

## A New Model for Cross Dock Scheduling Considering Product Arrangement

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**Abstract.** Cross docking is a concept that aims at minimizing the transportation cost in the supply chain. It is a kind of warehouse; however, it is not made for keeping inventory and usually incoming shipments are forwarded to their destinations in less than a day. One of the important problems, which affects on the performance of the cross dock system, is scheduling of incoming and outgoing trucks on the dock doors. In this paper, we study a new scheduling problem considering product arrangement in the incoming trucks, in which the items are loaded to the corresponding outgoing door instantly after unloading from incoming trucks. Thus, a non-linear programming model is presented that minimizes the total weighted number of delayed shipments.

**Keywords:** Supply chain management, Cross dock scheduling, Mathematical model.

### 1. Introduction

One of the important facts that make the manufacturers to try to be more efficient is the customer demand to deliver the requested shipment on time, in the right quantity, in the right place with affordable price. As you know transportation costs include about 10% of the total cost of a product [1], so defining strategies to reduce this cost can have significant effect on total cost of a product. One of the concepts which is defined for reaching the goal of on time delivery and lower transportation cost is cross docking. Incoming trucks usually contain different shipments for different destinations; they are unloaded at the cross docks and then items for each destination are loaded to the dedicated truck for the respective destination. Fig.1 shows a schematic view of a cross dock system.

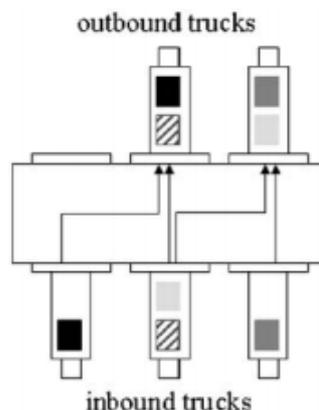


Fig.1 Schematic view of a cross docking warehouse

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The research on hand deals with truck scheduling problem in cross docks which deals with assigning trucks to dock doors in such a way that the performance of the system is improved. The specific constraints and objectives of this problem varies in different industries such as frozen food industry [2] which requires instant loading of products after unloading and zero inventory storage in the dock, or the JIT automobile industry and postal services which require on time deliveries of the shipments [3]. In the latter environment there may be a predefined sequence and departure time for the outgoing trucks, any item which does not arrive to the loading dock before the departure time will be delayed for the next day, so the performance measure in the problem will be (total weighted) number of delayed shipments.

In the next section the relevant literature is reviewed, then the proposed problem along with the mathematical model is presented; after that computational study and conclusion are described.

## 2. Literature Review

Previous research on cross docking can be divided to three categories based on the decision level and time horizon of the problem studied which are operational, tactical and strategic levels [4]. Based on this classification the scheduling problem falls in the first category of problems. McWilliams et al [5] considered scheduling a set of inbound trucks to a fixed number of unload docks in parcel hub minimizing the time span of transfer operation, a simulation based scheduling algorithm (SBSA) which uses genetic algorithm (GA) to search for new solution was proposed for the problem; McWilliams [6] proposed a dynamic load balancing algorithm (DLBA) for the same problem, the performance of the proposed algorithm was compared with an efficient static load balancing algorithm (SLBA), the study revealed that DLBA solutions which were 8.6% better than SLBA in shorter time. Boysen and Fliedner [3] presented a mathematical model for scheduling of inbound trucks where sequencing and departure time of outbound trucks are fixed in advance; they proved the problem to be NP-hard.

Yu and Egbelu [7] considered scheduling of trucks in a cross dock in which there were unlimited storage capacity, the proposed model does product assignment to trucks and scheduling of inbound and outbound trucks simultaneously minimizing total operation time. In contrast with this problem, there may be strict limitation on storage; Boysen [2] considered this kind of cross dock which usually occurs in frozen food and dairy industry, so that the products should be instantaneously loaded onto refrigerated outbound trucks after unloading, suited exact and heuristic algorithm were proposed for the problem. Álvarez-Pérez [8] considered a model which aimed to minimize transit storage time for a cargo based on the JIT concept, a metaheuristic method based on Reactive GRASP and Tabu Search (RGTS) was proposed for the model.

## 3. Model Description

Depending on the industry, facility, condition and management policy various models may be defined for cross dock scheduling problem. Boysen and Fliedner [3] proposed a classification of deterministic cross dock scheduling problem like what Graham et al [9] did for machine scheduling problems; also they presented a yet unexplored scheduling problem with fixed outbound schedule and its complexity was studied. The proposed problem in this research is based on the work of [3], as well we considered the arrangement of products in the incoming trucks based on the outgoing trucks which are assigned to destinations, this assumption results in a more efficient scheduling of trucks and a more realistic view about total operation time of the cross dock operation. Also we assume that unloading and transportation time is dependant of the volume of the shipments. The assumptions of the problem are the following:

- There are limited number of receiving doors so we need to sequence incoming trucks
- Outgoing trucks are sequenced in advance and each of them has a departure time, the truck will leave the dock on its predefined departure time; if there are any items which have not yet been loaded they will be delayed till the next day
- Incoming trucks are ready at time zero
- There is sufficient space inside the dock so we don't need to care about space limitation
- Unloading device is separated from transshipment device
- Each shipment for each outgoing truck will be transhipped immediately after it is unloaded, we don't need to wait until the incoming truck is fully unloaded and then start transshipment operation

- Unloading and transshipment time is dependant of the volume of the shipment
- Products are arranged inside each incoming truck for different outgoing truck (representing destination)

The notations used in the proposed mathematical model are described in Table 1.

