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A Mathematical Model for Optimal Production, Inventory and Transportation Planning with Direct Shipment

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Abstract: We develop the mixed integer linear programming model for an integrated decision of production, inventory and transportation planning problem. Our model combines the direct shipment into distribution decisions. In direct shipment, a manufacturer directly delivers the products to retailers by bypassing warehouse, thereby saving transportation cost from a plant to warehouse, and inventory holding cost at warehouse. The objective is to minimize overall cost comprising of production setup cost, inventory holding cost, transportation cost and reorder cost. The model is solved to optimality using CPLEX. From the numerical experiment, the results show that the firm could generate the cost saving of 28.93% and reduce the number of trucks used approximately 49.88% by incorporating the direct shipment. However, CPLEX takes large computation time, more than 10,000 seconds in many large size problems, to solve the problem optimally. In the future research, we will develop and propose a time-partitioning heuristic algorithm to efficiently solve the problem. The performance of the proposed heuristic will be compared with CPLEX's.

Keywords: Direct shipment, Production, Inventory and transportation planning

1. Introduction

Due to global economy downturn and competitive business environment, logistic and supply chain management plays an important role for any firms to reduce their operations cost and enhance their competitive advantage. However, logistic cost structure is still expensive because of ineffective planning. Transportation cost, the largest logistic cost component, is continuously increasing from year to year due to high and fluctuating gasoline price. A distribution strategy, called direct shipment, is considered a viable method to reduce the logistic cost. In direct shipment, the manufacturer ships products directly to customer by bypassing warehouse. The direct shipment is justified if the demand at buyer locations is close to a full truck load. The major advantages of a direct shipment are the elimination of intermediate warehouse and its simplicity of operation and coordination. The transportation distance from supplier to buyer is short because each shipment goes direct. [1] Direct shipment can be suitably applied to many industries such as food industry and grocery industry, where lead times are critical because of perishable goods.

In this research, we study an integrated decision of production, inventory, and transportation problem in which a manufacturer produces multiple products using multiple production lines and distributes products to retailers directly from plant or through warehouse by a fleet of vehicles. The objective is to minimize the total operations cost. Our study aims to develop a mathematical model and heuristic to solve the model efficiently.

The application of direct shipment is mentioned in a number of literatures. Comparing direct shipping with peddling is conducted by Burns et al [2]. They show that optimal shipment size is given by economic order quantity model for direct shipping, while for peddling the optimal shipment size is a full truck load. Gallego and Simchi-Levi [3] present that direct shipping is at least 94% effective whenever the minimum economic lot size overall retailers is at least 71% of the truck capacity. Liu et al [4] consider a mixed truck delivery system that allows both direct shipment and hub-and-spoke deliveries. They suggest that the mixed

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truck system can yield around 10% in distance saving compared with either of two pure systems. Chen [5], Barnes-Schuster and Bassok [6] analyze the application of direct shipment in the case of products under stochastic demand.

Our study differs from prior studies in the following areas: we address the inventory management decision at retailer (with fixed reorder cost), at manufacturer's warehouse and at manufacturer's loading area for an integrated decision of production, inventory and transportation problem with direct shipment.

The rest of the paper is organized as follows. Section 2 we formally describe our model. Section 3 we present numerical experiment. The last section we conclude the paper and suggest topics for future research.

2. Problem Definition and The Model

We consider a firm who owns multiple production lines and distributes multiple products to a number of retailers directly from a plant or through warehouse by a fleet of trucks with limited capacity. According to Fig. 1 the products are scheduled to produce at the production lines. At the end of production lines, the finished products (in pallets) are loaded onto the outbound truck and then shipped to warehouse. Then, they are transported to the retailers when retailers place the orders. With direct shipment, the finished products (in pallets) are sent to loading area, loaded onto outbound truck, and then directly shipped to retailer without going to the warehouse. We assume that demand is deterministic and must be fully satisfied. The manufacturer makes production, inventory and transportation decisions simultaneously in order to minimize the total costs. Then, we formulate the problem as a mixed integer linear programming model as follows:

