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Pricing Bidirectional hindsight Options in Binomial CEV Model

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[1]*Abstract*—**Bidirectional hindsight options are a kind of exotic options. In the constant elasticity of variance (CEV) model, a combining binomial tree was built up to approximate the non-constant volatility that is a function of the underlying asset. On this basis, a simple and efficient recursive algorithm was developed to compute the risk-neutral probability of each different node for the underlying asset reaching maximum or minimum price and the total number of maxima (minima) in the binomial tree. With help of it, the computational problems can be effectively solved arising from the inherent complexities of different types of bidirectional hindsight options when the underlying asset evolves as the CEV model. Numerical results demonstrate the validity and the convergence of the binomial approach for the different parameter values set in CEV model.**

Keywords -CEV mode ; ;binomia ltree; recursive algorithm;bidirectional hindsight options

I. INTRODUCTION

Cox(1975)[1] developed constant elasticity of variance (CEV) model by observing that the asset price behavior was often affected by volatility smile effect in the market reality and the origin of the volatility smile was the negative correlation between asset price changes and volatility changes. Beckers（1980）[2]andDavil（1982）[3] MacBeth , Merville (1980) [4] and Emanuel, Macheth. (1982) [5] provided theoretical arguments for and empirical evidences for the validity of CEV model that better described the evolution of asset price compared with the well-known Black-Scholes model [5]. Hence it is instructive to apply CEV model to option pricing.

The problem of pricing a standard European option when the underlying asset value is driven by a CEV model, was solved by Cox and Ross (1976) [7]who derived an analytical solution for the option value. Things are more complicated in the case of exotic options whose analytical solution is not available and numerical approximations must be used. Boyle and Tian(1999)[8],[9] used Monte Carlo simulations. Subsequently, he extended Babbs method to construct a trinomial tree for evaluating the barrier and lookback options under the CEV model. Davydov and Linetsky(2001) [10]derived the close-form formulae of barrier and lookback options under the CEV model with help of the numerical inversion of the Laplace transform of the option price following ordinary differential equation. Bin Peng and Fei Peng [11]applied the intuition binomials tree model to price Asian options under the constant elasticity of variance. Among these numerical techniques, the binomial tree approach plays a remarkable role both for its simple structure and implementation in solving the complex pricing problem of exotic options under the CEV model. However, in the framework of the binomial tree to approximate the constant elasticity of variance model, the transition probabilities are no longer constant over the tree. This property introduces a further complication in the evaluation process. In addition, there is little work on bidirectional hindsight options.

The objective of this paper is to study the application of the binomial tree approach to bidirectional hindsight options when the stock price evolves as a CEV process. Especially, .recursive algorithm based on a forward induction procedure is developed to compute the risk-neutral probability of each different payoff node in the binomial tree for bidirectional hindsight..The remainder of the paper is organized as follows. Section 2 illustrates the binomial method used to approximate the CEV model under the equivalent martingale measure. In section 3 the binomial approximation is used to price bidirectional hindsight options and numerical results are given. Conclusions are presented in the final section.

II. A BINOMIAL CEV MODEL

It is known that Black-Scholes model with costant volatility does not hold empirically for asset prices. Alternative stochastic models have been studied and applied to option pricing. For example, Cox(1975) and Davil(1982) study a general class of stochastic model known as the Constant Elasticity of Variance (CEV）diffusion model in a risk-neutral world.

$$dS = rSdt + \sigma SdB; \quad \sigma = \delta S^{0.5a-1} \tag{1}$$

where B is a standard Wiener process under the Q-measure，r is riskless rate of interest, the standard deviation of return σ is a function of the underlying price instead of a constant δ and a are constants, $0 \le a < 2$.

