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Effectiveness of an Improvised Abacus in Teaching Addition of Integers

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Abstract. This study was conducted to determine the effectiveness of an improvised abacus as remediation device in teaching addition of integers. This study employed the one-shot experimental design, which involved fifty low-performing prospective elementary teachers. Data were gathered using a multiple choice test for pretest and posttest, and completion test for the delayed posttest. Observation and interview were also conducted to find out their behavior while learning and the effects of using the improvised material. Findings showed that the use of the improvised abacus significantly reduced their errors and improved their performance and retention of skills in addition of integers, and improved their confidence in learning mathematics. Moreover, the learning styles of tactile and kinesthetic learners were addressed with the use of the material. Thus the improvised abacus promotes cognitive, affective and psychomotor learning. Results of this study verified the positive effects of manipulative materials on student achievement in mathematics.

Keywords: Improvised abacus, addition of integers, prospective elementary teachers

1. Introduction

John Dewey, early in the 20th century, stressed that learning comes from experience and active involvement of the learner. Jerome Bruner [1] advocated the use of physical manipulatives as providing scaffolding for abstract concepts. Even Jean Piaget was aware of the significance of manipulatives in a child's construction of logical-mathematical knowledge [2]. Similarly, Goldstone and Son [3] contended that "abstract understanding is most effectively achieved through experience with perpetually rich, concrete representations."

Manipulatives are physical models that represent concretely abstract concepts and appeal to the senses, can be touched or moved [4]. "Manipulatives can help children understand and develop mental images of mathematics concepts" [5]; they provide students a concrete basis from which abstract thinking develops.

Notable examples of concrete representations are Dienes blocks, spinners, number lines, geoboards (boards with a lattice of pegs and loose rubber bands to wrap around the pegs), Cuisenaire rods (colored wooden bars cut to integer lengths), algebra tiles, and balance beams.

In an effort to de-abstract mathematics, the use of manipulatives brings it to the concrete level. Recognizing the benefits of manipulative materials in learning Math, the National Council of Teachers of Mathematics in their 2000 Principles and Standards states that "concrete models can help students represent numbers and develop number sense; they can also help bring meaning to students' use of written symbols…" [6].

Researches, such as those conducted by Freudental [7], Janvier [8], Resnick [9], among others, have found out that students experienced extreme difficulties in conceptualizing and performing operations with negative numbers in the pre-algebraic and algebraic scope. Betaño [10] and Gamido [11] noted that college freshmen had difficulties in operations on signed numbers, particularly on addition and subtraction. Similar situation was experienced by the researcher wherein future elementary teachers had much difficulty on

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addition of integers. Their difficulty in this operation is quite alarming considering that they had studied this topic since first year high school. This can be attributed to their inability to comprehend the rules applied to this operation. They may need physical representations to see connections between the abstract and concrete which will help them understand and develop mental images of this process.

The main purpose of this study was to determine the effectiveness of an improvised abacus in teaching addition of integers among prospective elementary teachers. This study specifically aimed to measure the effectiveness of the manipulative material in terms of corrected students' errors, level of improvement it caused and retention of skills in adding integers, and attitude toward mathematics.

2. Methodology

This study made use of the one-shot experimental design. The subjects of this study were the Bachelor of Elementary Education (BEEd) freshmen in a teacher education institution in Tarlac City. Pretest in addition of two-digit integers, a 25-item multiple choice test, was administered to 250 BEEd freshmen and the 50 lowest performing students were selected as respondents. The pretest in addition of integers has a moderate difficulty level of 0.61, a discrimination index of 0.64, and a reliability index of 0.90.

After the administration of the pretest, the results were analyzed to determine students' common errors, which were addressed by the remedial teaching with the use of the improvised abacus. The respondents were taught by the researcher with the use of the manipulative for three days, one hour each day. They were guided in completing the learning activities because low performing students need their teacher's assistance to understand the lesson [12]. They were allowed to use the manipulative material even when answering formative tests if they had not yet reached the abstract learning stage. They were also given ample exercises to master addition of integers. Their answers were immediately checked by their peers with the guidance of the researcher in order to provide them feedback on their knowledge of the lesson.

The next day after the teaching period, the posttest, which is the same as the pretest, was administered to determine the effectiveness of the improvised abacus a remediation device. Then after a week, the delayed posttest, a 25-item completion test on two-digit and three-digit addition of integers, was conducted to measure the skills retained among the students. The manipulative was no longer used in the posttest and delayed posttest to determine whether the students can visualize the processes in addition of integers.

3. Results and Discussion

3.1. Corrected students' errors in addition of integers

Table 1 shows the performance of the students in addition of integers. In the pretest, the students obtained a mean percentage score (MPS) of 32.5%. This implies that the overall percentage of errors committed by the students was 67.5%, which reveals that they experienced much difficulty in addition of integers. This further shows that for every 10 items, they were able to answer only 3 items correctly.

Table 1. Students' Performance in Addition of Integers

Test	Mean	**Mean Percentage Score**	**Standard Deviation**
Pretest	8.12	32.5%	3.01
Posttest	18.96	75.8%	4.62
Delayed posttest	22.04	88.2%	3.53

In the posttest, their MPS was 75.8%. This means that their overall percentage of errors, 24.2%, was reduced by 43.3%. In the delayed posttest, their MPS of 88.2% indicates that their percentage of errors was only 11.8%. This indicates that the manipulative material was an effective device in correcting students' errors in addition of integers.

