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Analysing a modelling problem of the production structure

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***Abstract*— one of the models that surprise technological and production aspects are the one used for the determining of a production structure based on a given period. In the present paper we have worked based on an application, on the solving and than on the analyzing of such a problem. One follows the drafting of an optimal production program for the next months, according to the demands of the market and to the available resources that can maximise the total project of the firm, in the case of the establishment of three more work point. The program used in solving this application is WinQSB, and the model - Linear and Integer Programming.**

Keywords- objective function; restrictions; variables; resources; surplus; shadow price.

I. INTRODUCTION

The method for solving a linear programming problem depends on the complexity of the said issue, namely on the variables that define the problem.

If the number of variables is at most equal with two one can use the graphic method:

- The admissible field is presented in which two variables may take values and the family of axes given by the objective function is designed.
- The graphic interpretation linear programming of problems allows the classification of the solutions into admissible and optimal ones.
- There may be an optimal solution or infinity of optimal solutions.
- If the number of variables is higher than two, the graphic solution becomes more difficult or even impossible.

For the cases of the problems with a higher number of variables, the solving is done algorithmically, by computer. The best known algorithm is SIMPLEX, capable of solving problem of huge sizes (tens of thousands of variables). Starting from the basic variant developed in 1947 by G. Dantzig, many more variants of the algorithm have been developed, and among whom a tabular one that is suitable for problem of reduced sizes that can be solved even manually.

In the case in which the type of one of the variables is specified to be whole or binary, the solving method for the linear programming problem will be „Branch and bound" [1].

Linear programming deals with a special issues class of optimization, witch appears mostly in economical applications. The problems consist in maximizing or minimizing a linear function (objective function), whom variables must satisfy:

- A relation system given by some unstitch linear equations and/or in equations named restrictions
- The demand of taking only non negative numeric values.

The solving method depends on the complexity of the problem, meaning the number of variables that defines the problem.

Let's take into consideration the case in witch the problem of maximization is standard:

$$\begin{cases} (\max)f = cx \\ Ax = b \\ x \geq 0 \end{cases} \qquad (1)$$

Represents the optimizing model of company's activities.

The elements a_{ij} of the matrix „A" represents the consumption of resources; the components b_{ij} of free terms vector, „b", is the limit of available resources, and components c_j of objective function coefficients vector „c", may be profits or unit benefits.

It could occur that one of these elements not to be recognized, them moving around some probable values. In those situations it's utilized sensitivity analysis that has the objective the study of optimal solution stability in an optimization issue.

In the linear case, for each constant it could be determined a variation interval called stability interval, with the property that optimal base or associated solution to remain unchanged.

There are situations when the constants of the model remain unchanged and fixed for a period of time, part of them suffering small or big changes when passing through other period of time. What is the effect of those changes over the optimal solution? [5]

The optimising of the linear programming problems is an essential part of the linear programming. The post-optimising may lead to supplementary contribution of the problem. This includes, in fact, to improvement processes of the solutions for the linear programming problems favoured by the knowledge of the dual results and the sensitivity analysis.

From the dual side one retains the shadow prices that show the impacts of the modification of resources on the objective function and at the sensitivity analysis one can retain the capabilities of modifying the resources without affecting the feasibility of the linear programming problems solutions [2].

For the presentation of the transfixion module of a modelling problem of the fabrication structure in an enterprise to be as suggestive as possible, an example will be chosen, using the „Linear and Integer Programming" module, of the WinQSB program.

II. THE PROBLEM DATA

The leadership of an industrial enterprise wishes to draft an optimal program of production for the next months that will maximise the total profit of the firm.

The commercial society manufactures shoes for men (P1) and women (P2), following the opening of three more new work points in different locations, namely: Bocşa, Reşiţa and Buziaş.

In table 1 the following data is presented: the maximal capacities of production, the archiving on a work point, as well as the monthly demand and the necessary volume for stowage in the case of each model.

