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An Actuarial Approach for Aggregate Loss Assessment of the Critical Infrastructure Due to Natural Disasters

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Abstract. An actuarial model is proposed that will be used for aggregate loss assessment of the critical infrastructure elements due to natural disasters for a given time interval. This actuarial approach is based on the collective risk model. The main components of the collective risk model are discussed - the number of natural disasters (adverse events) and the negative consequences (severity of the losses). The risk assessment results can support the stakeholders to take more informed decisions regarding the efficient allocation of the available funding for the improvement of the critical infrastructure protection from natural disasters. Guidelines for implementing the collective risk model as a part of a Web Integrated Information System for risk management of natural disasters using by Cloud Computing technology are outlined.

Keywords: collective risk model, aggregate loss, natural disaster, cloud computing, critical infrastructure

1. Introduction

Nowadays the negative impact of natural disasters on sustainable development of the critical infrastructure increases. In this context it should be noted that the critical infrastructure generally includes all systems and assets, both physical and virtual, which make vital contributions to national security, economic stability, public health, or safety.

Scientific research and statistic data show a growth in number and severity of natural disasters compared to previous years [1]. Billions of dollars cost annual losses resulting from floods, hurricanes, earthquakes, tornadoes, etc. Natural disasters are impossible to avoid, and critical infrastructure elements cannot be made totally invulnerable. The only viable solution is to prepare cities, towns and communities through a combination of mitigation and adaptation strategies [2, 3].

Therefore it is necessary to develop long-term loss-reduction strategies and disaster preparedness which could improve the critical infrastructure resilience with respect to of natural disasters, resulting in less property damage and reduced rebuilding costs. Hence there is a need to propose integrated approaches to assess the possible aggregate loss for each critical element of infrastructure at occurrence of adverse events. The availability of an adequate assessment of the potential total loss would help taking more informed decisions for effective use of limited financial resources to activities in emergency situations.

On the other hand is well known that the so-called collective risk model is a fundamental model for insurance risk assessment. The collective risk model treats the aggregate loss (total claims) as a compound distribution with two main components. One is the claim frequency, i.e. the number of claims (accidences, adverse events) that which may be occur over a certain period. This is a discrete random variable taking nonnegative integer values. The second component describes the severity (size, amount) of the claim or the loss resulting from the occurrence of an adverse event. The stochastic nature of both components is a

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fundamental assumption of a realistic risk model [4, 5, 6]. The collective risk model is often used for modeling other noninsurance product risks, such as credit and operational risk [7].

The purpose of the paper is to propose an actuarial approach to assess the possible aggregate loss due to natural disasters for each element of critical infrastructure. This actuarial approach is based on the collective risk model. In particular, the main components of the collective risk model are the number of natural disasters (adverse events) and the negative consequences (severity of the losses).

The proposed collective risk model is envisaged to be implemented as a part of a Web Integrated Information System for risk management of natural disasters using the Cloud Computing technology.

2. Collective Risk Model Essentials

The collective risk model computes the aggregate loss as an independent sum of all losses (negative consequences from all occurred natural disasters) incurred over a certain period:

$$S = X_1 + X_2 + ... + X_N,$$

where S is the aggregate loss; N - the number (frequency) of losses; X_i - the severity of the *i*-th loss, for $i = 1,..., N$.

It is assumed that the loss severities X_i, $i = 1,..., N$ are independently and identically distributed (iid) as the loss severity random variable X. The loss-frequency N is itself a nonnegative integer-valued random variable distributed independently of X_i, $i = 1,..., N$.

The aggregate loss S is assumed to follow a nonnegative compound distribution. The loss-frequency random variable N represents the primary distribution and the loss-severity random variable X is secondary distribution of the compound distribution. Furthermore, N and X are assumed to be independent.

There are some advantages in modeling loss-frequency and loss-severity separately, and then combining them to determine the aggregate-loss distribution. In this study primary and secondary distributions are determined by non-negative discrete random variables. In practice for the computation of the aggregate-loss distribution are used both recursive and approximate methods.

The main properties of compound distributions are described below [6].

The moment generating function (mgf) of the random variable X as a function of t is denoted by $M_X(t)$. If the expectation exists then it is defined as follow

$$M_X(t) = \mathrm{E}\left(e^{tX}\right).$$

If the $M_X(t)$ exists for t in an open interval around $t = 0$, then the moments of X exist and can be obtained by successively differentiating the $M_X(t)$ with respect to t and evaluating the result at $t = 0$.

