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Group Solvency Optimization Model for Insurance Companies Using Copula Functions

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Abstract. The Global Financial Crisis highlighted an importance of insurance group's risk management in Enterprise Risk Management (ERM). In insurance sector, AIG's derivative subsidiary's enormous loss is still fresh in our minds, and Group Solvency as group risk management is recently a material issue. Insurance group adopt a methodology for concentration of group's capital on material subsidiaries using Capital and Risk Transfer Instruments (CRTIs) such as reinsurance when facing serious financial distress such as global financial crisis. CRTIs seem to be effective on ERM. In our paper, we model optimal dependency structure for asset and liabilities using some copula functions and calculate optimal risk transfer ratios. At the same time, we consider parent company's limited liability toward her subsidiary and evaluate a put option value to run-off an immaterial subsidiary facing financial distress for whole group diversification.

Keywords: Group Solvency, Copula Functions, CRTIs, Risk Capital, Global Financial Crisis

1. Introduction

Subprime loan problem and subsequent global financial crisis affected severe loss to financial institutions all over the world. In insurance sector, AIG's derivative subsidiary's enormous loss is still fresh in our minds, but on the other hand major insurance companies in Japan also suffered from variable annuity products with minimum guaranteed benefits. It is clear from these facts that insurance group's risk management, namely Group solvency is recently a material issue. The risk management on financial assets and real estate assets in normal economy relies on VaR or Expected Shortfall. On the other hand, the realization of liquidity risk or counterparty credit risk, appearance of credit derivatives or securitization products, and transmission of systemic risk severely have affected the management for banks and insurance companies. Group solvency is a methodology for maintaining the financial soundness of an insurance group as a while and its purpose is to minimize of the group's risk. When an insurance group faces severe financial distress, she concentrates her capital on the distressed but material subsidiaries within the group utilizing Capital and Risk Transfer Instruments (CRTIs) such as reinsurance, loan, and guarantee. Group solvency is popular among Europe, and also in US, AIG's management crisis strengthened the prudential rule for insurance companies. US regulator NAIC checked up the group capital requirement. Group solvency's regulatory framework has been already incorporated in Swiss Solvency Test (Keller (2007) and Luder (2007)). And International Association of Insurance Supervisors (IAIS) provides the standard of insurance groups' supervision for internationally active insurance groups (IAIS (2000)).

2. Group Solvency Optimization Model

2.1. Model Setup

We set up a group solvency model. The previous papers for group solvency are Filipović and Kupper (2007, 2008). Their papers explained parent's limited liability to her subsidiaries. Filipović and Kunz (2008) studied the parent's default effect in new bottom-up model. And Mayer et al. (2010) evaluated group risk by

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a linear model. We extend these researches and develop a new group solvency model for the ERM considering regulatory conditions too. Model's assumptions are the followings. 1) Our model explicitly considers the parent's credit risk, and if the parent defaults, reinsurance claim will not be paid. 2) Our model considers the parent's limited liability to her subsidiaries. 3) Our model explicitly considers Solvency Capital Requirement (SCR) on the subsidiary's entity level and the dependency between the capital and CRTI. 4) There is an order among subsidiaries in accordance with best practice in the insurance industry. If a subsidiary falls into financial distress, reinsurance claim will be paid in this order. 5) It is assumed that the parent company is financially sound only when her surplus exceeds Minimum Capital Requirement (MCR).