3.2. Level of improvement

Most students had low scores in the pretest. This verifies that the students are low performing in addition of integers (mean = 8.12). On the contrary, most of the scores are high in the posttest (mean = 18.96). This high performance of the students resulted from the use of the manipulative material. This confirms that the

use of the improvised manipulative material greatly improved their performance in addition of integers. Moreover, their performance became even better in the delayed posttest (mean = 22.04) than that in the posttest.

Table 2. T-Test between the Means of Pretest and Posttest in Addition of Integers

Statistics	Pretest	%	Posttest	%	Difference	%
Mean	8.12	32.48	18.96	75.84	10.84	43.36
Standard Deviation	3.01		4.62		4.88	
$t = 15.7218$		$df = 49$		Probability of $t = 8.8945 \times 10^{-21}$		

The computed *t*-value for the difference between the means (10.84) of the pretest and posttest was 15.7218, which was very significant at $p < 0.001$. Thus, there is significant difference between their scores before and after being taught with the use of the improvised manipulative material. The findings of the study that the use of manipulative material helps students concretize and understand abstract mathematical ideas which result to improved performance are similar to those of Chester, Davis and Reglin [13], Cotter [14], Fueyo and Bushel [15], Garrity [16], Leinenbach and Raymond [17], Mercer, Miller and Witzel [18], and Nalipay [19].

3.3. Retention of skills

Another indicator of the effectiveness of the improvised manipulative material is the retention of skills in addition of integers the students gained. Students' retention of skills is measured by comparing their posttest and delayed posttest scores.

Table 3. T-Test of Difference between Means of Posttest and Delayed Posttest in Addition of Integers

Statistics	Posttest	%	Delayed Posttest	%	Difference	%
Mean	18.96	75.84	22.04	88.16	3.08	12.32
Standard Deviation	4.62		3.53		2.91	
$t = 7.4775$		$df = 49$		Probability of $t = 1.2124 \times 10^{-9}$		

The statistical analysis in Table 3 shows the significance of the difference between means (3.08) in the posttest and delayed posttest in addition of integers. The computed *t*-value is 7.4775 and the probability of alpha accounted by *t* is 1.21×10^{-9}. This indicates that there is significant difference between their posttest and delayed posttest scores. The performance of the students in addition of integers still improved significantly even a week after the teaching session with the use of manipulative. Their scores further improved in the delayed posttest because the use of the improvised abacus promotes visualization and enhances retention of the concepts and processes of integer addition.

The use of manipulative materials allows students to learn by doing. Manipulative use has indeed been found to yield positive outcomes for learner's understanding in different levels of mathematics learning from elementary to college levels. The results of the study confirm the benefits of using manipulative materials. They help students understand and create mental images of concepts and processes involved in addition of integers [5] since it provides them a concrete basis in building, strengthening, and connecting representations of mathematical ideas [4]. Students' learning is enhanced since they are actively, physically and mentally involved in the learning process. The use of the improvised manipulative also supports students' transition from concrete to representational to abstract thinking (visualization) which enhanced their retention about addition of integers [18].

In addition to promoting conceptual understanding and lasting learning of the topic covered in the study, the students who used the improvised abacus were more engaged, motivated and participated actively during the discussion [19], [20]. Furthermore, they commented that they enjoyed using the manipulative because it is easy to use, fun, very helpful, very comfortable, and highly interactive [16], [17], [21] and that their confidence in adding integers improved [17].

4. Implications and Conclusion

The participants in this study who are college freshmen and have been taught how to add integers since their first year high school still have numerous errors and misconceptions. This means that their high school Math teachers failed to diagnose and correct their errors.

Low performers have the ability to learn provided that they are properly guided and appropriate materials and intervention strategies are used to address their difficulty [11], [12], [22], [23], [24]. In this study, the use of the manipulative material is effective in improving students' understanding of integer addition. This suggests that mathematics teachers should explore effective materials and methods of helping students, especially the low performers comprehend math concepts so that they will be enabled to succeed in their future math studies. One of the possible ways of helping them is through the use of concrete materials.

Students often describe Math as a boring and terrifying subject. This is due to their difficulty to visualize and understand its underlying symbolic concepts and processes. The use of appropriate manipulative is one way of actively engaging them in the learning process. They can meaningfully touch and move the concrete materials to make visual representations of mathematical concepts and to see connections between the abstract and concrete which promote systematic transition from concrete to pictorial to abstract levels of thinking. When students understand the mathematical abstractions, they will become confident and motivated in learning the subject which will make them realize that Math is also interesting and enjoyable. Thus the use of manipulative is a good device in promoting cognitive, psychomotor and affective learning.

Furthermore, the use of manipulative materials supports the learning style of kinesthetic learners who need to move some parts of their body to learn and of tactile learners who need something to manipulate to fully understand the lesson.

In general, the improvised abacus is effective in correcting students' errors and reducing their difficulties in addition of integers. The high performance of the students in addition of integers is due to the use of the manipulative material. This validates that the use of the improvised abacus highly improved their performance in addition of integers. The performance of the students in addition of integers still improved significantly a week after the teaching session with the use of manipulative. This verifies previous findings that the use of the manipulative promotes visualization and enhances retention of the concepts and processes involved in addition of integers.

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