The r-th derivative of the $M_X(t)$ is described by the following dependence:

$$M_X^r(t) = \frac{d^r M_X(t)}{dt^r} = \frac{d^r}{dt^r}\mathrm{E}\left(e^{tX}\right) = \mathrm{E}\left[\frac{d^r}{dt^r}\left(e^{tX}\right)\right] = \mathrm{E}\left(X^r e^{tX}\right).$$

Therefore, at the point $t = 0$ for the r-th derivative of the $M_X(t)$ is obtained

$$M_X^r(0) = \mathrm{E}\left(X^r\right) = \mu_r',$$

where by definition r-th initial moment of the random variable X in the general case is given by the formula

$$\mathrm{E}\left(X^r\right) = \sum_{i=1}^{\infty} x_i^r . f_{X_i}(x_i) = \mu_r'.$$

The probability generating function (pgf) of the nonnegative random variable X is denoted by $\mathrm{P}_X(t)$. If the expectation exists then it is defined as follow

$$\mathrm{P}_X(t) = \mathrm{E}\left(t^X\right).$$

It is known that the mgf $M_X(t)$ and pgf $\mathrm{P}_X(t)$ are related through the following equations

$$M_X(t) = \mathrm{P}_X\left(e^t\right).$$

The moment generating function (mgf) of the aggregate loss S as a function of t is denoted by $M_S(t)$.

If the primary distribution N has mgf $M_N(t)$ and the secondary distribution X has mgf $M_X(t)$, then the mgf of the compound distribution S is deduce as follow

$$M_S(t) = \mathrm{E}\left(e^{tS}\right) = \mathrm{E}\left(e^{t.(X_1+X_2+\ldots+X_N)}\right) = \mathrm{E}\left(e^{t.X_1+t.X_2+\ldots+t.X_N}\right) =$$

$$= \mathrm{E}\left(\mathrm{E}\left(e^{t.X_1+t.X_2+\ldots+t.X_N}\,|N\right)\right) = \mathrm{E}\left(\left(\mathrm{E}\left(e^{t.X}\right)\right)^N\right) =$$

$$= \mathrm{E}\left(\left(\mathrm{M}_X(\mathrm{t})\right)^N\right) = \mathrm{E}\left(\left(e^{\ln \mathrm{M}_X(\mathrm{t})}\right)^N\right) = M_N\left(\log M_X(t)\right).$$

If N has pgf $\mathrm{P}_N(t)$ and X is nonnegative integer valued with pgf $\mathrm{P}_X(t)$, then the pgf of S is

$$\mathrm{P}_S(t) = E\left(t^S\right) = E\left(t^{X_1+X_2+\ldots+X_N}\right) = E\left(E\left(t^{X_1+X_2+\ldots+X_N}\,\middle|N\right)\right) =$$

$$= E\left(\left(E\left(t^X\right)\right)^N\right) = E\left(\left(P_X(t)\right)^N\right) = \mathrm{P}_N\left(\mathrm{P}_X(t)\right).$$

Furthermore, the aggregate loss S is nonnegative and discrete, because the loss-severities take nonnegative discrete values.

The mean of the aggregate loss S is given by

$$\mathrm{E}S = \mathrm{E}\left(\mathrm{E}\left(\mathrm{S}|\mathrm{N}\right)\right) = \mathrm{E}\left(\mathrm{E}\left(X_1 + \ldots + X_N\,|N\right)\right) = \mathrm{E}\left(\mathrm{E}(N.X)|N\right) = \mathrm{E}\left(\mathrm{E}(N|N).\mathrm{E}(X|N)\right) =$$

$$= \mathrm{E}\left(N.\mathrm{E}(X)\right) = \mathrm{E}\left(N.\mu_X\right) = \mu_X \mathrm{E}(N) = \mu_N \mu_X = \mathrm{E}N.EX\ .$$

The variance of the random variables S is defined using the condition of independence between random variables X and N, and the following relations:

$$\mathrm{D}(S) = \mathrm{E}\left(S^2\right) - (\mathrm{E}S)^2 \quad \text{and} \quad \mathrm{E}\left(S^2\right) = \mathrm{E}\left(\mathrm{E}\left(S^2|N\right)\right),$$

$$\mathrm{E}\left(X|N\right) = \mathrm{E}X \quad \text{and} \quad \mathrm{D}\left(X|N\right) = \mathrm{D}X\ ,$$

$$\mathrm{D}\left(S|N\right) = \mathrm{E}\left(S^2|N\right) - \left(\mathrm{E}\left(S|N\right)\right)^2 \quad \text{and} \quad \mathrm{E}\left(S^2|N\right) = \mathrm{D}\left(S|N\right) + \left(\mathrm{E}\left(S|N\right)\right)^2.$$