A reinsurance claim paid by the parent to her subsidiary must be based on an explicitly defined insurance calculation principle. Namely an insurance premium must be evaluated on the market consistent base. We consider an insurance group composed of parent (i=0) and her directly owned subsidiaries (i=1,2,…,m). And we set a current available capital for entity i as follows.

$$v_i = a_i - l_i, \quad i = 0,1,\ldots,m \tag{1}$$

where a_i and l_i indicate best estimate for market value of assets and liabilities respectively and the value on t=0 are definite. Additionally, all entities' liabilities are composed of liability classes: j=1,2,…,n. Therefore, $l_i = \sum_{j=1}^n l_i^j,\ i = 0,1,\ldots,m$. And the termination value C_i of the portfolio is as follows.

$$C_i = A_i - \sum_{j=1}^n L_i^j,\ i = 0,1,\ldots,m \tag{2}$$

where A_i and L_i^j $(j = 1,2,\ldots,n)$ represent the discounted value for entity i's asset and liability and both are random variables observable at t=0. And for simplification we use the valuation principle $E(C_i)$=v_i.

2.2. Subsidiary model

We model subsidiaries' balance sheets. Net liability amounts for liability classes or business lines j=1,2,…,n of subsidiaries i=1,2,…,m are assumed to be G_i^j.

***Proportional*:** If a parameter $r_i^j \in [0,1]$ is assumed to be a ratio of proportional reinsurance for a liability class j undertaken by the parent from her subsidiary i, $R = (r_i^j)$ is the parameter matrix for reinsurance ratios (risk transferring ratios). Then net liability amount $\sum_{j=1}^n G_i^j$ for subsidiary i after the reinsurance is as follows.

$$\sum_{j=1}^n G_i^j = \sum_{j=1}^n r_i^j L_i^j,\ i = 1,2,\ldots,m \tag{3}$$

So the reinsurance amount $\sum_{j=1}^n (L_i^j - G_i^j)$ undertaken by the parent from her subsidiary i is as follows.

$$\sum_{j=1}^n (L_i^j - G_i^j) = \sum_{j=1}^n (1 - r_i^j) L_i^j \tag{4}$$

***Stop-loss*:** If we define mcr_i as MCR of subsidiary i, mcr_i is a definite parameter in accordance with risk amount the subsidiary i holds. An excess of the surplus to mcr_i, $\max(A_i - \sum_{j=1}^n L_i^j - mcr_i, 0)$ is transferable as a capital to the parent. Then the subsidiary's surplus $\tilde{C}_i$ is as follows.

$$\tilde{C}_i = \min(A_i - \sum_{j=1}^n L_i^j, mcr_i),\ i = 1,2,\ldots,m \tag{5}$$

And the reinsurance amount $\sum_{j=1}^n (L_i^j - G_i^j)$ undertaken by the parent from subsidiary i is as follows.

$$\sum_{j=1}^n (L_i^j - G_i^j) = \max(mcr_i - (A_i - \sum_{j=1}^n L_i^j), 0) \tag{6}$$

A net liability amount $\sum_{j=1}^n G_i^j$ after the reinsurance is as follows.

$$\sum_{j=1}^n G_i^j = \min(A_i - mcr_i, \sum_{j=1}^n L_i^j) \tag{7}$$

The subsidiary i's surplus V_i after reinsurance depends on a ratio on the reinsurance liability to the liability currently held by the parent. Therefore subsidiary i's surplus after reinsurance is as follows.

$$V_i = A_i + A_{rec,i} - \sum_{j=1}^n (G_i^j + P_i^j),\ i = 1,2,\ldots,m \tag{8}$$

where $A_{rec,i}$ is the subsidiary i's recovery amount and P_i^j is reinsurance premium the subsidiary i pays to her parent for liability class j at t=0 as follows.

$$P_i^j = (1-\delta) E[L_i^j - G_i^j] + \delta ES_\alpha [L_i^j - G_i^j],\ i = 1,2,\ldots,m,\ j = 1,2,\ldots,n \tag{9}$$

where δ (=10%) is a weight and also a cost of capital. Recovery amount is influenced by parent's payment default, so is calculated as;

$$A_{rec,i} = \max[\min(\sum_{j=1}^n (L_i^j - G_i^j), (\tilde{V}_0 - mcr_0)/m), 0],\ i = 1,2,\ldots,m \tag{10}$$

