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Assessing the Number Sense of Grade 6 Pupils

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Abstract. This study assessed the number sense of Grade 6 pupils, aged 12 years. The data on pupils' number sense were collected using a ten-item number sense test. Analysis of qualitative and quantitative results revealed that the pupils obtained a very low percentage of correct responses ranging from 0.71% to 28%. The pupils were not able to make appropriate mathematical judgments such as determining the appropriateness and sufficiency of information; they did not have a reasonable understanding of fractions and decimals and operations; and they did not utilize useful and efficient strategies, like estimation and number relationships, for managing numerical situations. Their over-reliance on algorithms may be attributed to their teachers' much emphasis on drill and practice, without profound understanding of mathematics concepts.

Keywords: Efficient strategy, number sense, mathematical judgment

1. Introduction

The strong mathematical foundation of students is usually developed during their elementary years. At this period, they are expected to acquire basic and essential critical thinking and reasoning skills to effectively solve more challenging problems. However, experienced teachers have noticed that children have a tendency to hand in homework exercises without checking whether the answers they obtained were sensible. In other words, children don't seem to be using number sense when they submit answers that are absurd or wrong. Number sense refers to "the general understanding of number and operations, along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful and efficient strategies for managing numerical situations" (Reys, Reys, McIntosh, & Emanuelsson, 1999).

Preservice and in-service mathematics teachers should be provided with empirical data to understand pupils' current number sense, which will prompt them to adapt approaches that will make sure that their pupils gain a profound understanding of mathematical concepts. Hence this study was conducted to assess the number sense of Grade 6 pupils so that mathematics teachers will be aware of children's number sense, in order to understand children's way of answering Math questions.

This study specifically aimed to examine and describe the responses of the pupils in each item of number sense test; and to analyze their performance in the three components of number sense.

2. Methodology

This study involved 47 randomly selected Grade 6 pupils, 19 boys and 28 girls, of a public elementary school in Tarlac City, Philippines. The pupils were 12 years old on the average and belonged to various levels of mathematics performance as reflected in their second quarter grade in mathematics.

The ten-item paper and pencil number sense test used in the present study was adopted from the study of Menon (2004), which, in turn, was partly based on items from McIntosh, Reys, Reys, Bana, & Farrell (1997). For each item, the pupils were given four choices and spaces below the choices to give written explanations for their answer. They were given 30 minutes to complete the test.

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The components of number sense, together with the corresponding item number assigned to test them, are as follows:

- To make mathematical judgments, for example by determining the appropriateness and sufficiency of information (Items #1 and #2)
- To develop useful and efficient strategies for managing numerical situations, for example, by using estimation and number relationships (Items #3 to #5)
- The general understanding of number and operations, especially those related to fractions and decimals (Items #6 to #10)

Content analysis was utilized in order to assess the Grade 6 pupils' number sense. The answers of the pupils to the number sense test were analyzed to obtain quantitative and qualitative data to describe their current number sense. Descriptive statistics like frequency, percent, mean and rank were used to facilitate analysis of data.

3. Results and Discussion

3.1. Pupils' responses in each item of number sense test

The number and percent of correct responses of the pupils, which ranged from 2 (4%) to 44 (94%), is presented in Table 1.

Table 1. Correct Responses of the Pupils in the Number Sense Test (N = 47)

Item #	Correct Response		
	Frequency	Percent	Rank
1	3	6.4	8
2	16	34.0	5
3	12	25.5	6
4	44	93.6	1
5	37	78.7	2
6	2	4.3	9.5
7	35	74.5	3
8	2	4.3	9.5
9	9	19.1	7
10	18	38.3	4

- Item #1 Only 4 passengers are allowed in one tricycle. How many tricycles would be needed to take 9 passengers?

  (A) 2 (B) $2\frac{1}{4}$ (C) 3 (D) Not enough information given

In item #1, almost 79% of them gave $2\frac{1}{4}$ as the correct answer. This result indicates that they just worked on the computational aspect of the problem, without paying attention to the context. They failed to assess that $2\frac{1}{4}$ tricycles is an unrealistic answer, a manifestation of poor number sense since they could not make appropriate judgment based on all the given information.

- Item #2 A tricycle driver picks up 4 passengers. The passenger's ages are 9, 11, 14 and 34. What is the age of the tricycle driver?