TABLE I. THE PROBLEM DATA

Work points	Maximum capacity of production (f.u.)	Archiving capacity (v.u)	Product / variable	Unitary profit (m.u. / f.u.)	Unitary volume (v.u. / f.u)
Bocşa	2000	2150	P1 / X1	24.0	1.1
			P2 / X2	21.5	0.9
Reşiţa	3000	2800	P1 / X3	24.0	1.1
			P2 / X4	21.5	0.9
Buziaş	2500	2450	P1 / X5	24.0	1.1
			P2 / X6	21.5	0.9
The estimated monthly demand (f.u.)		P1	3500		
		P2	4500		

III. THE SOLVING OF THE PROBLEM

Based on the numerical comprised in table 1, the following conclusions have been reached:

A. Maximum profit

In fig. 1 one may see the combined report of the solution for the problem. One may notice that the maximum profit is of 169375 m.u. This value, graphically represented in fig. 2, will represent a reference value in balance to which it is necessary to evaluate all other decisions regarding production.

This value represents the superior limit of the actual possibilities and could be obtained if there were no sort of problems related to the situation of sales and in general to all that hangs on the complex act of supplying, of producing and of marketing.

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	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	1.750,0000	24,0000	42.000,0000	0	basic	21,5000	26,2778
2	X2	250,0002	21,5000	5.375,0040	0	basic	19,6364	24,0000
3	X3	500,0002	24,0000	12.000,0100	0	basic	21,5000	26,2778
4	X4	2.500,0000	21,5000	53.750,0000	0	basic	19,6364	24,0000
5	X5	1.000,0000	24,0000	24.000,0000	0	basic	21,5000	26,2778
6	X6	1.500,0000	21,5000	32.250,0000	0	basic	19,6364	24,0000
	Objective	Function	(Max.) =	169.375,0000				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	2.000,0000	<=	2.000,0000	0	10,2500	1.954,5450	2.045,4550
2	C2	3.000,0000	<=	3.000,0000	0	10,2500	2.944,4440	3.045,4550
3	C3	2.500,0000	<=	2.500,0000	0	10,2500	2.444,4440	2.545,4550
4	C4	2.150,0000	<=	2.150,0000	0	12,5000	2.100,0000	2.200,0000
5	C5	2.800,0000	<=	2.800,0000	0	12,5000	2.750,0000	2.850,0000
6	C6	2.450,0000	<=	2.450,0000	0	12,5000	2.400,0000	2.500,0000
7	C7	3.250,0000	<=	3.500,0000	249,9999	0	3.250,0000	M
8	C8	4.250,0000	<=	4.500,0000	250,0001	0	4.250,0000	M

Figure 1. Combined report of initial problem

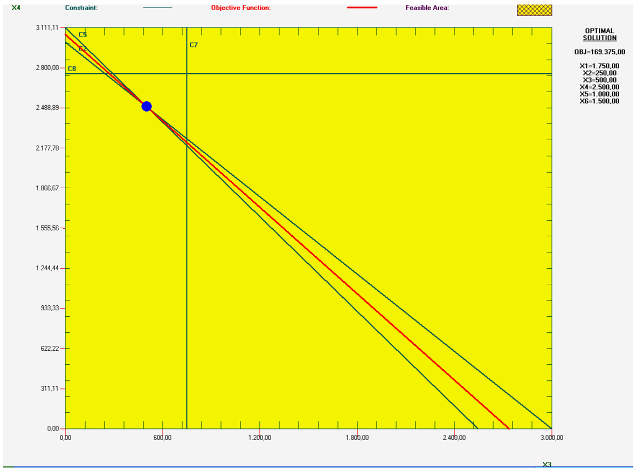

Figure 2. Graphic representation of the objective function

B. The optimal combination of products

The enterprise will obtain the maximum profit of 169375 m.u. if (fig.3):

- The work point in Bocşa produces 1750 male shoes and 250 female shoes.
- The work point in Reşiţa produces 500 male shoes and 2500 female shoes.
- The work point Buziaş produces 1000 male shoes and 1500 female shoes.

Decision Variable	Solution Value	Unit Cost or Profit C(j)	Total Contribution	Reduced Cost	Basis Status
X1	1.750,0000	24,0000	42.000,0000	0	basic
X2	250,0002	21,5000	5.375,0040	0	basic
X3	500,0002	24,0000	12.000,0100	0	basic
X4	2.500,0000	21,5000	53.750,0000	0	basic
X5	1.000,0000	24,0000	24.000,0000	0	basic
X6	1.500,0000	21,5000	32.250,0000	0	basic
Objective	Function	(Max.) =	169.375,0000		

Figure 3. Solution summary

C. *Resources consume*

There is a surplus of monthly demand (unused resource) for the restrictions C7 and C8 (fig.4) namely: 249, 9999 f.u., and respectively 250.0001 f.u.

Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price
C1	2.000,0000	<=	2.000,0000	0	10,2500
C2	3.000,0000	<=	3.000,0000	0	10,2500
C3	2.500,0000	<=	2.500,0000	0	10,2500
C4	2.150,0000	<=	2.150,0000	0	12,5000
C5	2.800,0000	<=	2.800,0000	0	12,5000
C6	2.450,0000	<=	2.450,0000	0	12,5000
C7	3.250,0000	<=	3.500,0000	249,9999	0
C8	4.250,0000	<=	4.500,0000	250,0001	0
Objective	Function	(Max.) =	169.375,0000		

Figure 4. Constraint summary

The restrictions C1, C2, C3, C4, C5 and C6 are no surplus. Through shadow price the information related to the increase of the value of the objective function by one unit of the respective resource [3] is offered, in our case of the production and stowage capacities.

IV. Analysis Of Results

As seen above the restrictions C1, C2, C3, C4, C5 and C6 are no surplus.

So, for example, the stowage space at the work point in Bocşa will increase from 2150 v.u. to 2151, the value of the objective function will increase with 12,5 v.u., thus becoming: 169375 + 12,5 = 169387,5 m.u., whereas if it increases with 40 v.u. we shall have: 169375 + (40 x 12,5) = 169875 m.u. (fig.5).

Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price
C1	2.000,0000	<=	2.000,0000	0	10,2500
C2	3.000,0000	<=	3.000,0000	0	10,2500
C3	2.500,0000	<=	2.500,0000	0	10,2500
C4	2.190,0000	<=	2.190,0000	0	12,5000
C5	2.800,0000	<=	2.800,0000	0	12,5000
C6	2.450,0000	<=	2.450,0000	0	12,5000
C7	3.450,0000	<=	3.500,0000	49,9999	0
C8	4.050,0000	<=	4.500,0000	450,0001	0
Objective	Function	(Max.) =	169.875,0000		

Figure 5. Constraint summary- modification of the archiving capacity C4

Therefore, the initial problem admits an optimal unique finite solution. The maximum profit that the enterprise may obtain is 169375 m.u.

Because of the period of crisis the unitary profit decreases at men shoes by 25% and at women shoes by 15%. One ma notice that the maximum profit decreases with 29306,2 m.u., but the optimal combination remains unchanged (fig.6).

Reiterating the initial problem and returning to figure 4, where one could notice that there is a surplus of monthly demand (unused resources) only for the restrictions C7 and C8, but not for the restrictions C1, C2, C3, C4, C5 and C6.

Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
X1	1.750,0000	19,2000	33.600,0000	0	basic	18,2750	22,3361
X2	250,0002	18,2750	4.568,7530	0	basic	15,7091	19,2000
X3	500,0002	19,2000	9.600,0050	0	basic	18,2750	22,3361
X4	2.500,0000	18,2750	45.687,5000	0	basic	15,7091	19,2000
X5	1.000,0000	19,2000	19.200,0000	0	basic	18,2750	22,3361
X6	1.500,0000	18,2750	27.412,5000	0	basic	15,7091	19,2000
Objective	Function	(Max.) =	140.068,8000				
Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
C1	2.000,0000	<=	2.000,0000	0	14,1125	1.954,5450	2.045,4550
C2	3.000,0000	<=	3.000,0000	0	14,1125	2.944,4440	3.045,4550
C3	2.500,0000	<=	2.500,0000	0	14,1125	2.444,4440	2.545,4550
C4	2.150,0000	<=	2.150,0000	0	4,6250	2.100,0000	2.200,0000
C5	2.800,0000	<=	2.800,0000	0	4,6250	2.750,0000	2.850,0000
C6	2.450,0000	<=	2.450,0000	0	4,6250	2.400,0000	2.500,0000
C7	3.250,0000	<=	3.500,0000	249,9999	0	3.250,0000	M
C8	4.250,0000	<=	4.500,0000	250,0001	0	4.250,0000	M