where mcr_0 is parent's MCR and only the parent fulfills her obligation in their sound financial conditions. And $\tilde{V}_0$ is a surplus of parent and parent's credit risk is not included in this value. When payment obligation for her subsidiary is evaluated, the credit of parent herself is not considered. Therefore,

$$\tilde{V}_0 = A_0 - \sum_{j=1}^n L_0^j + \sum_{k=1,k\neq i}^m [\tilde{A}_{sub,k} - \sum_{j=1}^n (L_k^j - G_k^j - P_k^j)] \tag{11}$$

where $\tilde{A}_{sub,k}$ is the surplus in which the recovery amount from the reinsurance for subsidiary k is not considered. If reinsurance agreement is claimed in a condition that the reinsurance amount for her subsidiary is smaller than parent's residual value, namely in case of the condition of $\sum_{j=1}^{n}(L_i^j - G_i^j) > (\tilde{V}_0 - mcr_0)/m$ in eq.(10), her subsidiary i's asset is not included in parent's recovery amount $A_{rec,i}$, so the subsidiary i is excluded from the parent's asset. As a subsidiary is generally thought as the asset of the parent, the subsidiary i's surplus $\tilde{A}_{sub,i}$ is defined as;

$$\tilde{A}_{sub,i} := A_i - \sum_{j=1}^{n}(G_i^j + P_i^j) \tag{12}$$

As the parent's limited liability to her subsidiary is not included in eq.(12), this equation must be modified to the equation including limited liability. Practically, there is an order among subsidiaries and in case of financial distress, the reinsurance claims are paid in accordance with this order. Therefore, if the order among subsidiaries is considered, eq.(12) is modified as follows.

$$A_{rec,i} = \max[\min(\sum_{j=1}^{n}(L_i^j - G_i^j), \tilde{V}_0 - mcr_0 - \sum_{k=1}^{i-1}\sum_{j=1}^{n}(L_k^j - G_k^j)), 0],\ i = 1,2,\ldots,m \tag{13}$$

Looking at this eq., the subsidiary's recovery amount is also influenced by amounts paid to other subsidiaries with higher priority orders. Then the probability p_i of parent's payment default is calculated in as follows.

$$p_i = P(\tilde{V}_0 - mcr_0 - \sum_{k=1}^{i-1}\sum_{j=1}^{n}(L_k^j - G_k^j) < \sum_{j=1}^{n}(L_i^j - G_i^j)),\ i = 1,2,\ldots,m \tag{14}$$

where P is physical probability measure.

2.3. Parent Model

We assume that the parent holds her own asset and liability, which are set as A_0 and $\sum_{j=1}^{n} L_0^j$ respectively. Then, her subsidiary i's surplus $A_{sub,i}$ is defined as;

$$A_{sub,i} = \max[A_i - \sum_{j=1}^{n}(G_i^j + P_i^j), 0],\ i = 1,2,\ldots,m \tag{15}$$

Eq.(15) reflects limited liability because of her subsidiary's firm value not being zero. In case her subsidiary i has excessive liabilities, the subsidiary's net asset value to her parent is not a minus but zero. And,

$$\Delta_i^{put} = A_{sub,i} - \tilde{A}_{sub,i},\ i = 1,2,\ldots,m \tag{16}$$

is a put option value for a run-off of her subsidiary i. The parent's surplus eq.(11) is modified as follows.

$$V_0 = A_0 - \sum_{j=1}^{n} L_0^j + \sum_{k=1,k\neq i}^{m}[A_{sub,k} - \sum_{j=1}^{n}(L_k^j - G_k^j - P_k^j)] \tag{17}$$

A premium P_k^j her subsidiary k pays for liability class j is not cancelled out in V_0 owing to parent's limited liability. Therefore parent's SCR depends on the premium her subsidiary pays for the reinsurance.