Thus the variance of the aggregate loss S is is deduce as follow

$$\mathrm{D}(S) = \mathrm{E}\left(S^2\right) - (\mathrm{E}S)^2 = \mathrm{E}\left(\mathrm{E}\left(S^2|N\right)\right) - (\mathrm{E}S)^2 = \mathrm{E}\left(\mathrm{D}\left(S|N\right) + \left(\mathrm{E}\left(S|N\right)\right)^2\right) - (\mathrm{E}S)^2 =$$

$$= \mathrm{E}\left(\mathrm{D}\left(S|N\right)\right) + \mathrm{E}\left(\left(\mathrm{E}\left(S|N\right)\right)^2\right) - \left(\mathrm{E}\left(\mathrm{E}\left(S|N\right)\right)\right)^2 = \mathrm{E}\left(\mathrm{D}\left(S|N\right)\right) + \mathrm{D}\left(\mathrm{E}\left(S|N\right)\right) =$$

$$= \mathrm{E}\left(\mathrm{D}\left(X_1 + ... + X_N|N\right)\right) + \mathrm{D}\left(\mathrm{E}\left(X_1 + ... + X_N|N\right)\right) =$$

$$= \mathrm{E}\left(N\mathrm{D}\left(X|N\right)\right) + \mathrm{D}\left(N\mathrm{E}\left(X|N\right)\right) = \mathrm{E}\left(N.\sigma_X^2\right) + \mathrm{D}\left(N.\mu_X\right) =$$

$$= \sigma_X^2 \mathrm{E}(N) + \mu_X \mathrm{D}(N) = \mu_N.\sigma_X^2 + \sigma_N^2.\mu_X^2 = \mu_N.\mathrm{D}X + \mu_X^2.\mathrm{D}N\,.$$

The collective risk model is described only theoretically. Further research is needed particularly regarding its implementation for aggregate loss assessment of the critical infrastructure elements due to natural disasters for a given time interval.

3. Guidelines for Implementing the Collective Risk Model by Cloud Computing

Cloud computing is an on-demand service model for IT provision based on virtualization and distributed computing technologies. Cloud computing is a model for enabling ubiquitous, convenient, on-demand network access to a shared pool of configurable computing resources (e.g., networks, servers, storage, applications, and services) that can be rapidly provisioned and released with minimal management effort or service provider interaction. This cloud model is composed of five essential characteristics**,** three service models, and four deployment models [8].

Cloud Computing is implemented through three service models that have matured from a broader set of related technologies - Software as a service (SaaS), Platform as a service (PaaS) and Infrastructure as service (IaaS).

SaaS is a software application delivery model where the responsibility for software delivery and application management lies within a certain service provider. In this service oriented model a software company develops a web-native software application for use by clients which is hosted and operated independently or through a third-party vendor over Internet. Clients do not own the software itself but pay for it on the basis of its actual usage. The service is used through an Application Programming Interface (API) which is Web accessible and the code is implemented by using Web Services.

The main advantage of the SaaS model is the IT operational cost reduction, especially small to medium-sized businesses where high investment costs could be saved in the areas of IT implementation and infrastructure maintenance.

A key feature of cloud computing is that information of the users is stored in multiple, geographically dispersed data centers that provide extensive backup, data archive and failover capabilities. This includes a multi-level backup strategy of disk-to-disk-to-tape data backups which ensure maximum recovery speed with minimum potential for data loss. Major suppliers of cloud computing infrastructure such as Salesforce.com provide very high levels of service availability through virtualized servers at multiple data centers. Users of web-based services have both their data and server availability protected in the event of a natural disaster.

The data involved in emergency management includes geographical data about the infected area, data about shelters and available transportation means, data about victims and relief personnel, available rescuing resources, and measurements from the field. The data may belong to multiple autonomous organizations, such as government organizations, non-governmental organizations, international non-governmental organizations, individuals, communities, and industries. Therefore, besides integrating and manage data from these different organizations, there is a need to coordinate these organizations by enabling efficient communication and collaboration.

Risk assessment as a service is a new paradigm for measuring risk as an autonomic method that follows the on-demand, automated, multi-tenant architecture of the cloud – a way to get a continuous risk score of the cloud environment with respect to a given tenant, a specific application or more generally, for use by new tenants and applications Risk assessment provides a significant value in increasing trust in a commercial

service, and thus appear particularly beneficial to the adoption of cloud computing. Traditional assessments developed for conventional IT environments do not readily fit the dynamic nature of the cloud [9].

The following issues must be resolved in order to implement the proposed collective risk model as a SaaS application:

- Multi-Tenancy - designing the application in such a way that it supports a huge number of potential users simultaneously.
- Security - customer data must be secure and completely separated from other user's data.
- Scalability - the application should operate with many individual users.
- Availability - the application must be always online.
- Performance – the application should provide for smallest processing times.
- Customizable GUI – the application should allow the customization the GUI.
- Customizable Business Logic – the application should provide the addition of the business logic from different users.
- Workflow - customers should be allowed to add their own workflows to the application.

Implementing the proposed collective risk model as a SaaS application is to make what is sometimes a complex process simpler, changing the dynamics of how software is purchased, consumed and maintained, and is quickly becoming an effective IT delivery option.

4. Conclusions

The collective risk model essentials are presented. This actuarial model is proposed to be used for aggregate loss assessment of the critical infrastructure elements due to natural disasters for a given time interval. Guidelines for implementing the collective risk model by Cloud Computing technology are outlined. The risk assessment results can support the stakeholders to take more informed decisions regarding the efficient allocation of the available funding for the improvement of the critical infrastructure protection from natural disasters.

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