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Exploring the Metacognitive Processes of Prospective Mathematics Teachers during Problem Solving

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Abstract. This qualitative research explored the metacognitive processes of 33 prospective mathematics teachers (PMTs) while solving individually a nonroutine problem. After attempting the problem the PMTs completed a questionnaire that asked them to describe the metacognitive actions they applied. The successful and unsuccessful problem solvers were also interviewed to provide further insight on the significant role of metacognition in problem solving. Analysis of their written work and questionnaire responses revealed that very few PMTs were able to monitor successfully their cognitive processes. They had metacognitive inability to monitor their comprehension effectively; to use correct representation of the problem conditions; and to properly respond to a situation that triggers risk of getting incorrect answer such as failure to detect and correct error and to verify the accuracy and sense of result.

Keywords: Metacognitive processes, problem solving, prospective mathematics teachers

1. Introduction

The art of problem solving is the heart of mathematics. The National Council of Teachers of Mathematics (NCTM, 1980) recommended that problem solving must be the focus of mathematics teaching. Kilpatrick (1985) noted that despite having necessary skills and concepts, students are not always able to successfully complete problems. A primary source of difficulties in problem solving has been suggested as a student's inability to actively monitor and regulate their own cognitive processes (Garofalo & Lester, 1985; Schoenfeld, 1987; Goos et al., 2000; Biryucov, 2004).

Metacognition is the knowledge and awareness of one's cognitive processes (Flavell, 1976) and the ability to monitor, regulate and evaluate one's thinking (Brown, 1987). Metacognition has been found by some researchers to be a key factor in successful problem solving (Schoenfeld, 1987; Gourgey, 1998).

In the review of progress on mathematics problem solving research from 1970–1994, Lester (1994) noted that research interest on this area appeared to be on decline. One issue highlighted by Lester was the role of metacognition in problem solving. To investigate the important role metacognition plays in problem solving, this study was conducted.

This study aimed to explore the metacognitive processes of prospective mathematics teachers (PMTs) during problem solving. Specifically, it intended to analyze the strategies they use in solving a nonroutine problem and to investigate the metacognitive processes they apply during problem solving activity.

2. Methodology

This qualitative study examined the metacognitive processes of PMTs with the use of the Episode-Based Model of Metacognitive Activity developed by Goos, Galbraith and Renshaw (2000). This study involved 33 second year mathematics major students in a teacher education institution in the Philippines. Among them: 18 were future elementary school mathematics teachers and 15 were prospective high school mathematics teachers.

Since the aim of the study was to examine the metacognitive processes rather than simply assess mathematics expertise, it was necessary to supply a nonroutine problem that would challenge the students

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and was suitable for the study of metacognitive process. Nonroutine or novel problem refers to a task the subject has not previously seen and which is not closely similar to a previously done problem. The nonroutine problem given is as follows:

In a class of 36 boys and girls, 10 pesos (P10) is to be divided among them such that each girl receives 8 centavos less than each boy. How much does each boy and each girl get?

Students were given written problem statement and allowed 45 minutes to solve the problem. They were instructed to show all their working and cross out, rather than erase, any working they think was incorrect.

The Self-Monitoring Questionnaire was used to implicitly investigate the students' metacognitive processes while working on a given nonroutine mathematics problem. The questionnaire was adopted from the study of Goos, Galbraith & Renshaw (2000). It consists of twenty statements to which students responded by checking columns marked Yes, No or Unsure. Only at the end was the questionnaire administered, to avoid cueing students on the strategies it listed. After their written works were analyzed, the two successful and two unsuccessful problem solvers were individually interviewed for about 10 minutes each to probe the metacognitive processes that occurred during problem solving.

The written solutions were scrutinized individually to identify the strategies used by the students in attempting to solve nonroutine problem. Likewise, questionnaire responses were examined particularly the responses to statements implicitly assessing the metacognitive processes expressed by the students while solving the problem. These replies were compared against the solution scripts to verify their accuracy.

3. Results and Discussion

3.1. Strategies students use in solving a nonroutine problem

Of the thirty-three PMTs who attempted the problem, only two got one correct answer, 31 (94%) gave incorrect answers and nobody got the more than one answer. Majority of them used "calculate mean value" strategy in solving the problem. The present study had similar findings with those of Goos, et al. (2000) in the sense that most of the students utilized the strategy "calculate the mean value."

Since answers alone do not reveal how students attacked the nonroutine problem, their written solution works were also examined to identify the strategies they had used. The three types of strategies that emerged are discussed below. (There were cases that some students tried more than one strategy, so their work was classified according to the strategy that produced their final answer.)

