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Some Risk Models for Loss Assessment Due to Natural Disasters

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Abstract. Various risk situations which describe the possible financial losses of the monitored objects due to occurrence of natural disasters, are considered. The main components of the risk assessment models are discussed: the number of natural disasters (adverse events) and the negative consequences (loss severity). The risk assessment results can support the stakeholders to take more informed decisions regarding the efficient allocation of the available funding for the improvement of the monitored objects protection from natural disasters. A concept for implementing the risk assessment models as a part of a Web integrated information system for risk management of natural disasters are outlined.

Keywords: risk situation, loss assessment, risk model natural disasters, Web integrated information system

1. Introduction

In recent years the number of the natural disasters and their negative consequences are increased [1, 2]. For this reason, the studies are needed to assess the possible losses due to occurrence of natural disasters [3]. Various risk models are used to describe the risk sources (natural disasters) and the related negative consequences (total losses) for the monitored objects (persons, firm, organization, geographical region, municipally, infrastructure, and so on) in the given time period [4-7]. There are great numbers of risk definitions [4]. In most of cases, the term "risk" is focused on quantitative aspects of the negative consequences. The risk can be defined the most common as a random variable (number), X , whose actual outcome (or realization) is unknown. In this case, a set of possible outcomes has to be specified, and probabilities over this set have to be assigned. In financial terms the variable X can represent the damages for monitored objects from natural disasters.

The purpose of the paper is to present various risk situations which are related with the possible financial losses of the monitored objects due to occurrence of natural disasters. Some risk assessment models are considered. In particular, the average number events and the average severity of losses are estimated. The described risk assessment models are envisaged to be implemented as a part of a Web integrated information system for risk management of natural disasters.

2. Various Risk Situations

The natural disasters cause various negative consequences of the monitored objects which can be described from a mathematical viewpoint as various risk situations [4, 6].

2.1. Possible loss with Fixed Amount for One-year Period

First it is assumed that the suffered total loss is with fixed amount. In particular, the potential loss of one object due to occurrence of one natural disaster within a given period is considered. Thus, if the negative event occurs, the amount of the loss is certain. The potential loss, X , is defined as follows:

$$X = \begin{cases} x & if \quad \eta \\ 0 & if \quad \overline{\eta} \end{cases}, \tag{1}$$

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where η is the natural disaster (the negative event) which causes the financial loss; x is the amount of the loss (lost severity). The loss is zero ($X = 0$) when the negative event is not occurred ($\overline{\eta}$).

2.2. Losses with Random Amounts for One-year Period

Second it is allowed that the loss is with random amount (severity) X . The possible loss realizations can be either discrete variables ($X: 0, x_1, ..., x_{\max}$) or continuous variables ($0 \le X \le x_{\max}$). The outcome $X = 0$ denotes the absence of damage for lack of natural disaster and the outcome $X = x_{\max}$ indicated the realization of the maximum loss amount. The set of discrete potential losses are described as follows:

$$X = \begin{cases} x_1 & if & \eta_1 \\ x_2 & if & \eta_2 \\ ... & ... & ... \\ x_m & if & \eta_m \\ 0 & if & \overline{\eta} \end{cases}, \tag{2}$$

where x_i , $i = 1,2,...,m$ is the i-th severity of the loss due to occurrence of η_i - the i-th variety of the natural disaster. The financial loss is zero ($X = 0$) when the natural disaster is not occurred ($\overline{\eta}$).

The negative events $\eta_1, \eta_2, ..., \eta_m$ (accidents) are scaled according to increasing severity of the consequences in terms of amount of the loss. The event η describes completely the natural disaster and it can be represented as the following union

$$\eta = \eta_1 \cup \eta_2 \cup ... \cup \eta_m , \tag{3}$$

whereas $\overline{\eta}$ still represents the absence of the negative event.