  (A) 20 (B) 48 (C) 68 (D) Not enough information given

More than half (51%) gave 68 as the age of the tricycle driver wherein only the ages of the passengers were given, which they arrived at by adding the ages of the 4 passengers. Only a third (34%) realized that there was insufficient information in the problem.

- Item #3 About how many days old are you?

  (A) 800 (B) 8000 (C) 80 000 (D) 800 000

Twelve or 26% answered item #3 correctly. However, scrutiny of the solutions and explanations of the pupils revealed that 11 out of 12 actually computed their age in days by multiplying 365 and their age, then rounded it down. Only one used estimation to arrive at the correct answer, showing that 98% were not using efficient strategy for managing numerical situations.

- Item #4 Given that 48 + 37 = 85, what is the answer to 49 + 36?
  (A) 85 (B) 86 (C) 715 (D) 4126
- Item #5 Given that 38 + 47 = 85, what is the answer to 85 – 47?
  (A) 38 (B) 42 (C) 132 (D) 917

More than 50% of the pupils obtained the correct answers in items #4 and #5. The high percent of pupils getting the correct answers is attributed to their mechanical use of algorithm or procedure, without making use of the relationships involved among the numbers, as revealed by the analysis of their written solution and explanation. All of them did not use 48 + 37 = 85 to deduce the answer to 49 + 36, and 38 + 47 = 85 to deduce the answer to 85 – 47. Instead, they just added 49 and 36, and subtracted 47 from 85, using the traditional addition and subtraction algorithms. Hence, all of the pupils used inefficient strategies for managing these numerical situations and were over-reliant on algorithms.

- Item #6 How many different fractions are there between $\frac{2}{5}$ and $\frac{3}{5}$?
  (A) None (B) One (C) A few (D) Many

Majority (28 or 60%) believed that there was one fraction between $\frac{2}{5}$ and $\frac{3}{5}$. By either adding $\frac{2}{5}$ and $\frac{3}{5}$ or by subtracting ($\frac{3}{5} - \frac{2}{5} = \frac{1}{5}$), they reasoned erroneously that there was one fraction between the given pair of fractions. Eleven (23%) thought that there were no fractions between $\frac{2}{5}$ and $\frac{3}{5}$ because the next whole number after 2 is 3.

- Item #7 Which is the largest fraction?
  (A) $\frac{499}{500}$ (B) $\frac{500}{501}$ (C) $\frac{4568}{4569}$ (D) $\frac{4569}{4570}$

Nearly three-fourths of them got the correct answer in item #7. Among the 35 pupils, 8 or 23% chose $\frac{4569}{4570}$ as the correct answer because 4569 is the highest number among the numerators and 4570 is the largest number among the denominators. They focused only on the numerator and denominator and obtained the right answer with the wrong reasoning. Their written solution revealed that nobody had a clue on how to find the largest of the four given fractions, and that they guessed and used wrong reasoning.

- Item #8 How many different decimal numbers are there between 1.52 and 1.53?
  (A) None (B) One(C) A few (D) Many

Similar to item #6, 24 or 51% stated that there was one decimal between 1.52 and 1.53 by subtracting the former from the latter getting a difference of 0.01, which they believed to be a decimal between the given decimal pair. About 23% reasoned that the next number after 1.52 is 1.53, so there were no decimals between the given decimals. In this case, they treated decimals as whole numbers.

- Item #9 Without calculating the exact answer, which is the best estimate for 292×0.96?
  (A) slightly more than 292 (C) 292
  (B) slightly less than 292 (D) cannot tell without computing it

As regards item #9, 9 (17%) admitted that they could not tell the product without computing it, whereas 5 (11%) did not attempt to answer. Fifteen (32%) actually multiplied and 5 got the correct answer while the other 10 committed errors in their computation. Those 14 pupils (30%) who answered that the product was slightly more than 292, had a misconception that multiplication results to a larger number. One pupil showed an understanding of the concept by explaining that: "when you multiply a whole number and a decimal [less than 1], the answer is less than the whole number."

The low percent of pupils getting the correct answers, which ranged from 4% to 19%, in items #6, #8 and #9 indicated that 81% to 96% did not understand the density of rational numbers, particularly fractions and decimals.