Strategy #1: Trial and Error (guessing and checking). Six students selected pairs of (b, g) and (c, $c - 8$) values that satisfied the explicit conditions of the problem and carried out trial substitutions into the rule. The students who utilized this strategy had little chance of finding a correct answer because it is difficult to systematically test all possible combinations of values for all the variables. Of the six who used this strategy, three almost accidentally found one of the four correct answers (their answer was: 19 boys received 24 centavos each and 17 girls received 32 centavos each). However, this answer violated the problem condition which required that each girl should receive 8 centavos less than each boy. Thus, 19 girls should have received 24 centavos each and 17 boys received 32 centavos each.

Strategy #2: Formulation of Equation. This method used by eight students resulted from their misinterpretation of the problem statement, which made them incorrectly represent the information it contained. All of them produced various incorrect answers.

Strategy #3: Calculate the Mean Value. Majority of the students who used this approach fixed the value of c and allowed b and g to vary. The mean value was P10 ÷ 36, or 27.8 centavos per student.

Finding a correct answer through this mean value suggested trial and error depending on first guessing of correct (c, $c - 8$) pair and then conducting a search for pairs of b and g. Although the mean value provided a fruitful starting point in this particular case (the correct c values were 32, 24; 30, 22; 28, 20; and 26, 18), the success of the two students who chanced to find one of the answers was due to luck rather than a correct formulation of the problem.

All of the students gave only one answer, either correct or incorrect. This event can be accounted for their belief that all mathematical problems have only one correct answer, which they have acquired from their experiences in learning mathematics.

3.2. Metacognitive processes students apply during mathematical problem solving

A high frequency of Yes responses was recorded for almost all Self-Monitoring Questionnaire statements referring to metacognitive processes demonstrated by the PMTS while solving the problem. These results seemed to suggest that they were immersed in metacognitive activity but it is unwise to accept self-reports of this kind of information relating to regulation of cognition which is not necessarily statable (Brown, Bransford, Ferrara and Campione, 1983). Therefore, their questionnaire responses must be interpreted in the light of their actual problem solving process which was manifested in their written solution work.

In the following section, PMTs' written solutions were examined and where needed, compared with their responses to the questionnaire statements, to reveal the metacognitive processes of the respondents while solving mathematical problem.

Clarifying task requirement is the first essential sub-component of metacognitive process needed to successful problem solving. Unless a student fully understood what the problem is asking and its implied conditions, he cannot start to derive a solution to the problem. It is necessary to ask and assure oneself if he truly understood the problem.

Analysis of the students' written solution attempts revealed that only 6 (18%) fully understood the problem and 27 (82%) did not totally understand what the problem was asking. Based on the questionnaire responses, 9 (27%) were unsure whether they understood the problem or not while 18 (55%) who claimed to have understood the problem reported their thinking inaccurately.

Of the six students who fully understood the problem, only two found a correct answer by calculating the mean value. The other four who used the same strategy accepted an answer that was either insensible (non-integral number of centavos per student) or inaccurate (the total did not come to □10).

Reviewing progress means asking oneself whether he is getting closer to a solution. If no progress is found, one should think of other methods and try a different approach. Recognizing error relates to checking work step by step as the student goes through the problem to detect error in computation and strategy execution, after which he has to redo some working to correct any identified error.

Successful self-monitoring is difficult to detect if it merely confirms that satisfactory progress is being made (Goos, et al., 2000). However, the students' solution scripts provided proof of self-monitoring where difficulties or errors forced a change of strategy. One of the two students who found a correct answer changed strategy due to lack of progress in formulating the problem algebraically.

As their questionnaire responses and written works revealed, all of the students reviewed progress since they had produced an answer to the problem, either correct or incorrect, and majority (82%) recognized and corrected error.

Contrary to the questionnaire responses reporting that 22 (67%) checked their answer for accuracy and 22 checked the result for sense, analysis of students' written work suggested otherwise. Eighteen (55%) had no evidence of verification in their written solution. Two students who used faulty verification procedure utilized incorrect formulation of equation. Of the five students who verified integral centavos whose sum was P10, two of them produced a correct answer using "calculate the mean value" strategy whereas the three used "trial and error." Although the number of centavos received by the boys and girls were integral (whole number), it violated the problem condition stating each boy should receive 8 centavos more than each girl because they accepted the answer wherein the number of centavos received by each girl was 8 centavos greater than each boy (an opposite of the problem requirement). This shows that they did not fully understand the problem.

The three students who used "calculate the mean value" strategy verified and accepted non-integral number of centavos amounting to P10. Although their answer satisfied the problem condition, they should have discarded the non-integral values which did not make sense. Likewise, the two students who verified non-integral number of centavos not totaling to exactly P10 should have rejected their answer and thought of a different approach to solve the problem. The five students accepted non-integral values because they focused only on the computational aspect of the problem without considering its context and implied conditions.