2.3. Random Number of Events Each with Deterministic Loss for One-year Period

In this case a random number of events (accidents due to natural disaster) may occur within the stated period and each event implying a deterministic loss. The time horizon is one year.

If random variable Z denotes the random number of accident due to natural disaster within a given year then the total loss is described by

$$X = BZ , \tag{4}$$

where B is the amount of the loss for each object; n – the number of the monitored objects, exposed to the risk of the natural disaster; the possible outcomes of Z (the number of potential accidents) are $0,1,...,n$, so that the corresponding outcomes of X are $0, B,..., nB$. The expected value, $\mathrm{E}X$, of the total financial loss is

$$\mathrm{E}(X) = B.\mathrm{E}(Z). \tag{5}$$

If for the j-th monitored object, $j = 1,2,...,n$, the amount of the loss is B_j then, the total loss does not depend on the number of accidents only, as it also depends on which objects affect the natural disaster. In formal terms, with reference to j-th object the random loss amount X_j is defined as follows

$$X_j = \begin{cases} B_j & in\ the\ case\ of\ accident \\ 0 & otherwise \end{cases}. \tag{6}$$

Then, the total financial loss is given by

$$X = \sum_{j=1}^{n} X_j . \tag{7}$$

It is necessary to mention that the risk, which leads to the random total loss X , is actually a set of individual risks, each one represented by the related loss is B_j (or B).

2.4. Random Number of Events Each with Deterministic Loss for Multi-year Period

In this case a random number of accidents due to natural disaster may occur for the multi-year period and each event implying a deterministic loss. The time-value of money cannot be disregarded when a longer time horizon is addressed.

It is assumed that the time horizon consists of m years. In particular, it is interested in setting $m > 1$ (for example, $m = 3$, $m = 5$ or $m = 10$). It is still assumed more that the number of the monitored objects is n. Moreover, it is supposed that each monitored object who suffered an accident implying permanent loss any given year is replaced, at the beginning of the following year, by another object. Further new objects are not allowed. Hence, *n* objects are exposed to risk at the beginning of each year.

The individual loss, at the end of the year in which the accident occurs, is B, whatever the year may be (within the stated period). The random loss amount at time *t*, exactly at the end of year *t* is given by

$$X_t = B.Z_t, \tag{8}$$

where Z_t is the random number of accidents occurring in the various years for $t = 1,2,...,m$.

It is necessary to note that if the total financial loss as following sum

$$X = X_1 + X_2 + ... + X_m \tag{9}$$

then the time-value of the money is disregard, i.e. a zero interest is assumed.

2.5. Random Number of Events with Random Loss for One-year Period

It is assumed that a monitored object can be damaged one or more times within the stated period (one-year period), by natural disaster. In each occurrence of natural disaster, the amount of the related damage (financial loss) is random.

In this case the randomness of the loss and random number of negative events are merged together.

In formal terms, it is defined the random number N as the number of occurrences of the natural disaster within the stated period. Then, it is denoted with X_k the damage (loss severity) caused by the *k*-th natural disaster. Hence, the total random damage X (total loss) is defined as follows:

$$X = \begin{cases} 0 & if \quad N = 0 \\ X_1 + X_2 + ... + X_N & if \quad N > 0 \end{cases}, \tag{10}$$

where $X_k > 0$ for each $k = 1,2,...,N$, when $N > 0$. The zero damage ($X = 0$) is described by $N = 0$.

It can be determined $x_{\min} = Min\ (X_k)$ and $x_{\max} = Max\ (X_k)$, for each $k = 1,2,...,N$. In particular, the maximum loss severity, $x_{\max}$, could be the cost of the monitored object. However, it is unlikely that, in the case of multiple occurrence of the natural disaster, each event completely destroys the object (which, in the meanwhile, should have been completely recovered. For this reason, it very important to correctly determine the probabilities of random variables $X_1, X_2,..., X_N$ and N.

