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A Note on Inverted Yield Curve and Recession

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Abstract - **This note attempts to explain why an inverted yield curve can be a leading indicator of a recession. It develops a modified version of the extended IS-LM model with the term structure of interest rates ([3] Blanchard and Fischer, 1989) and provides a phase-diagram analysis to illustrate how an adverse shock may result in inverted yield curve as well as a subsequent recession. It demonstrates that the occurrence of inverted yield curve is an off-equilibrium phenomenon after an adverse shock in the adjustment process of interest rate and output, and that an inverted yield curve may lead, but does not lead to, a recession.**

***Keywords:* Inverted yield curve, recession, IS-LM model, term structure**

I. INTRODUCTION

It has been well documented in the literature that the yield curve serves as a leading indicator of output with most of them being empirical studies (e.g., [6] and [7], among others. See also [2] for an excellent survey on this issue, and the Federal Reserve Bank at New York provided an extensive bibliography.[1] Theoretical research to explain such a very interesting observation seems to have lagged behind until Estella [5], to the best of our knowledge. It is a subject of macroeconomics that examines the connection between interest rates and real economic activities. Thus, it suggests that the IS-LM setup is a natural candidate in modeling to investigate the issue. Since it involves the discrepancy between the short-term interest rate and the long-term interest rate, it also proposes that the term structure of interest rates should be incorporated into the traditional IS-LM model to study why an inverted yield curve may signal a following recession.

This note studies how and why the yield curve may serve as a leading indicator of real economic activities. It employs and modifies the extended IS-LM model with the term structure of interest rates introduced in [3] (pp.523-6). As a complement to [5], it attempts to provide an explicit visual setup to illustrate why an inverted yield curve may be a signal of a subsequent recession. We demonstrate that the occurrence of inverted yield curve is an off-equilibrium phenomenon during the process of adjustment in output and interest rates after an adverse shock, in particular, in the money market. As in standard equilibrium and comparative static analyses, an adverse shock, real or monetary, makes the IS or the LM curve shift. While the consequent equilibrium stands for the recession that follows, it is the adjustment path of the triplet – output, long-term interest rate and short-term interest rate – that illustrates how and why the yield curve could become inverted *prior to* the succeeding recession. Therefore, an inverted yield curve may lead, but does not lead to, a recession.

Our analysis differs from the previous studies with a similar framework (e.g., [3] and [4]) in several aspects. First, we explicitly maintain both the long-term interest rate and the short-term interest rate in the picture and equations, instead of eliminating the short-term interest rate to examine a reduced form of the model with the long-term interest rate only. Second, to show the "shape" of yield curve, we assume that the short-term interest rate is *always* on the LM curve, though the long-term interest rate is on the IS curve only at equilibrium and can be away from the IS curve when it is off equilibrium. This point of view allows us to see directly the relative positions between the two interest rates that characterize the "shape" of the yield curve in the model. Third, the dynamic adjustment process in the long-term interest rate is driven by the resultant of two component forces: the market forces in the loanable funds market as well as the connection between the long-term interest rate and the short-term interest rate tied by the term premium and the expectations (see, e.g., [1, p.105] and [9, pp. 144-50], among others). The resultant induces the long-term interest rate to adjust toward the equilibrium, and hence, it is globally stable. This property is in contrast to what have been obtained in previous studies that involve a saddle point in equilibrium, because they only have one component force to adjust the long rate in their models. Fourth, our analysis does not rely on the assumption that the long-term bond is a consol; rather, the long-term interest rate can be of any kind of long-term bonds.

A key assumption in our analysis is that the short-term interest rate is always on the LM curve. It is plausible and can be easily justified by the fact that recently most central banks have employed the short-term interest rate rather than money supply as a tool in conducting monetary policy. Hence, once the targeted short-term interest rate is announced the central bank will adjust money supply to ensure the interest rate as aimed at. This is exactly conducted by the Federal Reserve System of the Unite States through the Open Market Operations. Theoretically, the LM curve in our model essentially becomes the "monetary policy curve" as labeled in more recent macroeconomics and monetary economics literature and textbooks (See [9, p. 227]

[1] See http://www.ny.frb.org/research/capital_markets/biblio.pdf.

for a discussion on the equivalence between the two treatments in this regard).