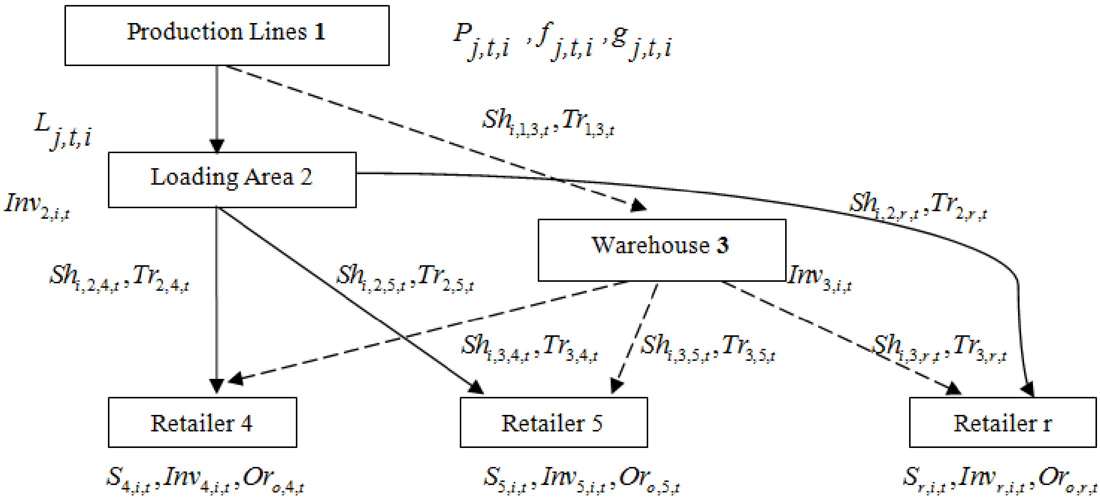

Fig.1: Flow of Products and Decision Variables at each location

The following tables present the notations used in this paper

Table 1: Decision Variable and Auxiliary Decision Variable

Decision Variable		Auxiliary Decision Variable	
Notation	Description	Notation	Description
$L_{j,t,i}$	Quantity of product i produced by production line j and sent to loading area in period t	$f_{j,t,i}$	1 if product i is produced from production line j in period t , 0 otherwise
$Inv_{n,i,t}$	Inventory of product i at node n at the end of period t	$g_{j,t,i}$	1 if product i is produced from production line j at the end of period t and at the beginning of period t+1 , 0 otherwise
$Tr_{o,d,t}$	Number of truck used to ship the product from o to d in period t	$Or_{o,r,t}$	1 if product is ordered by retailer r in period t 0 otherwise
$Sh_{i,o,d,t}$	Quantity of product i being shipped from o to d in period t	$g_{j,t,i}$	1 if product i is produced from production line j at the end of period t and at the

$P_{j,t,i}$	Production quantity of product i from production line j in period t		beginning of period t+1 0 otherwise

Table 2: The firm requires the following data to make decisions

Notation	Description	Notation	Description
$S_{r,i,t}$	Demand for product i at retailer r in period t	Rc	Fixed reorder cost in period t
$Pr_{i,j}$	Production rate of product i at production line j	$\overline{A}$	Maximum number of pallets that can be stored in the loading area
$Ct_{i,j}$	Changeover time from product i to product j at production line j	$\overline{C}$	Maximum number of pallets a truck can carry in one container
$\overline{P}$	Upper bound of production quantity for all production lines	$h_{n,i}$	Inventory holding cost of product i per pallet per period at node n
$\underline{P}$	Lower bound of production quantity for all production lines	$Vc_{tr,o,d}$	Variable cost incurred from using truck to ship one container from o to d
$\overline{H}_t$	Number of available production time (hours) per period per one production line	$Sc_{i,j}$	Setup cost incurred from changeover from product i to j

Mathematical Model:

$$\text{Minimize Total Cost Function } z =$$

$$\sum_{t=1}^{T}\sum_{o\in N_m}\sum_{r\in N_r} Rc\times Or_{o,r,t} + \sum_{t=1}^{T}\sum_{o\in N}\sum_{d\in D_o} Vc_{tr,o,d}\times Tr_{o,d,t} + \sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{n=2}^{N} h_{n,i}\times Inv_{n,i,t} \quad (1)$$

$$+\sum_{j=1}^{J}\sum_{t=1}^{T}\sum_{i=1}^{I}\left(f_{j,t,i}-g_{j,t,i}\right)\times Sc_{i,j}$$