- Item #10 If a “broken” calculator displays 6858 as the answer to 15.24 × 4.5, where should you place the decimal point in the answer?
  (A) 0.6858 (B) 6.858 (C) 68.58 (D) 685.8

Eighteen (38%) answered item #10 correctly. Eight actually worked out the product and placed the decimal point correctly without using estimation, while the other 10 just guessed on the proper placement of the decimal point. Thirteen (28%) counted the total number of decimal places in the factors, which is 3, and moved the decimal point 3 places to the left of the product, getting a result of 6.858, without thinking that this product is less than the product of the whole number factors (15 × 4) which is equal to 60. Again, the over-reliance on algorithms was evident, with no reliance on estimation and making sense.

3.2. Pupils’ performance in the three components of number sense

Table 2 shows the performance of the pupils in the three components of number sense, namely: ability to make mathematical judgments (MJ), ability to develop useful and efficient strategies for managing numerical situations (ES) and the general understanding of numbers and operations (GU).

Table 2. Correct Responses to the Components of Number Sense

Number Sense Component	Item Number	No. of Correct Response	Maximum No. of Correct Responses	Percent
MJ	1 – 2	19	94	20.21
ES	3 – 5	93 (1)	141	65.96 (0.71)
GU	6 – 10	65	235	27.66

It can be inferred from Table 2 that majority of the pupils were unable to make appropriate mathematical judgments (80%), but were able to develop useful and efficient strategies for managing numerical situations (66%). However, given that two of the three items under ES, items #4 and #5 were correctly obtained by not using relationship by all of the pupils who got the correct answer (as well as those who did not get the right answer), but by direct computation using algorithms. Likewise, item #3 was answered correctly by 11 out of the 12 pupils by not using estimation but by actually multiplying, thus a reevaluation of 66% was in order. Then, the total number of correct responses for items #3, #4 and #5 would be only 1. This would result to one correct (number sense) response for component ES, giving the percent of correct response in ES to be 0.71% and not 66%. As to the component GU, only about 28% of the pupils seem to have a reasonable understanding of fractions and decimals.

Overall, the percent of correct number sense responses ranges from 0.71% to 28%. This indicates that the Grade 6 pupils have poor number sense since majority of them were unable to make appropriate mathematical judgments, did not utilize useful and efficient strategies for managing numerical situations and did not have a reasonable understanding of numbers and operations.

4. Implications and Conclusion

Math teachers must understand learners’ current number sense, by using number sense test and other appropriate diagnostic tests, so that they can help their pupils develop deep understanding of math concepts and processes.

Since the results of the study clearly reveal that the pupils have poor number sense, Math teachers must reflect on their ways of teaching the basic concepts to children. The emphasis on drill and practice, without profound understanding of fundamental mathematics as suggested by Ma (1999), might have caused this problem. If that is the case, we may be preventing them from acquiring “greater facility with analyzing and making sense of data and deeper conceptual understanding of mathematics.” Math teachers should use concrete materials, pictorial representations and authentic problems within the learner’s experiences and expose them to a wide variety of useful and efficient strategies such as estimation and number relationships to make sure that learners gain deep understanding of Math concepts.

One of the major goals of mathematics instruction is aimed at conceptual understanding rather than at mere mechanical skills, and at developing in students the ability to apply the skills with flexibility and resourcefulness (Schoenfeld, 1992). Likewise, the National Council of Teachers of Mathematics *Principles and Standards for School Mathematics* (NCTM, 2000) suggests that teachers need to emphasize number sense. However, it seems that teachers are unwilling to let go of the adage that "practice makes perfect," with practicing algorithms made synonymous with "practice" (Menon, 2004).

The pupils who are about to finish their elementary education do not have good number sense. Thus, similar studies should be conducted to different levels of learners to make more conclusive results of students' number sense. In case similar findings are obtained, further investigation is needed to identify the causes of students' poor number sense and the solutions to effectively address this problem. Preservice and in-service Mathematics teachers' number sense, content knowledge and teaching practices should also be examined to identify possible causes and determine effective solutions to pupils' poor number sense. A comparative study on the number sense of students taught with an emphasis on conceptual understanding and of those taught with an emphasis on drill and practice may also be conducted.

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