Another twist in our model is how we characterize the dynamic adjustment process of the long-term interest rate. We borrow the concepts of component and resultant forces from physics. One driving force that moves the long-term interest rate is the market mechanism of supply and demand in the market for loanable funds, while the other one is the term structure that links the long-term interest rate to the short-term interest rate in terms of the preferred habitat theory and the expectations theory. This treatment helps illustrate when and why a monetary policy may be as effective on the long-term interest rate as intended and when it is not so. Either way, it allows us to manifestly get the picture why an inverted yield curve may arise *prior to* a recession.

II. AN EXTENDED IS-LM MODEL WITH A TERM STRUCTURE

It is assumed that the price is rigid as in typical short-run macroeconomic models when analyzing business cycle and the stabilization policy. There are two markets: the loanable funds (or equivalently, the product or goods) market and the short-run money market. Formally, we have

Product market (IS):

$$Y = C(Y - T) + I(R) + G \tag{1}$$

Money market (LM):

$$M/P = L(r, Y) \tag{2}$$

Term structure of interest rate:

$$R = \tau + (1-\alpha)\, r + \alpha\, E(r) \tag{3}$$

where Y = output, R = real long-term interest rate, and r = real short-term interest rate, T = tax, G = government spending, M = nominal money supply, P = price level, τ = term premium (> 0), $\alpha \in (0, 1)$ and E(r) = the expectation of r. Also, $C(\cdot)$ = consumption function with $0 < C' < 1$, $I(\cdot)$ = investment function with $I' < 0$, and L(r, Y) = demand function for real money with $L_r < 0$ and $L_Y > 0$. Equations (1) and (2) represent the standard IS and LM curves, and (3) describes the term structure of interest rates that links the long-term interest rate with the short-term interest rate in terms of the expectations theory as well as the liquidity premium theory (a.k.a. preferred habitat theory). We refer to it as an extended IS-LM model with the term structure of interest rates, as characterized by (1)-(3). (To the best of our knowledge, [3, pp.523-526] developed such a model to study the effects of a change in fiscal policy such as the 1981-1983 tax cuts. But they did not explicitly label both of the long-term interest rate and the short-term interest rate in their graphic analysis; instead, the short-term interest rate was eliminated and indirectly represented by the long-term interest rate in terms of the term structure and Fisher equation. Hence, their treatment did not explicitly present yield curve in the picture.)

Given the values of all exogenous variables and parameters, the equilibrium Y^*, r^* and R^* can be obtained from the model as described above. As illustrated in Figure 1, r and R are unequal in equilibrium; their gap is determined by the term premium τ and the expectation E(r) as indicated in (3). Hence, the equilibrium output is not as usually given at the intersection between the IS and LM curves. Instead, it looks like the equilibrium in a demand-and-supply model with a sales tax.

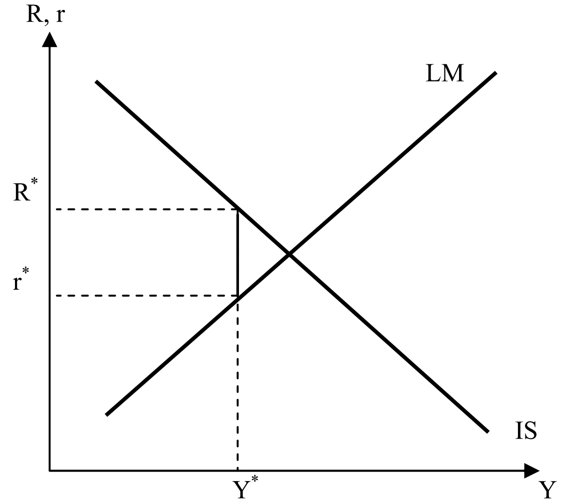

Figure1. The extended IS-LM model with term structure

With a shock or a policy change in the product market or in the money market, it causes IS or LM curve to shift and a possible change in E(r) that lead the economy to a new equilibrium eventually. The comparative static analysis in this extended IS-LM model is similar to the standard one; it compares the succeeding new equilibrium with the initial one to check what happens in Y, R and r.