We consider a discrete approximation for the asset price evolution described in (1) using the binomial tree. Let n be the number of time intervals between the time t and the maturity T, $[t,T]$ is divided into n equal pieces, each of width Δt. The underlying asset may move up a level, down a level. In the case of the asset price dynamics driven by CEV model, the volatility is not a constant but varies with the level of the

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underlying price. This implies that the approximation binomial tree is non-recombining and the number of nodes produced at each vertical layer is 2^i i=0,,,,,,,,n, thus computational complexity becomes unmanageable even with a small number of time steps. In order to effectively solve the computational problems arising from the inherent complexities of the constant elasticity of variance model, we need to transform the variable S governed by (1) with non-constant volatility so that the transformed process has constant volatility. Considering the transformed process

$$X_t = S^{1-0.5a} /(1-0.5a)\delta$$

using the Ito's Lemma, we have:.

$$\begin{aligned} dX &= \frac{\partial X}{\partial S}dS + \frac{\partial X}{\partial t}dt + \frac{1}{2}\frac{\partial^2 X}{\partial S^2}(\delta S^{0.5a})^2 dt \\ &= \frac{S^{-0.5a}}{\delta}dS - \frac{1}{2}\frac{0.5aS^{-0.5a-1}}{\delta}(\delta S^{0.5a})^2 dt \qquad (2) \\ &= \frac{S^{-0.5a}}{\delta}dS - \frac{0.5a\delta S^{0.5a-1}}{2}dt \end{aligned}$$

Substituting equation (1) into the above equation and using the fact that $S = [\delta(1-0.5a)X]^{1/(1-0.5a)}$. Equation (2) becomes:

$$dX = \left\{ r(1-0.5a)X - \frac{a^2}{4(1-0.5a)X} \right\} dt + dB \qquad (3)$$

Now, it is easy to build up a computationally simple binomial tree to approximate the X-process. The value X of the process at time t, after one period at time t+1, can rise to $X_t + \sqrt{\Delta t}$, or decrease to $X_t - \sqrt{\Delta t}$. Continuing in this way, we see that the value of the binomial X-process is equal to

$$X^j_{t+i\Delta t} = X_t + (2j-i)\sqrt{\Delta t}\,, \qquad (4)$$

where $X^j_{t+i\Delta t}$ represents the value of the binomial X-process at time $t+i\Delta t$ after j up steps and i-j down steps, and i=0,1,………,n, j =0,1,…….,i.

Given the values on the X lattice, we can recover the dynamics of the underlying price on its lattice. As before, consider the value $S_t{}^j$ of the process at time t, after one period at time t+1, can rise to $S^{j^+}_{t+ilt} = f(X_{tt} + \sqrt{\Delta t}\,)$, or decrease to $S^{j^-}_{t+ilt} = f(X_{tt} - \sqrt{\Delta t}\,)$ Continuing in this way, we note that the value of the binomial approximating S-process at time $t+i\Delta t$ after j up steps and i-j down steps. is expressed as follows:

$$S^j_{t+i\Delta t} = f(X^j_{t+i\Delta t}) = f(X_t + (2j-i)\sqrt{\Delta t}\,) \qquad (5)$$

Once we have developed the binomial tree that approximates the S-process, it remains to compute the probabilities of an upward and downward move for the evolution of the asset price in the binomial tree. The probability of an upward move with the current underlying price S_tj depends on the up level $S^+{}_{t+1}$ and down level $S^-{}_{t+1}$ at the next period time. Thus the probability that the underlying asset with price $\mathrm{S}^{\mathrm{j}}_{\mathrm{t+i\Delta t}}$ makes an up move is

$$p^j_{t+i\Delta t} = \frac{S^j_{t+i\Delta t}e^{r\Delta t} - S^{j^-}_{t+(i+1)\Delta t}}{S^{j^+}_{t+(i+1)\Delta t} - S^{j^-}_{t+(i+1)\Delta t}} \qquad (6)$$

Obviously, the probability that the underlying asset with price S makes down move is 1-p. The above calculation of the transition probabilities represents legitimate probabilities which allows for multiple moves in the approximating binomial S-process. This is important because, in the region near to S=0, the magnitude of each move could be very small and , as a consequence, the transition probabilities could exceed one or smaller than zero. With the definition biven above p represents a legitimate probability for the evolution of the underlying price in the approximating binomial tree.