2.4. Group Solvency Model

We set up an insurance group solvency model. The group's purpose is the minimization of whole group's SCR. Here we adopt as a risk measure "expected shortfall" which is also used in Swiss Solvency Test. Then we solve following minimization problem subject to subsidiary's SCR for optimal risk transferring vectors **R** (for proportional reinsurance).

$$\min_{\mathbf{R}} \sum_{i=0}^{m}[ES_\alpha(-V_i) - E(-V_i)]\quad s.t.\ ES_\alpha(-V_i) - E(-V_i) \geq \beta_i,\ i = 1,2,\ldots,m \tag{18}$$

where β_i is subsidiary i's SCR and ES_α is calculated at C.I. α=99%. When we get an optimal solution for parent's and her subsidiary's surpluses V_i (i=1,2,…,n), then group level SCR is calculated as follows.

$$SCR_{CRT} = \sum_{i=0}^{m}[ES_\alpha(-V_i^{opt}) - E(-V_i^{opt})] \tag{19}$$

3. Numerical examples for group solvency capital

3.1. Examples of group level solvency capital for a case of the parent and one subsidiary

We calculate group level SCR for a case of the parent and one subsidiary. Then, the subsidiary's liabilities are aggregated into L_1. And reinsurance liability undertaken by the parent is amounted to;

$$L_1 - G_1 = \begin{cases} (1-r_1)L_1 & ;\text{Proportional reinusrance} \\ \max(mcr_1 - A_1 + L_1, 0); & \text{Stop-loss reinsurance} \end{cases} \tag{20}$$

Then we model future values of assets and liabilities. For example, if we assume asset value processes follow lognormal processes, they are expressed as follows.

$$dA_i(t)/A_i(t) = \mu_{A_i}dt + \sigma_{A_i}dW_{A_i}(t),\ i = 0,1 \tag{21}$$

Solving eq.(21) and set a_i (i=0,1) as an initial value at time t=0, we can derive following eq.

$$A_i(1) = a_i \exp((\mu_{A_i} - \sigma_{A_i}^2 / 2) + \sigma_{A_i} x_i),\ x_i \sim N(0,1) \tag{22}$$

And if we assume parent's and her subsidiary's future liability values independently follow lognormal distribution, in the same way we can derive the following by setting l_i (i=0,1) as an initial value at time t=0.

$$L_i(1) = l_i \exp((\mu_{L_i} - \sigma_{L_i}^2 / 2) + \sigma_{L_i} y_i),\ y_i \sim N(0,1) \tag{23}$$

On the other hand, we express future liability values by other distributions. As Non-life insurance companies hold catastrophic liabilities such as seismic insurance, we express subsidiary's liability distribution as t-distribution with d.f. ν and parent's liability by Gamma distribution $G(a,b)$ (a,b>0).

$$L_0(1)/l_0 \sim G(a,b),\ L_1(1)/l_1 \sim t_\nu \tag{24}$$

where pdfs $f(x)$ and $g(x)$ are respectively expressed by Gamma func $\Gamma(a)$ and Beta func $B(\nu/2,1/2)$ as follows.

$$f(x) = \frac{1}{b^a \Gamma(a)} x^{a-1} e^{-\frac{x}{b}},\ x \geq 0, a, b > 0,\quad g(x) = \frac{1}{\sqrt{\pi}\sigma B(\nu/2, 1/2)} \left[1 + \frac{1}{\nu}\left(\frac{x-\mu}{\sigma}\right)^2\right]^{-\frac{\nu+1}{2}} \tag{25}$$

Next we assume that subsidiary's MCR mcr_1 is fixed in accordance with the country's regulation and parent's MCR mcr_0 is expressed as $mcr_0 = 0.2 \times (v_0 + ES_\alpha(-V_0))$, where mcr_0 depends on initial capital v_0 and asset-liability portfolio's future value change $ES_\alpha(-V_0)$, and the coefficient 0.2 is a parameter which is fixed in the country. Solvency ratio is defined by SCR in eq.(19) as follows.