In relation to the random variable N, it is essentially assumed that the possible outcomes are all the integer numbers 0,1,2,... . Conversely, it can accepted a maximum (reasonable) outcome $n_{\max}$, so that the possible outcomes are $0,1,2,...,n_{\max}$.

If it is assumed that all the random amounts (damages or loss severities) X_k, $k = 0,1,2,...,n_{\max}$, have the same expected value, i.e.

$$\mathrm{E}(X_1) = \mathrm{E}(X_2) = ... = \mathrm{E}(X_k) = ... = \mathrm{E}(X_{n_{\max}}), \tag{11}$$

where $\mathrm{E}(X_k)$ is the expected value of the damage resulting from the *k*-th occurrence, then

$$\mathrm{E}(X) = \mathrm{E}(X_1).\mathrm{E}(N), \tag{12}$$

where $\mathrm{E}(X)$ is the expected value of the total damage (total loss), $\mathrm{E}(N)$ is the expected value of the random number of occurrences of the natural disaster in the given period.

3. Some Risk Assessment Models

Many risk assessment models are known [6,7]. Here, some models are considered only [4].

3.1. Model for Assessment of the Possible Loss with Fixed Amount

The random loss with fixed amount (1) is expressed. In the case, the determination of the natural disaster (negative event) probability, η is only required in order to design the risk assessment model. Let it is denoted this probability as $p = P(\eta)$. The expected value of the potential loss X is then given by

$$\mathrm{E}(X) = x.p,$$

and the variance, $\mathrm{D}(X)$ is expressed by

$$\mathrm{D}(X) = x^2.p.(1-p)$$

The standard deviation, $\sigma(X)$ is defined as the square root of the variance:

$$\sigma(X) = \sqrt{\mathrm{D}(X)} = \sqrt{x^2.p.(1-p)} = x\sqrt{p.(1-p)}$$

3.2. Model for Assessment of the Losses with Random Amounts for One-year period

The discrete probability distribution of the random variable X, describing the severity of the loss ($X: 0, x_1, ..., x_{\max}$) is expressed as follows

$$p_i = P(X = x_i), \qquad i = 1,2,...,m, \qquad x_m = x_{\max}$$

and the main constraint $\sum_{i=0}^{m} p_i = P(X = x_i)$ is fulfilled. The expected value of the potential damage X is

$$\mathrm{E}(X) = \sum_{i=0}^{m} x_i.p_i, \quad \text{under constraint} \quad \sum_{i=0}^{m} p_i = 1,$$

and the variance and the standard deviation are given as

$$\mathrm{D}(X) = \sum_{i=0}^{m} (x_i - \mathrm{E}(X))^2.p_i \quad \text{and} \quad \sigma(X) = \sqrt{\mathrm{D}(X)}.$$

From condition (3) the complete probability of an accident, regardless of the corresponding severity of the natural disaster, is expressed by

$$p = \mathrm{P}(\eta) = \mathrm{P}(\eta_1 \cup \eta_2 \cup ... \cup \eta_m) \quad \text{and} \quad p = \sum_{i=1}^{m} p_i$$

whereas the probability of not occurring of the natural disaster is $p_0 = \mathrm{P}(\overline{\eta})$ and $p_0 = 1 - p$.

According to the theorem of conditional probabilities, the following condition is satisfied

$$P(X = x_i) = P(X = x_i | \eta).P(\eta), \qquad i = 1,2,...,m$$

Then, the conditional probability of the loss severity, when the natural disaster is occurred, is given as

$$\mathrm{P}(X = x_i | \eta) = \frac{\mathrm{P}(X = x_i)}{\mathrm{P}(\eta)} = \frac{p_i}{p}, \qquad i = 1,2,...,m$$

and the conditional expected value of the potential damage X is

$$\overline{x} = \mathrm{E}(X|\eta) = \frac{1}{p}.\sum_{i=1}^{m} x_i.p_i.$$

3.3. Risk Model for Assessment of the Random Number of Events

A finite discrete distribution of the random number N is investigated. In practice the Poisson distribution is often used. As a first step a reasonable maximum outcome, $n_{\max}$ is selected. Then, the following probabilities are assigned