Our main theme here is not the comparative static analysis *per se* in this extended IS-LM model. Instead, we want to examine the adjustment process of both the long-term interest rate and the short-term it=nterest rate as well as the output between the two equilibria *before* and *after* a shock or a policy change, allowing τ and E(r) to vary in the adjustment process as well. We analyze this issue in the next section.

III. THE DYNAMIC ADJUSTMENT PROCESS: A PHASE-DIAGRAM ANALYSIS

The adjustment processes of three variables, Y, R and r are described as follows:

$$dY/dt = \varphi\,[C(Y - T) + I(R) + G - Y], \qquad \varphi > 0, \tag{4}$$

$$dr/dt = [dM/dt - L_Y\, dY/dt]/L_r, \tag{5}$$

$$dR/dt = dR_1/dt + dR_2/dt \tag{6}$$

where

$$dR_1/dt = \eta\,[I(R) - (Y - C(Y - T) - G)], \qquad \eta > 0, \tag{7}$$

and

$$dR_2/dt = d\tau/dt + (1 - \alpha)\, dr/dt + \alpha\, dE(r)/dt. \quad (8)$$

The adjustment process for output employs the standard Keynesian approach that "spending determines the income", as indicated in (4) with $\varphi > 0$. It is worth noting that the adjustment process for the short-term interest rate as characterized in (5) is derived from (2), since it is assumed that r is always on the LM curve. This assumption can be justified by the fact that the targeted federal funds rate is actually set by the Fed and the money supply is adjusted through the Open Market Operations to keep the federal funds rate as announced. However, as Y or M changes, r may change by either moving along the LM curve or jumping as the curve shifts. Since the short-term interest rate r is always on the LM curve, whenever an off-equilibrium combination of (Y, R) is below the LM curve, it implies that the corresponding yield curve becomes inverted. In particular, when the (Y, R) combination is on the right hand side of the IS curve and below the LM curve, the output tends to decrease with the yield curve inverted.

Equation (6) indicates that the dynamics of the long-term interest rate R are driven by two forces. One is the typical price adjustment in the loanable funds market as described by (7); surplus pulls the interest rate R down and shortage pushes it up. The other is the term structure that links R to r based on the preferred habitat theory and the expectations theory as shown in (8). The two forces, as represented by dR_1/dt and dR_2/dt, respectively, may move the long-term interest rate R differently in mechanisms as well as in directions. While dR_1/dt always steers R *vertically* toward the IS curve as if water in a river flows a raft, dR_2/dt may compel R to the same direction as, or opposite direction to, dR_1/dt like a screw propeller of a yacht. The former force is enduring and one-way, whereas the latter power may be on, off, or U-turned. Hence, dR_1/dt and dR_2/dt may have the same or opposite sign. Nevertheless, it is their resultant that determines the direction and the magnitude of dR/dt, as how component forces shape their resultant in physics. Setting $dR/dt = 0$, we bring in an "R-rest" curve in the Y-R space. By construction, this curve must pass through the equilibrium (Y^*, R^*), as illustrated in Figure 2.

The phase diagram can be drawn from the above discussion on the dynamic adjustment process of Y and R. The IS curve and the "R-rest" curve (determined by $dR/dt = 0$) divide the Y-R space into four areas, labeled I, II, III and IV in Figure 2. Equation (4) reveals that at any point off the IS curve the corresponding output Y would move toward the IS curve *horizontally*. On the other hand, at any point off the "R-rest" curve, the long-term interest rate R is inclined to adjust toward the "R-rest" curve *vertically*. This is because the IS curve pulls the long-term interest rate R toward it, i.e., dR_1/dt from (7), while the curve that is parallel to the LM curve and crosses over (Y^*, R^*) also drags the long-term interest rate R toward it, i.e., dR_2/dt from (8). Also, each of these two magnetic powers gets weaker as R moves closer to the corresponding curve. Hence, the long-term interest rate R tends to move toward the "R-rest" curve as constructed. As a result, the equilibrium (Y^*, R^*) is globally stable.