$$SR_i = E[-V_i] / SCR_{CRT_i},\ i = 0, 1, whole\ group \tag{26}$$

Correlation matrix is shown on Table 1. Matrix 1 is for Valuation models of NO.1 to NO.3 in Table 2 and Matrix 2 is for ones of NO.4 to NO.7 in Table 2. Initial values of the asset and liability (unit: trillion yen) are set as the parameters (Parent: a_0=8, l_0=6; Subsidiary: a_1=4, l_1=3). Simulation number is 100 thousand, and C.I. for risk valuation is 99%.

Proportional: The group level SCR has a minimum value owing to subsidiary's SCR convex downward (for example, see left below of Fig. 1: Base model). On the other hand, as for models which don't have a minimum (t-copula model, Meta-Gaussian copula model, and Meta-t-copula model), both subsidiary's SCR and parent's one are monotonically increasing, and aggregated group level SCR is also monotonically increasing. Therefore whether group level SCR's having minimum or not mainly depends on the dependence structure between a parent's liability value and her subsidiary's one.

Stop-loss: The reason that group level SCR has a minimum value is the same as one for reinsurance cases. In stop-loss reinsurance, a payoff function is non-linear to a ratio mcr_1/v_1. So parent's SCR increases till halfway and hereafter become constant. This fact show there is little put option value. On the other hand, in case of financial distress (parameter set; Parent: a_0=8, l_0=7.5; Subsidiary: a_1=4, l_1=3.5), parent's SCR is not constant, and reflects a put option value.

3.2. Probability that parent defaults payment of reinsurance claim

The probability p_0 that parent defaults the payment of reinsurance claim is calculated by setting m=n=1 in eq.(14) as follows.

$$p_1 = P\left(\tilde{V}_0(R) - mcr_0 < L_1 - G_1\right) \tag{27}$$

Proportional: For $(1-r_1)$ over 40%, parent's default probability sharply increases, but there is no large difference between Reference model and Lognormal model. On the other hand, Base model's probability is much higher than ones of other two models at the risk transfer ratio being over 70% (see left side of Fig. 2).

Stop-loss: Parent's default probability is much smaller compared to ones for proportional reinsurance, but Base model's probability is much higher than ones of other two models (see right side of Fig. 2).

4. Conclusion

We presented a general capital allocation model for minimizing insurance group's risk and analyzed various models' features on a simple set of a parent and one subsidiary. Major findings are as follows.

- Different dependency structures result in different risk transferring ratios. Therefore insurance companies must analyze an exact dependency structure based on their own surplus data.

- Diversification effect for group risk is different depending on CRTIs. For practical purposes, available CRTIs must be considered for the group as a whole.
- We incorporated a parent's limited liability to her subsidiary and offered the information when the subsidiary cannot hold MCR in severe financial distress and parent runs off the subsidiary to survive as whole group.

Table 1: Linear correlation matrix

Matrix 1	A_0	A_1	L_0	L_1	Matrix 2	A_0	A_1	L_0	L_1
A_0	1.0	1.0	0.0	0.0	A_0	1.0	0.8	0.2	0.2
A_1	1.0	1.0	0.0	0.0	A_1	0.8	1.0	0.2	0.2
L_0	0.0	0.0	1.0	0.0	L_0	0.2	0.2	1.0	0.3
L_1	0.0	0.0	0.0	1.0	L_1	0.2	0.2	0.3	1.0

Table 2: Valuation models

NO.	Models	Marginals of assets*	Marginals of liabilities*	Corr. Matrix
1	Reference	Normal	Lognormal	Matrix 1
2	Lognormal	Lognormal	Lognormal	
3	Base	Normal	P: Gamma / S: t	
4	Gaussian copula	Normal	Normal	Matrix 2
5	t-copula	t	t	
6	Meta-Gaussian copula	Normal	P: Gamma / S: t	
7	Meta-t-copula	Normal	P: Gamma / S: t	