$$\nu_i = P(N = i), \qquad i = 0,1,...,n_{\max}.$$

The expected value of the random number, N is

$$\overline{n} = \mathrm{E}(N) = \sum_{i=0}^{n_{\max}} i.\nu_i$$

and the variance is expressed as follows:

$$\mathrm{D}(N) = \sum_{i=0}^{n_{\max}} (i - \overline{n})^2.\nu_i.$$

3.4. Risk Model for Assessment of the Random Loss for One-year Period

The probability distribution of the total loss and the related typical values are of great interest. The variable X usually represents the random cost referred to the stated period (one year). The total random damage X , (10), is a random sum, since the number N of terms in the summation as well as the individual values of the terms are random variables.

The following probabilistic assumptions about the random variables N and X_k , $k = 0,1,2,..., n_{\max}$ are adopted for calculating the expected value $\mathrm{E}(X)$:

- The random variables X_k are independent of the random number N ;
- Whatever the outcome n of N , the random variables $X_1, X_2,..., X_n$ are mutually independent and identically distributed with a common expected value (11).

The expected value of the loss severity, X is described as

$$\mathrm{E}(X) = \sum_{i=0}^{n_{\max}} \nu_i .\mathrm{E}(X|N=i) = \sum_{i=1}^{n_{\max}} \nu_i .\mathrm{E}(X|N=i) = \sum_{i=1}^{n_{\max}} \nu_i .\left(\sum_{k=1}^{i} \mathrm{E}(X_k|N=i) \right).$$

From the first assumption follows

$$\mathrm{E}(X_k|N=i) = \mathrm{E}(X_k), \quad \text{for each} \quad k = 0,1,2,..., n_{\max},$$

and taking into account the second assumption and the condition (11) it is obtained

$$\sum_{k=1}^{i} \mathrm{E}(X_k|N=i) = i.\mathrm{E}(X_1)$$

Therefore, the expected value of the loss severity is expressed as

$$\mathrm{E}(X) = \sum_{i=1}^{n_{\max}} \nu_i .i.\mathrm{E}(X_1) = \mathrm{E}(N).\mathrm{E}(X_1).$$

The assumption of independence between the random variables X_k and the random number N is is too idealized. More realistic are the situations in which a very high total number of damages is likely associated to a prevailing number of damages with small amounts.

4. Conclusions

Various risk situations associated with the possible financial losses of the monitored objects due to the occurrence of natural disasters are described. Some risk assessment models are considered. In particular, the average number events and the average severity of losses are estimated. A concept for implementing those models in a Web integrated information system for risk management of natural disasters is outlined.

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6. References

[1] J. Pollner, J. Kryspin-Watson, S. Nieuwejaar. *Disaster Risk Management and Climate Change Adaptation in Europe and Central Asia*. Global Facility for Disaster Reduction and Recovery: The World Bank, 2010.

[2] J. Padli, M. Habibullah, A. Baharom. Economic impact of natural disasters' fatalities, *International Journal of Social Economics*, 2010, **37** (6): 429 – 441.

[3] Prevention Web - Strengthening climate change adaptation through effective disaster risk reduction, http://www.preventionweb.net/english/

[4] A. Olivieri, E. Pitacco. *Introduction to Insurance Mathematics*, Springer, Berlin, 2011.

[5] R. Kaas, M. Goovaerts, J. Dhaene, M. Denuit. *Modern Actuarial Risk Theory: Using R*, Springer, Berlin, 2008.

[6] S. Klugman, H. Panjer, G. Willmot. *Loss Models: From Data to Decisions* (3rd ed.), Wiley, New York, 2008.

[7] Y.-K. Tse. *Nonlife Actuarial Models: Theory, Methods and Evaluation*, Cambridge Univ. Press, Cambridge, 2009.