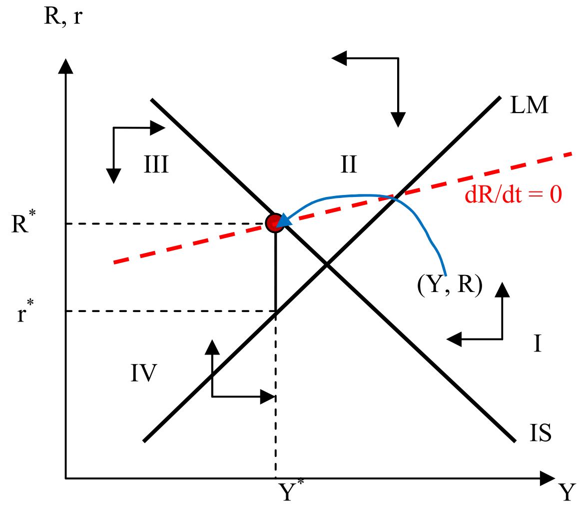

Figure 2. The phase diagram in the extended IS-LM model

It is of particular interest when an off-equilibrium (Y, R) is in area I or II and below the LM curve. The output Y is going to decrease with the yield curve inverted; what is more, it may take quite a while for it to pass through the LM curve and then reach the new equilibrium on the IS curve. If the new equilibrium represents a final position after the economic contraction hits the bottom, the above phase diagram and the path of (Y, R) can well explain why an inverted yield curve may lead a recession a while in advance. Empirical evidences show that an inverted yield curve had occurred about twelve months before a recession actually arrived. By official definition of a recession, it occurs (and is announced or realized) after the output consecutively goes down for at least two quarters. Hence, when (Y, R) combination is still in area I or II and below the LM curve under an adverse shock, the yield curve starts to be inverted but the recession has not been "official" or obviously observed, yet. It would take some time for (Y, R) to move from area I, maybe via area II, to finally reach the new equilibrium with recession.

IV. Concluding Remarks

This note attempts to explain why an inverted yield curve can be a leading indicator of a recession. We developed a modified version of the extended IS-LM model with the term structure [3]. With such a framework, we provide a phase-diagram analysis to illuminate how an adverse shock may result in an inverted yield curve as well as a subsequent recession. Our phase diagram analysis of off-equilibrium (Y, R) is different from those in previous studies. We model the adjustment process for long-term interest rate based on two component forces, instead of only one. It is their resultant force that determines how the long rate is adjusted. This treatment allows us not to rely on an assumption that the long-term bond be a consol, as usually adopted in the literature. What is more, the resultant force helps us draw a different phase diagram that results in a globally stable equilibrium (Y^*, R^*), rather than the one with a saddle point as in [3]. Another key technical assumption made in this note is that the short-term interest rate is always on the LM curve, which helps us easily see the "shape" of the yield curve along the path of an off-equilibrium (Y, R). Finally, it is worthwhile to emphasize that the occurrence of inverted yield curve is an off-equilibrium phenomenon in the dynamic adjustment process after an adverse shock. That is, inverted yield curve itself is a by-product of an adverse shock. Hence, it only leads, but does not lead to, the succeeding recession.

With the model developed here, we can further investigate other related questions concerning yield curve and recessions. For example, does inverted yield curve only occur under an adverse shock in the money market? Would it also arise with an adverse shock in the goods market? With the LM curve shifting leftwards, is inverted yield curve caused by a contractionary monetary policy exclusively? Or can it also be due to an adverse shock in money market for other reasons? Can a recession arise without following inverted yield curve? Does inverted yield curve always lead a recession? Can yield curve be inverted without any shock? We plan to explore these questions along the line of the research conducted in this note.

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