III. Pricing Bidirectional Hindsight Options

We now apply the binomial CEV model depicted in above section to pricing the bidirectional hindsight options Invoking the no-arbitrage principle, the value at time t of bidirectional hindsight options is given by discounting at the risk-free interest rate the sum of all the option payoffs multiplied by the corresponding probability of occurring. Thus we need to see the .payoff corresponding to each node of the tree at maturity. For floating striking bidirectional hindsight options, its payoff at maturity is equal to the difference between the current underlying asset and the minimum price registered by the underlying asset during the option life time plus the difference between the maximum underlying asst price registered during the option life time and current asset price. Conversely, the fixed strike bidirectional hindsight options pay off the maximum between zero and the difference between the maximum underlying asset price and a fixed strike price plus the maximum between zero and the difference between a fixed strike price and the minimum asset price. However, things are complicated by the fact that, to each terminal node of the tree correspond, in general, different values of the option payoff. This is because, even when some trajectories of the underlying asset price end with the same terminal value, they could have registered a different minimum and maximum price during the option lifetime

In the framework of the binomial CEV model developed by section 2 we see that the transition probabilities are not constant over the tree, and as a consequence, all the paths of the underlying asset with the same terminal value may have the different probabilities of occurring. In this framework, the evaluation problem is not solved even if we can compute the number of paths with the same terminal value but with a different minimum or maximum price registered during the option life. This property introduces a further complication in the evaluation process. To copy with this problem, we present a recursive algorithm based on a forward induction procedure that allows to simply compute the risk-neutral probability of each different payoff of the exotic options at maturity in the binomial CEV model. The algorithm works as follows: at first, we write (i, j) for the binomial tree where the underlying asset has a price $S^{j}_{t+i\Delta t}$ (i=0,1,….,n; j=0,1,…..,i). Note that i-2j is the difference between the down steps and the up steps taken by the underlying asset price Further, we use index k and h respectively to specify the lowest and highest layer of horizontal nodes reached by the underlying asset price after i time steps. In other words, the asset with current price $S^{j}_{t+i\Delta t}$ has registered minimum price equal to $S^{0}_{t+k\Delta t}$,*and* has registered maximum price equal to $S^{h}_{t+h\Delta t}$ we use the triplet (i ,j, k) and (i ,j, h) to respectively specify the state of the world in which the underlying asset reached a minimum price equal to $S^{0}_{t+k\Delta t}$ and a maximum price equal to $S^{2h}_{t+h\Delta t}$,.over a path with j up steps and i-j down steps. Clearly, given i and j, the value of the index k ranges over the interval $[max(i-2j,0),i-j]$; and the value of the index h ranges over the internal $[max(2j-i,0),j]$. With help of these important relations given in Pressacco.Santanrossa and Bianchi (1994)[12]. we can identify all the different minimum prices, $S^{0}_{t+k\Delta t}$ and all the different maximum prices, $S^{h}_{t+h\Delta t}$ registered by the underlying asset price that has a current value $S^{j}_{t+i\Delta t}$, as is illustrated in Figure.1 for the case when the time step n=4. we use the bold to denote (i,j,h),the arrow node denote i-2j<0, and the solid dot node denote i-2j>0 noting that the state of nature (i,j,k) is equivalent to (i,j,h) when i= 2j.