To gain additional insights on the metacognitive processes of the four students while solving the nonroutine problem, the two successful (S1 and S2) and two unsuccessful (U1 and U2) problem solvers were interviewed by the researcher (R).

The following questions investigate clarifying task requirement.

R: Did you think whether there was something in the information given in the problem that needed special attention?
S1: Yes, sir.
R: What was that?
S1: I need to find out the number of boys and girls to know how many centavos each student would receive.

Both students who produced a correct answer and one who was almost successful in solving the problem clarified task requirement. They observed that it was important to determine the number of girls and boys before finding the amount each student received. The other unsuccessful student did not notice this fact that's why while he was solving, he was not sure whether what he was doing was right and he did not know how he was going (This student formulated an equation, which was incorrect, in solving the problem). On one hand the three students were certain that what they were doing was working and knew how they were going since they fully understood the task.

The following questions explore reviewing progress.

R: What did you do when you got stuck on the problem?
U1: I read the problem again and made sure I had not missed anything the problem gave.
S1: I tried different approaches and started again.
R: What prompted you to change your approach?
U2: I was not able to find an answer.
S1: The strategy did not lead to any solution.

The one successful and two unsuccessful solvers changed their way of working when they were not getting a solution while the other successful problem solver arrived at a solution without changing her strategy.

The next question examines recognizing error.

R: What did you do to identify your mistakes?
S2: I checked my calculation a few times.
U2: I paused at some stages in the problem to check what I had done before going on.

The four students easily detected and corrected very few errors in computation as if it was a spontaneous act while solving problem.

The following questions investigate assessing result for accuracy and sense.

R: How could you tell whether you solved the problem correctly?
S1: I checked my answer to find out whether the solution satisfied the conditions.
S2: I made sure that the result was sensible.
R: When you found out that the result was incorrect, what did you do?
S1: I assessed the correctness of my strategy.
U2: I changed my strategy and tried a new approach.

The four students verified whether their answer met the problem requirement. Upon knowing that their answer did not satisfy the problem condition, the three students (1 successful and two unsuccessful) considered other ways of working until the number of centavos each student received totaled exactly to P10. One unsuccessful solver checked his answer using a formulated equation, which was incorrect, that resulted to his acceptance of wrong answer. This student was the one who did not fully understood the problem.

In assessing result for sense, the four students were sure their answer was sensible. However, the one unsuccessful problem solver who used "calculate the mean value" strategy accepted a non-integral value of centavos, which is in fact, has no sense. This student could have found a correct answer if only he discarded non-whole number of centavos whose sum was P10, and searched for other values that would make sense. The other unsuccessful student who did not fully understand what the problem was asking resulted to formulation of (incorrect) equation which produced an incorrect answer though result verification was done using incorrectly formulated mathematical sentence.

4. Implications and Conclusion

The strategies used by the PMTs are categorized as inappropriate strategies. This means that they have poor background in using systematic approach and efficient strategies such as algebraic reasoning and verbal reasoning in solving problems. Mathematics teachers should show different ways of solving a problem so that students will not have misconception that there is a definite formula or procedure in solving problem. Using various approaches in attacking some mathematics problems could also lead to more than one answer. This will erase the students' incorrect notion that there is only one answer to some problems.

Majority of the PMTs concentrated only on the computational part of the problem and failed to consider its context. This result is not unexpected because most of the problems students solve in mathematics classes are devoid of real-life connections and only meant to cultivate analytical thinking. So when students are exposed to problems that involve realistic situations, they frequently give accurate but not sensible answers. This poses a great task among teachers to design challenging, relevant and real-world problems that nurture students' metacognitive ability.

Very few PMTs were able to monitor successfully their cognitive processes. This implies that they have metacognitive inability (a) to monitor their comprehension effectively, that is, they do not recognize when they don't fully understand a task; (b) to use correct representation of the problem condition; and (c) to properly respond to a situation that triggers risk of getting incorrect answer such as failure to detect and correct error and to verify the sense and accuracy of result. Since a student cannot successfully solve a problem which he does not fully understand, he must ask himself whether he fully grasped what the problem was asking and recognize when he does not fully understand a task. Equally important, he should verify if the result is accurate and makes sense. To encourage students to monitor and regulate their thinking, teachers should expose them to various nonroutine problems that activate their metacognitive thinking.

Teachers as well as would-be teachers of mathematics should be given opportunities to analyze their own mathematical thinking and consider implications for classroom practices that will promote the development of students' metacognitive skills.

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