91		-29.69		-11.92	
6.	ADF(d_i^r)	-22.27		-31.76		-17.72		-23.29		-30.40		-31.53		-24.42	
7.	ADF(ε_i)	-42.31		-28.85		-30.73		-31.27		-33.35		-29.03		-16.04	
8.	R^2	0.14		0.25		0.21		0.27		0.18		0.16		0.14	
								B. Approximation							
9.	α_i	1,705	(0)	1,231	(0)	983.02	(0)	729.91	(0)	527.65	(0)	377.07	(0)	268.07	(0)
10.	β_i	9.25	(0)	6.14	(0)	3.04	(0)	2.01	(0)	1.71	(0)	1.60	(0)	1.54	(0)
11.	$F(\alpha_i=0,\beta_i=1)$	3,046	(0)	1,261	(0)	2,646	(0)	5,064	(0)	9,271	(0)	15,596	(0)	23,534	(0)
12.	$F(\beta_i=1)$	322.66	(0)	33.84	(0)	13.89	(0)	8.10	(0)	8.13	(0)	10.50	(0)	13.07	(0)
13.	ADF(d_i^s)	-24.37		-35.92		-16.25		-15.46		-10.03		-4.09		0.50	
14.	ADF(d_i^r)	-23.15		-9.62		-6.30		-4.60		-2.76		-3.12		-2.71	
15.	ADF(ε_i)	-48.94		-33.92		-15.78		-16.40		-11.65		-5.22		0.00	
16.	R^2	0.49		0.10		0.07		0.07		0.10		0.15		0.20	

Notes:
1. For a level-n decomposition, n+1 regressions need to be examined, i.e., n regressions for details and 1 regression for approximation. For illustration purpose, take the example of a level-3 results for Canada. In Panel A, we need to look at rows 1 to 8 of all last three columns corresponding to level 1-3 details as well as the rows 9-16 of the third last column corresponding to the approximation component. In total, we have n+1=4 sets of regression and tests results.
2. The regression results for the level-n detail or approximation coefficients include two parameter coefficient estimates and their corresponding p values in parenthesis, two F-tests on coefficient restrictions and the p values in parenthesis, the ADF unit root test statistics for the dependent variable, independent variable, and the regression residual for each level, and R^2 of the regression.
3. The F test of α =0, β =1 is the test for absolute PPP, and that of β =1 is for relative PPP.
4. The 1%, 5% and 10% critical values for the ADF test are -3.46, -2.87 and -2.59, respectively, for which we use “***”, “**” and “*” next to an ADF statistic to denote the corresponding significance.

TABLE II. SUMMARY OF THE WAVELET ANALYSIS OF THE RELATIONSHIP BETWEEN THE NOMINAL EXCHANGE RATE AND RELATIVE PRICE

Country	Sample period	Does absolute PPP hold?	If yes, at what level?	If yes, at what horizon?	Does relative PPP hold?	If yes, at what level?	If yes, at what horizon?
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Canada	1973M1-2008M6	No	-	-	No	-	-
Euro area	1994M3-2008M6	No	-	-	Yes	> 7	> 10 years
Japan	1973M1-2008M6	Yes	3, 4	1~3 years	Yes	3, 4	1~3 years
Switzerland	1973M1-2008M6	Yes	1 ~ 4	< 1 year	Yes	1 ~ 4	< 1 year
UK	1973M1-2008M6	Yes	4 ~ 6	1.5 ~ 5 years	Yes	5 ~ 6	3 ~ 5 years

TABLE III. UNIT ROOT TESTS OF THE DECOMPOSED REAL EXCHANGE RATE

Country	D or A?	Level 7 (128-256 mths)	Level 6 (64-128 mths)	Level 5 (32-64 mths)	Level 4 (16-32 mths)	Level 3 (8-16 mths)	Level 2 (4-8 mths)	Level 1 (2-4 mths)
A. ADF test statistics								
Canada	D	-2.71*	-3.41**	-6.57***	-8.50***	-10.22***	-25.14***	-25.44***
	A	-1.29	-1.69	-1.52	-1.90	-1.78	-1.70	-2.39
Euro area	D	-3.50***	-4.76***	-6.07***	-8.46***	-10.21***	-25.09***	-25.59***
	A	-1.16	-1.93	-1.54	-1.51	-1.47	-1.49	-2.06
Japan	D	-3.07**	-3.36**	-6.57***	-8.56***	-10.23***	-25.07***	-25.56***
	A	-1.45	-1.83	-1.63	-1.90	-1.78	-1.80	-2.50
Switzerland	D	-2.38	-3.84***	-7.31***	-7.87***	-10.26***	-25.78***	-25.27***
	A	-1.74	-1.76	-2.06	-2.46	-2.51	-2.17	-3.16**
UK	D	-3.03**	-4.45***	-6.46***	-8.78***	-10.13***	-24.96***	-25.85***
	A	-1.70	-1.87	-2.20	-2.53	-2.32	-2.29	-3.24**
B. PP test statistics								
Canada	D	-4.38***	-21.64***	11.47	-37.99***	36.16	163.15	1354.10
	A	-25.00***	-5.08***	-24.24***	-5.72***	-36.60***	-9.47***	-37.22***
Euro area	D	-3.69***	-9.54***	-3.88***	-22.69***	2.55	33.40	462.48
	A	-15.10***	-3.99***	-8.84***	-6.70***	-18.36***	-8.24***	-19.37***
Japan	D	-4.70***	-11.63***	-4.56***	-20.05***	-1.73	16.47	336.90
	A	-10.16***	-4.61***	-10.10***	-6.71***	-16.09***	-8.15***	-17.65***
Switzerland	D	-20.55***	-27.59***	-54.89***	-82.26***	133.34	519.35	3631.20
	A	-46.83***	-23.46***	-35.33***	9.77	-72.07***	-11.49***	-98.39***
UK	D	-20.28***	-32.80***	-39.49***	-61.76***	116.57	489.52	3813.70
	A	-89.28***	-25.62***	-45.39***	-2.59	-84.91***	-14.97***	-70.40***

Notes: 1. In column 2, “D” refers to details and “A” approximation.
2. The 1%, 5% and 10% levels of significance are indicated by “***”, “**” and “*”, respectively, next to a test statistic.