* Common among both parent and subsidiary unless otherwise noted

Table 3: Parameters for marginals

Type	Marginals	Parameters
Assets	Normal/Lognormal	$\mu_{A0}=\mu_{A1}=0.01$, $\sigma_{A0}=\sigma_{A1}=0.02$
	t	*d.f.* $v=5$, $\mu_A= 0.01$, $\sigma_A=0.02$
Liabilities	Normal/Lognormal	$\mu_{L0}=\mu_{L1}=0$, $\sigma_{L0}=0.1$, $\sigma_{L1}=0.16$
	t	*d.f.* $v=5$, $\mu_{L0}=\mu_{L1}=0$, $\sigma_{L0}=0.1$, $\sigma_{L1}=0.16$
	Gamma	$a=3$, $b=0.01$

Table 4: Optimal risk transferring ratios for proportional reinsurance

NO.	Models	Optimal $1\text{-}r_1$	
		SCR(%)	*SR*(%)
1	Reference	15	14
2	Lognormal	15	14
3	Base	6	4
4	Gaussian copula	18	17
5	t-copula	—	—
6	Meta-Gaussian copula	—	—
7	Meta-t-copula	—	—

Table 5: Optimal MCR ratios for stop-loss reinsurance

NO.	Models	Optimal mcr_1/v_1	
		SCR(%)	*SR*(%)
1	Reference	74	79
2	Lognormal	71	76
3	Base	—	—
4	Gaussian copula	75	73
5	t-copula	—	—
6	Meta-Gaussian copula	—	—
7	Meta-t-copula	—	—

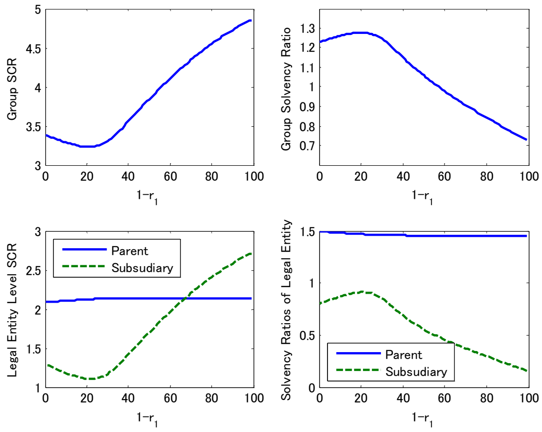

Fig. 1: Example of SCR and SR (Base model)

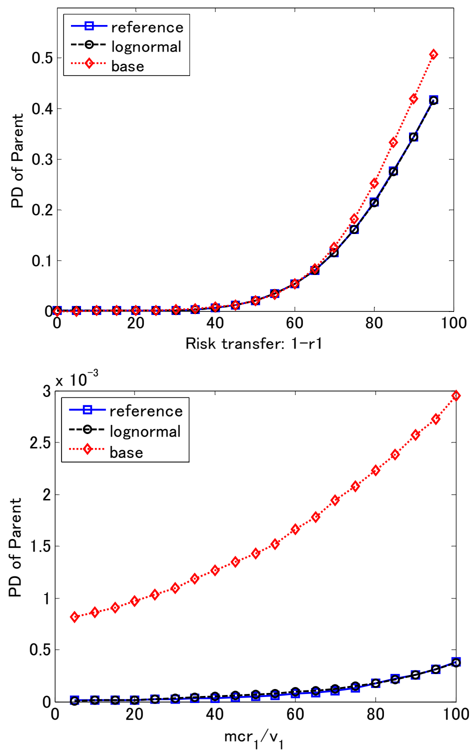

Fig. 2: Parent's default probability changes (Left: Proportional, Right: Stop-loss), Note: Each line corresponds to Reference model, Lognormal model, or Base model on Table 2.

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