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Ant colony optimization approach to portfolio optimization

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Abstract. The purpose of this paper is to apply ACO approach to the portfolio optimization mean–variance model. The problem of portfolio optimization is a multiobjective problem that aims at simultaneously maximizing the expected return of the portfolio and minimizing portfolio risk. Present study is a heuristic approach to portfolio optimization problem using Ant Colony Optimization technique. The test data set is the monthly prices since 2008/20/3 up to 2011/20/03 from Tehran stock exchange. Reliability of proposed algorithm is evaluated. The performance of ACO is compared with *frontcon* function of *MATLAB* software as an exact method. The results show that proposed ACO approach is reliable but not preferred to an exact method.

Keywords: Portfolio Optimization, Ant Colony Optimization (ACO), Multiobjective

1. Introduction

One of the vital problems in finance is Portfolio Optimization Problem (POP) that has received a lot of attention in recent decades. Harry M. Markowitz was the first to come up with a parametric optimization model to this problem which meanwhile has become the foundation for Modern Portfolio Theory (MPT) [5]. The problem usually has two criteria: expected return is to be maximized, while the risk is to be minimized [2]. In the other words POP presents a multiobjective problem that could be solved with metaheuristic approaches inspired biologically.

In present paper we will use the standard Markowitz model as a multiobjective problem solving model, in order to design an algorithm that is based on Ant Colony Optimization approach. This paper is structured as follows. First importance of this study and a summery of POP is presented. A brief literature review is provided in section 2, followed in section 3 by problem description. In section 4 ACO approach is explained and proposed algorithm is presented. Empirical study is presented in section 5. The paper concludes in section 6 with conclusion.

2. Literature Review

- Maringer addressed the issue of finding an optimal portfolio structure when there is a limit on the number of different assets that may be included. He used Ant systems, empirical studies were performed for NYSE, FTSE and DAX data. The results confirmed that small portfolios can indeed be very well diversified – provided the asset and weight selection has been done with a suitable method [6].
- Wang and Yang worked on securities-investment market's situation of China, proposed an objective model of the securities investment combination optimization under conditions of the nonnegative investment ratio, and designed ant group algorithm to solve this model's continual optimization.

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Through the example of computer simulation, they could see that this algorithm was effective in solving the multiobjective programming and in optimizing portfolio investment's application [8].

- Eslami Bidgoli et al used Ant Colony Optimization to solve portfolio optimization problem with cardinality constrain on the maximum number of assets on Tehran Exchange Market. The results showed that a small portfolio of assets could be found having a comparable performance with much diversified portfolios [4].

3. Portfolio Optimization

In 1952, Harry Markowitz published a paper on portfolio selection. He divided portfolio selecting process in two stages. The first stage starts with observation and experience and ends with beliefs about the future performances of available securities. The second stage starts with the relevant beliefs about future performances and ends with the choice of portfolio [7].

3.1. Problem Description

The basic mean–variance portfolio selection problem we consider in present paper can be formalized as follows:

$$\max \quad R_p = \sum_{i=1}^{n} r_i x_i \tag{1}$$

$$\min \quad \sigma_p = \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij}} \tag{2}$$

Subject to:

$$\sum_{i=1}^{n} x_i = 1 \quad and \quad \begin{cases} x_i > 0 & \forall_i \in p \\ x_i = 0 & \forall_i \notin p \end{cases} \tag{3}$$

$$p \subset M \tag{4}$$

Where:

R_p : return of portfolio, r_i : expected return of stock i, x_i : weight of stock i in portfolio, σ_{ij}: covariance between stock i and j, $M = \{1,2,\dots,N\}$ that N is the number of assets in market.

Equation (1) maximizes the profit associated with the portfolio. Equation (2) minimizes the total standard deviation (the risk) associated with the portfolio. The purpose is to determine value of x_i, that optimize the objective functions.

4. Ant Colony Optimization Approach

One of the first behaviors studied by entomologists was the ability of ants to find the shortest path between their nest and a food source. From these studies and observations followed the first algorithmic models of the foraging behavior of ants, as developed by Marco Dorigo [3].

4.1. Applying the Algorithm

Solution space could be described as a vector with n members which points to number of stocks in the market. This vector called stock vector. Object is allocating proper coefficients to each member of stock vector. Therefore coefficient vector would be introduced as c = [0, 1, 2, …, k]. Allocating 0 to each stock means that stock does not participate in portfolio and more than 0 means that stock is a participant.

The proposed algorithm is based on pheromone trail. Saved information about density of solution fragments, guide ants to proper choose. This collective memory is known as pheromone matrix. Pheromone matrix contains n culms and m rows. Where n is the number of stocks and m points to coefficient vector members.