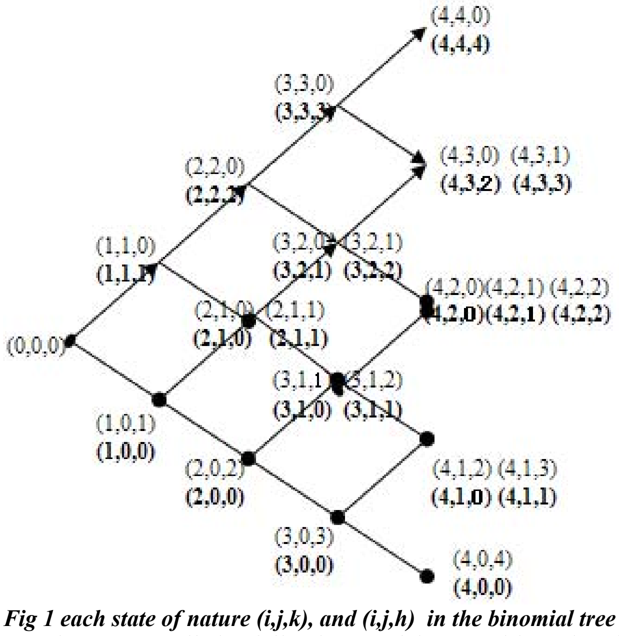

Fig 1 each state of nature (i,j,k), and (i,j,h) in the binomial tree

Then we recall the following useful proposition already proven in Massimo Costbile (2006)[13]

Proposition 1: in two case of k=i−2j and k≠i−2j, the probability of each state of nature represented by the triplet (i ,j, k) , respectively, is

$$prob(i,j,k)=$$
$$\begin{cases} [prob(i-1,j,k)+prob(i-1,j,k-1)]\times q^{j}_{t+(i-1)\Delta t}; k=i-2j \\ prob(i-1,j,k)\times q^{j}_{t+(i-1)\Delta t}+prob(i-1,j-1,k)\times p^{j-1}_{t+(i-1)\Delta t}; \\ i-2j<k\le i-j \end{cases}$$

Where $q^{j}_{t+(i-1)\Delta t}$ represents the probability that the last time step taken by the underlying asset price $S^{j}_{t+i\Delta t}$ is a down step, and $p^{j-1}_{t+(i-1)\Delta t}$ represents the probability that the last time step taken by $S^{j-1}_{t+i\Delta t}$ is up step, which can be calculated by the equation (6)

From the proposition 1, we can see that the last time step of the asset price must be a down step and the minimum may be reached with the last time step when $S^{0}_{t+k\Delta t}$ is equal to the minimum value of the asset price, i.e. k=i−2j. On the contrary, when the minimum underlying asset price is greater than the current asset price, i.e, k>i-2j, the last time step of the asset price may be a down or an up step and the minimum may be reached with the last time step. Further, the recursive algorithm assigns the risk-neutral probability 1 to the state of nature (0,0,0) and a risk-neutral probability 0 to the states of natures that can never occur, i.e. when k<0 or k>i−j. thus in light of figure 1, we can derive the general expression for the probability reaching the each state of nature (i,j,k).if $i-2j<k<i-j$

$$prob(i,j,k)=\prod_{g=0}^{k}q^{0}_{t+(g-1)\Delta t}\{\prod_{\substack{g=k\\ l=0}}^{\substack{j+k-1\\ j-1}}p^{l}_{t+g\Delta t}\prod_{g=j+k}^{i-1}q^{j}_{t+g\Delta t}$$
$$+\prod_{g=j+k+1}^{i-1}q^{j}_{t+g\Delta t}\sum_{m=1}^{j-1}[\prod_{g=k}^{m}p^{g-k}_{t+g\Delta t}q^{m}_{t+(m+k)\Delta t}\prod_{\substack{g=m+k+1\\ l=m}}^{\substack{j+k\\ j-1}}p^{l}_{g}] \quad (7)$$
$$+\sum_{m=1}^{j-1}[\prod_{g=k}^{m}p^{g-k}_{g}q^{m}_{m+k}\prod_{l=m+k}^{j-1}q^{l}_{t+2l\Delta t}\prod_{s=m}^{j-1}p^{s}_{t+(2s+1)\Delta t}]\};$$