Table 1. Notation

<b>I</b>	Set of incoming trucks $I = \{1, 2, \dots, n\}$
<b>O</b>	Set of outgoing trucks $O = \{1, 2, \dots, m\}$
<b>D</b>	Set of receiving doors available for unloading incoming trucks $D = \{1, 2, \dots, k\}$
<b>p</b>	Unloading time for each unit (pallet) of product
<b>d<sub>o</sub></b>	Departure time of outgoing truck o
<b>t<sub>ko</sub></b>	Transshipment time from receiving door k to dock outbound where outgoing truck o is being loaded
<b>w<sub>io</sub></b>	Number of pallets in incoming truck i brought for outgoing truck o
<b>z<sub>ioo'</sub></b>	1, if products inside incoming truck i for outgoing truck o is placed before products of outgoing truck o'
<b>M</b>	Big integer
<b>cu<sub>i</sub></b>	Continuous variable: Completion time for unloading incoming truck i
<b>c<sub>io</sub></b>	Continuous variable: Completion time for unloading and moving products from incoming truck i to outgoing truck o
<b>x<sub>ij</sub><sup>k</sup></b>	Binary variable: 1, if incoming truck j is unloaded immediately after incoming truck i at door k; 0, otherwise
<b>x<sub>0i</sub><sup>k</sup></b>	Binary variable: 1, if incoming truck i is the first truck being unloaded at door k; 0, otherwise
<b>x<sub>i,n+1</sub><sup>k</sup></b>	Binary variable: 1, if incoming truck i is the last truck being unloaded at door k; 0, otherwise
<b>y<sub>io</sub></b>	Binary variable: 1, if products inside truck i for outgoing truck o could not be loaded in it before its departure time

Using the mentioned notations the mathematical model is as follows.

$$\text{Min } Z = \sum_{i \in I} \sum_{o \in O} w_{io} \cdot y_{io} \quad (1)$$

$$\sum_{k \in D} \sum_{\substack{i \in I \cup \{0\} \\ i \neq j}} x_{ij}^k = 1 \quad \forall j \in I \quad (2)$$

$$\sum_{i \in I} x_{0i}^k \leq 1 \quad \forall k \in D \quad (3)$$

$$\sum_{\substack{i \in I \cup \{0\} \\ i \neq j}} x_{ij}^k = \sum_{\substack{i \in I \cup \{n+1\} \\ i \neq j}} x_{ji}^k \quad \forall j \in I; k \in D \quad (4)$$

$$cu_i = \sum_{k \in D} \left( x_{0i}^k \cdot p \sum_{o \in O} (w_{io}) + \sum_{\substack{j \in I \\ j \neq i'}} x_{ji}^k \cdot (cu_j + p \cdot \sum_{o \in O} w_{io}) \right) \quad \forall i \in I \quad (5)$$

$$c_{io} = p \cdot \left( w_{io} + \sum_{\substack{o' \in O \\ o' \neq o}} w_{io'} \cdot z_{io'o} \right) + \sum_{k \in D} (x_{0i}^k \cdot t_{ko} \cdot w_{io} + \sum_{\substack{j \in I \\ j \neq i}} x_{ji}^k (cu_j + w_{io} \cdot t_{ko})) \quad \forall i \in I; o \in O \quad (6)$$

$$y_{io} \cdot M \geq c_{io} - d_o \quad \forall i \in I; o \in O \quad (7)$$

$$x_{ij}^k \in \{0, 1\} \quad \forall i \in I \cup \{0\}; j \in I \cup \{n + 1\}; k \in D \quad (8)$$

$$y_{io} \in \{0, 1\} \quad \forall i \in I; o \in O \quad (9)$$

The objective function minimizes number of delayed products. Constraint (2) considers assigning each machine to only one door. Constraint (3) proposes that each door is utilized at most once by restricting number of start up trucks to one. Constraint (4) prevents the congestion of trucks on each door. Constraint (5) calculates unloading time of each truck while constraint (6) computes the time of releasing the last batch of each outgoing truck. Constraint (7) ensures that shipments of inbound truck  $i$  won't reach outbound truck  $o$  if the batch for outgoing truck  $o$  doesn't reach to it before its departure time. Constraints (8) and (9) determine variable types.

## 4. Conclusion

In this paper an unexplored problem of cross dock scheduling was considered, and then a NLP mathematical model was proposed for the problem. In the future we will linearize the model and solve it using commercial software, such as LINGO or GAMS, also heuristic methods could be employed if necessary.

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