To continue, each member of stock vector should be assigned a weight in the set. Algorithm does this duty with employing coefficient vector. The probability of coefficient allocation could be determined as:

$$p_c = \frac{\tau_{cs}}{\sum_{\forall c} \tau_{cs}} \tag{7}$$

Ants use this strategy to select a proper portfolio. After, all the ants have constructed solutions to the problem, the quality of each of these solutions is assessed, and this information is used to update the pheromone trails. For each portfolio, a fitness value would be calculated as following:

$$\text{Fitness function} = \frac{R_p}{\sigma_p} \tag{8}$$

The best fitness value points to the best portfolio. System could recognize the best value and starts to update the pheromone trail information according to the best portfolio. The update process could be presented as follows:

$$\tau_{cs}(t+1) = \tau_{cs}(t)(1-\gamma) + \delta_{cs} \tag{9}$$

And

$$\delta_{ci} = Q \cdot \frac{\frac{R_p}{\sigma_p}}{\frac{R_{p^*}}{\sigma_{p^*}}} \tag{10}$$

Where:

γ: evaporation rate, p^* : best portfolio, Q : fixed amount

The algorithm ends after a prefixed number of iterations.

5. EMPIRICAL STUDY

5.1. Data

The empirical study in this paper is based on data set in Tehran Stock Exchange (TSE). Data set is the monthly prices over the period 2008/20/03 up to 2011/20/03. Out of quoted companies in TSE, 30 companies, participating in TEFIX 30, were chosen in order to test the performance of algorithm. Brabozan and O'Nell (2007) underlined the effect of the collected data quality on a model success.

Out of 30 companies, 6 of them were not fully active in predefined time period. The expected return computed for the period of 3 companies, were negative. So these companies were ignored.

5.2. Algorithm implementation

Proposed algorithm was programmed using *MATLAB* software. After several experiments, the numerical parameters have been determined as following: Ants number: 168, Q = 0.1, evaporation rate (γ): 0.01 and repeat count: 500 times.

Figures 1 and 2 show the performance of proposed algorithm on objective functions.

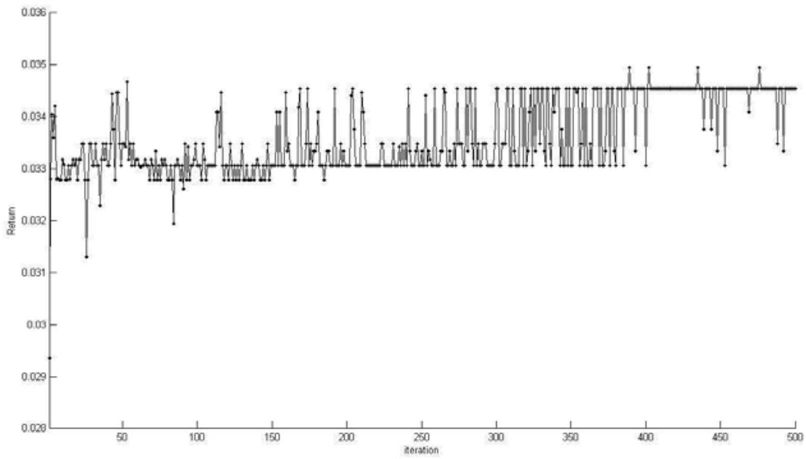

Fig.1: return of optimal portfolio for each iteration.

As figure 1 shows return of optimum portfolio increases as a whole in a converging trend.

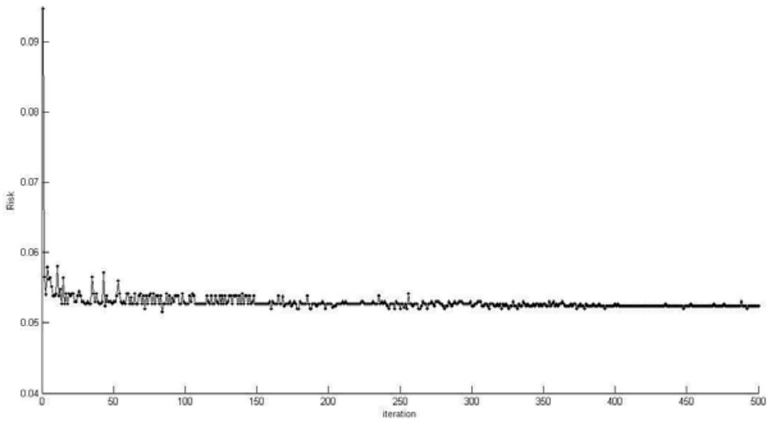

Fig.2: risk of optimal portfolio for each iteration.