And $k\neq i-2j$

$$prob(i,j,k)=\prod_{g=0}^{k}q^{0}_{t+(g-1)\Delta t}\{\prod_{\substack{g=k\\ l=0}}^{\substack{j+k-1\\ j-1}}p^{l}_{t+g\Delta t}\prod_{g=j+k}^{i-1}q^{j}_{t+g\Delta t}$$
$$+\prod_{g=j+k+1}^{i-1}q^{j}_{t+g\Delta t}\sum_{m=1}^{j-1}[\prod_{g=k}^{m-1}p^{g-k}_{t+g\Delta t}q^{m}_{t+(m+k)\Delta t}\prod_{\substack{g=m+k+1\\ l=m}}^{\substack{j+k\\ j-1}}p^{l}_{g}] \quad (8)$$
$$+q^{j}_{t=(i-1)\Delta 他}\sum_{m=1}^{j-1}[\prod_{g=k}^{m}p^{g-k}_{g}q^{m}_{m+k}\prod_{l=m+k}^{j-1}p^{ll}_{t+2s\Delta tt}\prod_{s=m+1}^{j-1}q^{s}_{t+(2s+1)\Delta t}]\};$$

It is worth noting that in the particular situation k=i−j, and i−2j=k≤j, the probability of the state of nature (i,j,k) is given by

$$prob(i,j,k)=\prod_{g=0}^{k-1} q_g^0 \prod_{\substack{g=k\\ l=0}}^{\substack{i-1\\ j-1}} p_g^l \text{ , for } k=i-j \quad (9)$$

$$prob(i,j,k)=\prod_{g=0}^{k-1} q_g^0 \prod_{\substack{g=k\\ l=0}}^{\substack{j+k-1\\ j-1}} p_g^l \prod_{g=j+k}^{i-1} q_g^j \text{ , for } i-2j=k\le j (10)$$

As before we have the following proposition.

Proposition2: in two case of $h= 2j- i$ and $h\neq 2j- i$ .the probability of each state of nature represented by the triplet (i ,j,h) , respectively, is

$$prob(i,j,h)=\begin{cases} [prob(i-1,j-1,h-1)+prob(i-1,j-1,h)]\times p_{t+(i-1)\Delta t}^{j}; \\ h=2j-i \\ prob(i-1,j,h)\times q_{t+(i-1)\Delta t}^{j}+prob(i-1,j-1,h)\times p_{t+(i-1)\Delta t}^{j-1}; \\ 2j-i<h\le j \end{cases}$$

Where $q_{t+(i-1)\Delta t}^{j}$ and $p_{t+(i-1)\Delta t}^{j}$ represent the probability that the last time step taken by the underlying asset prive $S_{t+i\Delta t}^{j}$ is a down and up step.

Analogously, we can see that the last time step of the asset price must be an up step and the maximum may be reached with the last time step when $S_{t+h\Delta t}^{h}$ is equal to the maximum value of the asset price, i.e. $k= 2j- i$. On the contrary, when the maximum underlying asset price is greater than the current asset price, i.e, $h>2j-i$, the last time step of the asset price may be a down or an up step and the maximum may be reached with the last time step. Further, the recursive algorithm assigns the risk-neutral probability 1 to the state of nature (0,0,0) and a risk-neutral probability 0 to the states of natures that can never occur, i.e. when $h<0$ or $h>j$. thus in light of figure 1, we can derive the general expression for the probability reaching the each state of nature (i,j,h). if $2j-i<h<j$

$$\begin{aligned} prob(i,j,h)=&\prod_{g=0g}^{h-1} p_{t+(g-1)\Delta t}^{g-1}\prod_{g=h}^{i} q_{t+g\Delta t}^{j-1}\prod_{\substack{r=j+1\\ s=h}}^{\substack{i-1\\ j-1}} p_{t+r\Delta t}^{s}+\prod_{g=0}^{h} p_{t+(g-1)\Delta t}^{g-1} \\ &\{\prod_{g=j+h+1}^{i-1} p_{t+g\Delta t}^{j-1}\sum_{m=j-h}^{j}[(\prod_{g=l}^{m-1} q_{t+g\Delta t}^{h})p_{t+m\Delta t}^{h}\prod_{g=m+1}^{j+l} q_{t+s\Delta t}^{j-l}] \\ &+\sum_{m=j-h}^{j}[(\prod_{g=h}^{m-1} q_{t+g\Delta t}^{h})p_m^h\prod_{s=m}^{j-1} p_{t+2s\Delta t}^{s}\prod_{s=m}^{j} q_{t+(2s-1)\Delta t}^{s}] \end{aligned} \quad (11)$$