Figure 6. Analysing the data following the decrease of unitary profits

Modifying the value of the resources C1, C2 and C3 with 45 f.u., and the ones of the resources C4, C5 and C6 with 50 v.u., one obtains an increase of the maximum profit by 3258, 8 m.u., as well as a modification of the optimal combination of products (fig.7)

Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution
X1	1.797,5000	24,0000	43.139,9900
X2	247,5002	21,5000	5.321,2540
X3	547,5002	24,0000	13.140,0100
X4	2.497,5000	21,5000	53.696,2500
X5	1.047,5000	24,0000	25.140,0000
X6	1.497,5000	21,5000	32.196,2500
Objective	Function	(Max.) =	172.633,8000

Figure 7. the value of the objective function following the modification of the resources value C1÷C6

A. *The dual problem*

From the duality theorem, in the case in which the prime and the dual are admissible solution, they admit optimal finite solution and the values of the objective functions coincide. [4]

Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
C1	10,2500	2.000,0000	20.500,0100	0	basic	1.954,5450	2.045,4550
C2	10,2500	3.000,0000	30.750,0100	0	basic	2.944,4440	3.045,4550
C3	10,2500	2.500,0000	25.625,0100	0	basic	2.444,4440	2.545,4550
C4	12,5000	2.150,0000	26.874,9900	0	basic	2.100,0000	2.200,0000
C5	12,5000	2.800,0000	34.999,9900	0	basic	2.750,0000	2.850,0000
C6	12,5000	2.450,0000	30.624,9900	0	basic	2.400,0000	2.500,0000
C7	0	3.500,0000	0	249,9999	at bound	3.250,0000	M
C8	0	4.500,0000	0	250,0001	at bound	4.250,0000	M
Objective	Function	(Min.) =	169.375,0000				
Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
X1	24,0000	>=	24,0000	0	1.750,0000	21,5000	26,2778
X2	21,5000	>=	21,5000	0	250,0002	19,6364	24,0000
X3	24,0000	>=	24,0000	0	500,0002	21,5000	26,2778
X4	21,5000	>=	21,5000	0	2.500,0000	19,6364	24,0000
X5	24,0000	>=	24,0000	0	1.000,0000	21,5000	26,2778
X6	21,5000	>=	21,5000	0	1.500,0000	19,6364	24,0000

Figure 8. Combined Report of the dual problem

One may notice that the value of the objective function in the case of the dual problem is identical with the one in the case of primal problem namely 169375 m.u.

Any modification of the resources (increase/ decrease) influences the value of the objective function, be it primal or dual.

V. THE SENSITIVITY ANALYSIS OF THE OTIMAL SOLUTION

The stability intervals for the optimal solution in report to each of the coefficients of the objective function, presented in figure 9, are as follow:

- $21,5 \le 24 \le 26,2778$
- $19,6364 \le 21,5 \le 24$
- $21,5 \le 24 \le 26,2778$
- $19,6364 \le 21,5 \le 24$
- $21,5 \le 24 \le 26,2778$
- $19,6364 \le 21,5 \le 24$

Decision Variable	Solution Value	Reduced Cost	Unit Cost or Profit C(j)	Allowable Min. C(j)	Allowable Max. C(j)
X1	1.750,0000	0	24,0000	21,5000	26,2778
X2	250,0002	0	21,5000	19,6364	24,0000
X3	500,0002	0	24,0000	21,5000	26,2778
X4	2.500,0000	0	21,5000	19,6364	24,0000
X5	1.000,0000	0	24,0000	21,5000	26,2778
X6	1.500,0000	0	21,5000	19,6364	24,0000

Figure 9. Sensitivity Analysis for OBJ coefficients

Due to the fact that the actual values, 24 and 21, 5 m.u., of these coefficients are situated within the corresponding stability intervals, one may reach the conclusion that the optimal solution is stable in balance to each of them.

If, for example, the profit increases from 21, 5 to 23 m.u., the total profit will be: 169375 + (23 – 21, 5) (250, 0002 + 2500 + 1500) = 175750, 0003 (fig.10).

Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution
X1	1.750,0000	24,0000	42.000,0000
X2	250,0002	23,0000	5.750,0040
X3	500,0002	24,0000	12.000,0100
X4	2.500,0000	23,0000	57.500,0000
X5	1.000,0000	24,0000	24.000,0000
X6	1.500,0000	23,0000	34.500,0000
Objective	Function	(Max.) =	175.750,0000

Figure 10. the objective function after the modification of the unitary profit at the variables X2, X4 and X6

For resources, the corresponding stability intervals are (fig.11):

- $1954,5450 \le 2000 \le 2045,4550$
- $2944,4440 \le 3000 \le 3045,4550$
- $2444,4440 \le 2500 \le 2545,4550$
- $2100,0000 \le 2150 \le 2200,0000$
- $2750,0000 \le 2800 \le 2850,0000$
- $2400,0000 \le 2450 \le 2500,0000$
- $3250,0000 \le 3500 \le M$
- $4500,0000 \le 4500 \le M$

Constraint	Direction	Shadow Price	Right Hand Side	Allowable Min. RHS	Allowable Max. RHS
C1	<=	10,2500	2.000,0000	1.954,5450	2.045,4550
C2	<=	10,2500	3.000,0000	2.944,4440	3.045,4550
C3	<=	10,2500	2.500,0000	2.444,4440	2.545,4550
C4	<=	12,5000	2.150,0000	2.100,0000	2.200,0000
C5	<=	12,5000	2.800,0000	2.750,0000	2.850,0000
C6	<=	12,5000	2.450,0000	2.400,0000	2.500,0000
C7	<=	0	3.500,0000	3.250,0000	M
C8	<=	0	4.500,0000	4.250,0000	M

Figure 11. Sensitivity analysis for RHS

If one modifies the stowage capacity at the work point in Reşiţa, in this case the objective function increases with the value: 12, 5 x (2850-2800) = 625 and becomes 169375 + 625= 170000 m.u.

VI. THE PARAMETRIC ANALYSIS OF THE COEFFICIENTS OF THE OBJECTIVE FUNCTION AND OF THE FREE ITEMS OF RESTRICTIONS

If one increase the unitary profit only at women shoes in the work point Reşiţa, from 21, 5 to 24 m.u., the total profit rises from 169375 to 175625 m.u., with a 2500 slope (fig.12).

From Coeff. of X4	To Coeff. of X4	From OBJ Value	To OBJ Value	Slope	Leaving Variable	Entering Variable
21,5000	24,0000	169.375,0000	175.625,0000	2.500,0000	Slack_C8	Slack_C5
24,0000	25,8636	175.625,0000	180.750,0000	2.750,0000	X3	Slack_C1
25,8636	M	180.750,0000	M	3.000,0000		
21,5000	19,6364	169.375,0000	164.715,9000	2.500,0000	Slack_C7	Slack_C2
19,6364	17,5909	164.715,9000	160.227,3000	2.194,4440	X1	Slack_C4
17,5909	17,5909	160.227,3000	160.227,3000	55,5556	X4	Slack_C6
17,5909	-M	160.227,3000	160.227,3000	0		

Figure 12. Parametric analysis – Objective function

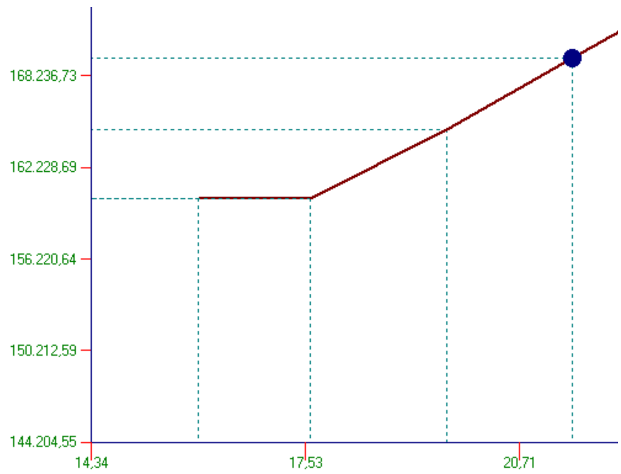

Figure 13. Graphic parametric analysis - Objective function coefficient of X2

Figure 13 shows the change in total profit whichever unit profit women's shoes at the point of working in the area of plant.