As figure 2 shows risk of optimum portfolio decreases as a whole in a converging trend.

To evaluate the reliability, algorithm was run 30 times. Figure 3 presents the distribution graph of the results of algorithm run for 30 times.

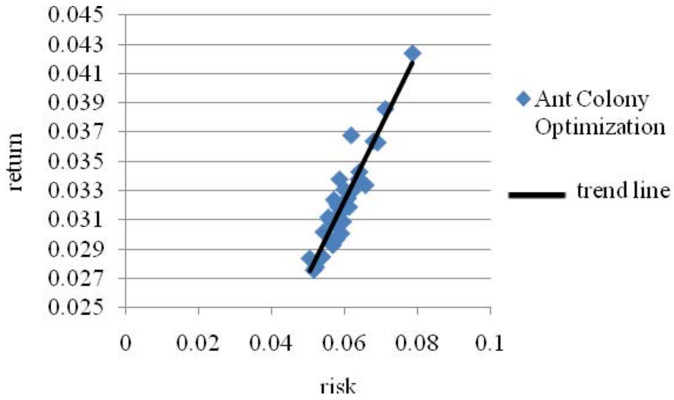

Figure.3: distributions of results for ant colony optimization algorithm.

Correlation coefficient of scattered points on the graph was calculated by correlation coefficient analysis. Reliability of proposed algorithm is confirmed by score of 93% for correlation coefficient.

In order to compare the performance of ACO algorithm with an exact method, efficient frontier line is drown using the *frontcon* function of *MATLAB* software. As figure.4 shows the results of ACO points, are located under the efficient frontier of *frontcon* function. In other words, performance of *frontcon* function is preferred to ACO approach.

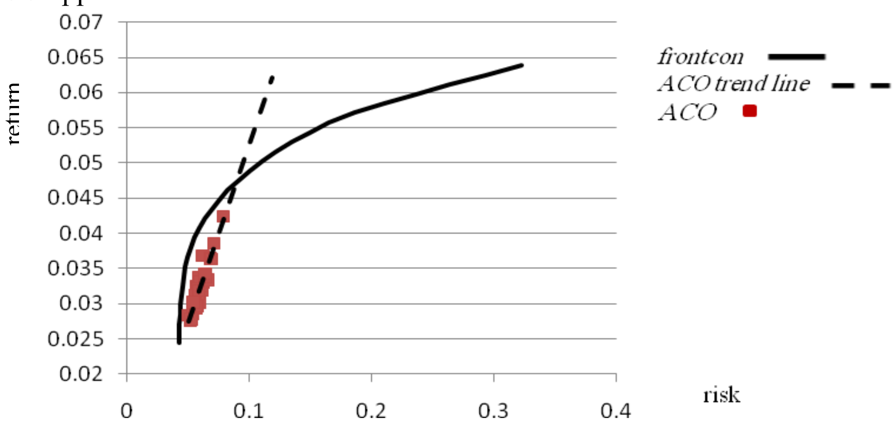

Figure.4: comparing exact method with ACO approach.

To ensure that there is significant difference between the results obtained by ACO approach and exact method, *Wilcoxon Signed-Rank Test* was performed. The *Wilcoxon Signed-Rank Test,* compare two sets of scores that come from the same participants. The distribution of the differences between the scores of the two related groups, were tested for normality. Because of abnormally distribution, *Wilcoxon Signed-Rank Test* was used as a nonparametric test. Table 1, points to test result.

Table.1: Wilcoxon Signed-Rank Test

	frontcon - ant
Z	-4.783[a]
Asymp. Sig. (2-tailed)	.000

a. Based on negative ranks.

b. Wilcoxon Signed Ranks Test

A *Wilcoxon Signed Ranks Test* shows that there is significant difference between the results obtained by ACO approach and exact method.

6. Conclusion

Nowadays, studies on ACO approach are extending in several fields of finance, like banking, ecommerce, marketing, risk management and etc. Following the pheromone trail and exploring the shortest path is interpreted as effectiveness and efficiency in biological life. It shows the potential ability of ACO to be used in finance.

In present study, an ACO algorithm was proposed for POP. This algorithm, search the solution space using ants for optimum rate of return on risk. Algorithm performs in two major steps, coefficient allocation and updating.

Convergence of the algorithm is a good feature. Despite the stochastic condition of Ant Colony Algorithm, its reliability is proved at a high level.

The results confirmed the preferred of *frontcon* function as an exact method with significant difference. It means that more attempts are needed to improve the ACO algorithm toward a more directed one.

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