It is also worth noting that in the situation $h=j$, and $i-j<h=2j-i$, the probability of the state of nature (i,j,k) is given by

$$prob(i,j,h)=\prod_{g=0}^{h-1} p_{t+g\Delta t}^{g}\prod_{g=h}^{i-1} q_{t+g\Delta t}^{h} \text{ , } h=j \quad (12)$$

$$prob(i,j,h)=\prod_{g=0}^{h-1} p_{t+g\Delta t}^{g}\prod_{g=h}^{i-1} q_{t+g\Delta t}^{g}\prod_{g=j}^{i-1} p_g^h \quad i-j<h=2j-i (13)$$

And if $h=2j-i$

$$\begin{aligned} prob(i,j,h)=&\prod_{g=0g}^{h-1} p_{t+(g-1)\Delta t}^{g-1}\prod_{g=h}^{j-1} q_{t+g\Delta t}^{h}\prod_{\substack{r=j\\ s=h}}^{\substack{i-1\\ j-1}} p_{t+r\Delta t}^{s} \\ &+(\prod_{\substack{r=j+h+1\\ s=h}}^{\substack{i-1\\ j+h-1}} p_{t+r\Delta t}^{s})q_{t+j\Delta t}^{h+1}\sum_{m=1}^{j-1}[p_{t+m\Delta t}^{h}\prod_{g=0}^{m-1} q_{t+g\Delta t}^{h} \\ &+\prod_{s=j+h-1}^{j} p_{t+(2s-1)\Delta t}^{s}\sum_{m=h}^{j-1}[p_{t+m\Delta t}^{h}\prod_{g=0}^{m-1} q_{t+g\Delta t}^{h}\prod_{s=m}^{j-1} q_{t+2s\Delta t}^{s}] \end{aligned} \quad (14)$$

Once we compute the risk-neutral probability of eaxh state of nature at maturity, (n ,j ,k) and (n ,j, h) ,j=0,1,….n, $k=max(i-2j,0),......., \ i-j$ $\quad h=max(2j-i,0),........ j$; we are ready to evaluate the bidirectional hindsight options since to each possible state of nature at maturity (n,j,k) and (n, j, h) are respectively associated the option payoff $S_{t+n\Delta t}^{j}-max(S_m,S_{t+k\Delta t}^{0})$ and $max(S_M,S_{t+h\Delta t}^{h})-S_{t+n\Delta t}^{j}$; hence , the price at time t of a floating strike option is

$$\begin{aligned} FC=&\sum_{j=9}^{n}\{\sum_{k=max(n-2j,0)}^{n-j}\{prob(n,j,k)[S_{t+n\Delta t}^{j}-max(S_m,S_{t+k\Delta t}^{0}]\} \\ &+\sum_{h=max(0,2j-i)}^{j}\{prob(n,j,h)[max(S_M,S_{t+h\Delta t}^{h})-S_{T+n\Delta t}^{j}\}\} \end{aligned} \quad (15)$$

And the fixed strike option is

$$\begin{aligned} fC=&\sum_{j=9}^{n}\{\sum_{k=max(n-2j,0)}^{n-j}\{prob(n,j,k)[K-S_{t+k\Delta t}^{0}]\} \\ &+\sum_{h=max(0,2j-i)}^{j}\{prob(n,j,h)[S_{t+h\Delta t}^{h}-K\}\} \end{aligned} \quad (16)$$