Subject to:

$$Inv_{2,i,t} = Inv_{2,i,t-1} + \sum_{j=1}^{J} L_{j,t,i} - \sum_{d\in D_2} Sh_{i,2,d,t} \quad \text{for all } i\in \text{Set } i \text{ and } t\in \text{Set } t \quad (2)$$

$$Inv_{3,i,t} = Inv_{3,i,t-1} + Sh_{i,1,3,t} - \sum_{d\in D_3} Sh_{i,3,d,t} \quad \text{for all } i\in \text{Set } i, t\in \text{Set } t \quad (3)$$

$$Inv_{r,i,t} = Inv_{r,i,t-1} + \sum_{o\in N_m} Sh_{i,o,r,t} - S_{r,i,t} \quad \text{for all } i\in \text{Set } i, t\in \text{Set } t, r\in \text{Set } r \quad (4)$$

$$Inv_{n,i,0} = 0 \quad \text{for all } i\in \text{Set } i \quad (5)$$

$$L_{j,t,i} \le P_{j,t,i} \quad \text{for all } i\in \text{Set } i, j\in \text{Set } j, t\in \text{Set } t \quad (6)$$

$$Sh_{i,1,3,t} = \sum_{j=1}^{J}\left(P_{j,t,i} - L_{j,t,i}\right) \quad \text{for all } i\in \text{Set } i, t\in \text{Set } t \quad (7)$$

$$\sum_{i=1}^{I} Inv_{2,i,t}\times space \le \overline{A} \quad \text{for all } t\in \text{Set } t \quad (8)$$

$$\sum_{i=1}^{I} Sh_{i,o,d,t}\times space \le Tr_{o,d,t}\times \overline{C} \quad \text{for all } t\in \text{Set } t, o\in N, d\in D_o \quad (9)$$

$$P_{j,t,i} \le f_{j,t,i}\times \overline{P} \quad \text{for all } i\in \text{Set } i, j\in \text{Set } j, t\in \text{Set } t \quad (10)$$

$$P_{j,t,i} \geq f_{j,t,i} \times \underline{P} \text{ for all } i \in Set\ i, j \in Set\ j, t \in Set\ t \quad (11)$$

$$\sum_{i=1}^{I} \left(P_{j,t,i} / Pr_{i,j} \right) + \sum_{i=1}^{I} \left(f_{j,t,i} \times Ct_{i,j} \right) - \sum_{i=1}^{I} \left(g_{j,t,i} \times Ct_{i,j} \right) \leq \bar{H} \text{ for all } j \in Set\ j, t \in Set\ t \quad (12)$$

$$g_{j,t+1,i} \leq f_{j,t,i} \text{ for all } i \in Set\ i, j \in Set\ j, t \in Set\ t1 \quad (13)$$

$$g_{j,t+1,i} \leq f_{j,t+1,i} \text{ for all } i \in Set\ i, j \in Set\ j, t \in Set\ t1 \quad (14)$$

$$\sum_{i=1}^{I} g_{j,t,i} \leq 1 \text{ for all } j \in Set\ j, t \in Set\ t \quad (15)$$

$$g_{j,t+1,i} \leq 1 - g_{j,t,i} \text{ for all } i \in Set\ i, j \in Set\ j, t \in Set\ t1 \quad (16)$$

$$\sum_{i=1}^{I} Sh_{i,o,r,t} \leq \mathrm{Or}_{o,r,t} \times M \text{ for all } r \in Set\ N_r, t \in Set\ t, o \in SetN_m \quad (17)$$