Parametric analysis to the period is free of restrictions in a similar way (Fig. 14 and 15).

From RHS of C3	To RHS of C3	From OBJ Value	To OBJ Value	Slope	Leaving Variable	Entering Variable
2.500,0000	2.545,4550	169.375,0000	169.840,9000	10,2500	Slack_C8	Slack_C3
2.545,4550	M	169.840,9000	169.840,9000	0		
2.500,0000	2.444,4440	169.375,0000	168.805,5000	10,2500	Slack_C7	Slack_C4
2.444,4440	2.227,2730	168.805,5000	164.136,4000	21,5000	X6	Slack_C6
2.227,2730	1.250,0000	164.136,4000	143.125,0000	21,5000	Slack_C4	Slack_C7
1.250,0000	0	143.125,0000	113.125,0000	24,0000	X5	
0	-Infinity	Infeasible				

Figure 14. Parametric analysis –Right-hand-side

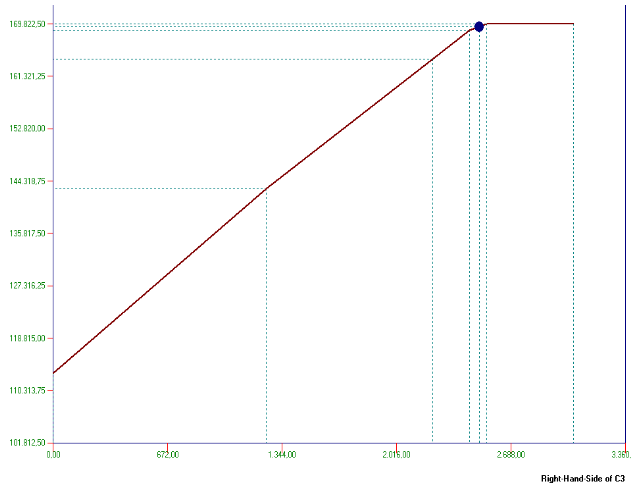

Figure 15. Graphic parametric analysis - Right-hand-side of C3

VII. CONCLUSIONS

The enterprise will obtain the maximum profit of 169375 m.u. if:

- The work point in Bocşa produces 1750 male shoes and 250 female shoes.
- The work point in Reşiţa produces 500 male shoes and 2500 female shoes.
- The work point Buziaş produces 1000 male shoes and 1500 female shoes.

The restrictions C1, C2, C3, C4, C5 and C6, the ones which represent the production and stowage capacities, have been used at maximum, but the restrictions C7 and C8 represent a surplus in the monthly demand (unused resource) namely: 249,9999 f.u, respectively 250.0001 f.u.

When the unitary profit decreased by 25% at male shoes and by 15% at female shoes, the maximum profit decreased with 29306,2 m.u., but the optimal combination remained the same.

The value of the objective function in the case of the dual problem is identical with the one in the case of the primal problem, namely: 169375 m.u.

The sensitivity analysis established that the objective function coefficient values are located inside the corresponding stability intervals, so that the optimum solution is stable in relation to each of them.

REFERENCES

[1] O.I. Amariei, „Aplications of WinQSB software in production systems simulation", Eftimie Murgu Press, Reşiţa, 2009, pp. 44-66

[2] I. Stăncioiu, "Cercetări operaţionale pentru optimizarea deciziilor economice", Editura Economică, Bucureşti, 2004, pp.79

[3] C. Raţiu-Suciu, „Modelarea & simularea proceselor economice. Teorie şi practică", Ediţia a treia, Ed. Economică, Bucureşti, 2003, pp.109

[4] C. Raţiu-Suciu, „Modelarea & simularea proceselor economice. Teorie şi practică", Ediţia a treia, Ed. Economică, Bucureşti, 2003, pp.110

Article in a conference proceeding:

[5] O.I. Amariei, C. Dumitrescu , G. Popovici , C.O. Hamat, „Presentation of Transporting and Solving Mode of an Optimisation Problem ", The 5th International Vilnius Conference „Knowledge-Based Technologies and or Methodologies for strategic Decisions of Sustainable Development" September 30 - October 3, 2009, Vilnius, Lithuania; (KORDS-2009), pages 483-488, ISBN 978-9955-28-482-6