K is the striking price

Finally we turn to the evaluation of the computational cost of the recursive algorithm for pricing bidirectional hindsight options in binomial CEV model. we determine the number of possible minima or maxima reached by the underlying asset at each node $(i,$ j) (i=0,1,….,n; j=0,…….,i) of the binomial tree. Without loss of generality, we consider a binomial tree with an even number, n, of time steps. We need distinguish two cases. The first considers the states of nature characterized by the triplet (i,j,k) and (i,j,h) such that $i-2j\geq 0$. In Fig 1, we can easily observe that each node reached by a trajectory with j up steps or i-j up steps is characterized by j+1 minima or maxima in the case of nature states characterized by the triplet (i,j,k) and (i,j,h) such that $i-2j\geq 0$. Since in the tree there exist $n-2j+1$ nodes characterized by j+1different minima or maxima, and these nodes correspond to(n−2j+1)*(j+1) minima or maxima. The second case considers the states of nature characterized by the triplet (i,j,k) and (i,j,h) such that $i-2j<0$, in the tree there exist $2j-n$ nodes characterized by$i$- j+1different minima or maxima, and these nodes correspond to $(2j-n)*(i-j+1)$ minima or maxima. Thus the total number of minima or maxima in a binomial tree with n time steps is equal to

$$\sum_{j=0}^{i/2}(i-2j+1)(j+1)+\sum_{j=i/2+1}^{i}(i-j+1)(2j-i)$$
$$=\frac{n^3}{12}+\frac{7}{8}n^2+\frac{11}{12}n+1 \quad (17)$$

To compute the risk-neutral probability of each minimum or maximum in the tree, according to the forward induction procedure developed above, we have to implement two multiplications and one addition, hence, $n^3/6+7n^2/4+11n/6$ multiplications and $n^3/12+7n^2/8+11n/12$ additions(the node (0.0.0) has a risk-neutral probability equal to 1). Moreover, to the risk-neutral probability of each possible minimum price at the last time step, n, we have to associate the corresponding option payoff at maturity and then sum them up.

Table 1 and table 2 illustrate the numerical results of the algorithm described above for evaluating floating strike and fixed strike bidirectional hindsight options in the binomial CEV model. The focus of the table is to check the algorithm performance and convergence. we choose the parameters: S_t=100, T=6 months, the annualized risk-free interest rate r equals 10% and the volatility of the underlying asset rate of return is 25% per annum. the strike price may be 90, 100, or 110, the current minimum value of the underlying asset may be 90, 95, or 100and the current maximum value may be 100, 105, or 110. The number of time steps n used ranges among 100, 250 and 5000.the elasticity factor a=0.5, 1 or 1.5.

INSERT TABLE 1

IV. CONCLUSIONS

A binomial tree approximation for the CEV model was constructed to describe the evolution of the underlying asset with non-constant volatility. Once the binomial tree has been built up, we proposed a simple and efficient algorithm based on forward induction scheme to calculate transition probability of possible minimum or maximum reached by the underlying asset at each node over the binomial tree when volatility varied with asset price level, further we use it to value different types of bidirectional hindsight options. Numerical results show that the recursive algorithm has satisfactory convergence and produces accurate prices for a wide range of parameter values of the binomial CEV model compared with other numerical methods such as close-form solution method. In addition, for bidirectional hindsight options that depend on the extreme of the process, the prices are quite sensitive to the specification of the process.

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TABLE I. **VALUATION OF BIDIRECTIONAL HINDSIGHT OPTIONS IN THE BINOMIALCEV MODEL**

	Floating strike option					fixed strike option			
n	**S_M**	**S_m**	***a*=0.5**	***a*=1**	***a*=1.5**	**K**	***a*=0.5**	***a*=1**	***a*=1.5**
100	100	90	30.1459	30.2299	30.3291	90	20.3697	20.3937	20.4647
	105	95	28.6135	28.7147	28.8511	100	19.0725	19.2022	19.3697
	110	100	29.1786	29.2291	29.3975	110	18.1462	18.1375	18.4731
250	100	90	30.1452	29.7188	30.3438	90	20.3734	20.3954	20.4681
	105	95	28.6128	28.7143	28.8548	100	19.0827	19.2009	19.3722
	110	100	29.1399	29.2511	29.4146	110	18.1439	18.1299	18.4724
5000	100	90	30.1437	30.2185	30.3107	90	20.3723	20.3945	20.4644
	105	95	28.6078	28.7071	28.8355	100	18.9898	19.1925	19.3707
	110	100	29.1232	29.2486	29.4063	110	18.1405	18.1285	18.4699