The objective function (1) is to minimize total cost comprising of reorder cost, transportation cost, inventory holding cost and production changeover cost. (2), (3) and (4) sets up the inventory at each location. (5) sets the initial condition inventory.(6) states that the quantity of products sent to loading area cannot exceed the quantity of products produced from the production lines. (7) calculates the quantity of products to be transported to warehouse. (8) sets the upper limit on the number of pallets stored at the loading area at the end of period. (9) calculates the numbers of containers to be transported from origin node to destination node. (10) and (11) put constraints on the lower bound and upper bound of production batch sizes. (12) stipulates that the sum of total production time and production changeover time must not exceed the total time available in a period. (13), (14), (15) and (16) set up the auxiliary binary variable $g_{j,t,i}$ to indicate which product *i* is produced at the conjunction between period *t*-1 and *t*. If $g_{j,t,i}$ is set to 1, it means that the product *i* is being produced at the end of period *t* -1 and the beginning of *t* and thus the changeover time and changeover cost should not be charged only once not twice in the production time (14) and objective function (1) respectively. (17 states that if there is shipment from warehouse or loading area to any retailers, the auxiliary binary variable $\mathrm{Or}_{o,r,t}$ cannot be zero.

3. Numerical Experiment

In the first experiment, we compare the total cost of direct shipment with no direct shipment's to show the percentage of saving by using direct shipment. With no direct shipment, the products are loaded and unloaded at warehouse before being sent to retailer. The direct shipment allows a manufacturer directly ships the product to retailer by bypassing warehouse, thereby saving one trip of transportation cost and inventory holding cost at warehouse. Using seven planning periods, a manufacturer produces 3 product types by two production lines and distributes the products to 3 retailers by trucks with limited capacity. The transportation cost is charged at 1,500 Bath for shipment from plant to warehouse, and 3,000 Bath for shipment from warehouse to retailer or from loading area to retailer. The unit holding cost at loading area, warehouse and retailer is 2, 6 and 10 Bath per pallet per period respectively. The reorder cost is charged at 2,000 Bath whenever the retailer places an order. The mean of demand is randomly generated from 50 pallets to 100 pallets per period. The problems are all solved by IBM ILOG CPLEX version 12 to find the optimal cost for both cases. The average % saving is shown in Table 3.

Table 3. The impact of the mean of demand on the percentage of saving by using direct shipment with 3 retailers, 3products and 7 periods.

Mean of demand	Direct shipment		No direct shipment		% cost saving	% truck saving
	Total cost	Trucks	Total cost	Trucks		
50	270,920	64	352,008	127	23.04	49.60

100	478,990	127	653,244	253	26.68	49.80
264.9	1,115,000	334	1,601,640	668	30.38	50
500	2,016,360	635	2,959,840	1269	31.88	49.99
1000	3,898,620	1262	5,789,950	2524	32.67	50
Average	1,555,978	485	2,271,336.4	969	28.93	49.88

Table 3 shows that the average cost saving is 28.93% and the number of trucks used is reduced to only 49.88% by incorporating the direct shipment. This highlights the benefit of using direct shipment together with a warehouse. Then, the performance of CPLEX is tested with many problem sizes to show the limitation of CPLEX. We consider the case that a manufacturer produces and distributes the products to 3 retailers along 4 planning periods. We vary the number of product type from 10 to 60 product types to observe the computation time that CPLEX requires. From the experiment, we found that CPLEX take about 1,033.29 seconds to solve the problem optimally for 40 products. However, for problem of 60 products it could not find the optimal solution within 10,000 seconds. The larger problem size is, the more computation time is required. CPLEX cannot suitably be applied to real case, normally large size.

Due to large computation time required by CPLEX, in the future research, we will develop and propose a time partitioning heuristic to efficiently solve the problem with lower computation time while maintaining the quality of solution close to optimality. The performance of the proposed heuristic will be compared with CPLEX's. Various numerical experiments will be conducted to test the performance of the proposed heuristic.

4. Conclusion and Future Research

In this research, we develop the mixed integer linear programming model for an integrated decision of production, inventory and transportation planning problem. The products can be distributed directly from a plant or through warehouse by a fleet of truck each of which has a limited capacity. The objective is to minimize the total cost comprising of production setup cost, inventory holding cost, transportation cost and reorder cost. The model is solved to optimality using CPLEX. The computational experiment shows that the average cost saving is 28.93% and the number of trucks used is reduced to only 49.88% by incorporating the direct shipment. In the future research, we will develop and propose a time-partitioning heuristic algorithm to efficiently solve the problem. Then, we will compare the performance of the proposed heuristic to CPLEX's. Various numerical experiments will be conducted to show the performance of the proposed heuristic. We expect that the proposed heuristic will provide the good solution within acceptable time.

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