2011 3rd International Conference on Advanced Management Science
IPEDR vol.19 (2011) © (2011) IACSIT Press, Singapore

Discriminating efficient candidates of MEA model

Fuh-hwa Franklin LIU [1 +] and Cheng-Li CHEN [2]

[1, 2] Department of Industrial Engineering and Management, National Chiao Tung University
Box 17, 1001 Daxue Road, Hsin Chu City, Taiwan 30010, Republic of China.

Abstract. Multi-directional efficiency analysis (MEA) has the advantage of separating the issue of benchmark selection from the issue of efficiency measurement. Like most of DEA models, MEA model often identifies plural candidates to be efficient. Due to absence of weights and infeasibility, we are unable to create cross-efficiency and super-efficiency models of MEA for discriminating efficient candidates. The recent paper suggests a proper technique and proposes a worst-practice MEA model for discriminating efficient candidates. An empirical example is used for illustrative purposes.

Keywords: data envelopment analysis, multi-directional efficiency analysis, worst-practice frontier.

1. Introduction

The potential improvements approach and an associated efficiency index was introduced by Bogetoft and Hougaard [1]. The idea of the potential improvement approach is to let the estimated production possibilities determine the improvement direction. This is in contrast to the proportional improvements approach suggested by the traditional data envelopment analysis (DEA) models with radial efficiency index. As demonstrated in [1], the potential improvements approach has several attractive features: it distinguishes weakly efficient units from strongly efficient ones, it is invariant not only to linear scale changes but to affine translations as well, and it imposes a considerably less demanding symmetry assumption.

A DEA-like approach for the determination of the potential improvements and associated index is suggested by Asmild et al. [2]. The approach is called multi-directional efficiency analysis (MEA).

Assume a set of n decision-making units (DMU$_j$, j = 1, ..., n) produce s outputs (y_r; r = 1, ..., s) using m inputs (x_i; i = 1, ..., m). Consider the production plan ($\mathbf{x}^k$, $\mathbf{y}^k$) of DMU$_k$ (k = 1, ..., n). To calculate the value of the potential improvements inefficiency index $E^{PI}(\mathbf{x}^k, \mathbf{y}^k)$ for DMU$_k$, the ideal input reference point $\mathbf{x}^*(\mathbf{x}^k)$ is found by solving m linear programming problems (one for each input dimension):

$$
\begin{aligned}
&\underline{x}_{ik}^* = \min \underline{x}_{ik} \quad (i = 1, \ldots, m) \\
&\text{s.t. } \sum\nolimits_{j=1}^{n} \lambda_j x_{ij} \le \underline{x}_{ik}, \\
&\sum\nolimits_{j=1}^{n} \lambda_j x_{i'j} \le x_{i'k}, \quad i' = 1, \ldots, i-1, i+1, \ldots, m, \\
&\sum\nolimits_{j=1}^{n} \lambda_j y_{rj} \ge y_{rk}, \quad r = 1, \ldots, s, \\
&\sum\nolimits_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \ge 0, \quad j = 1, \ldots, n.
\end{aligned} \tag{1}
$$

The ideal output reference point $\mathbf{y}^*(\mathbf{y}^k)$ of DMU$_k$ (k = 1, ..., n) is found by solving s linear programming problems (one for each output dimension):

+ Corresponding author. Tel.: + 886-0910-652525.
E-mail address: fliu@mail.nctu.edu.tw.

$$\bar{y}_{rk}^* = \min \bar{y}_{rk} \quad (r = 1,\ldots,s)$$

$$\begin{aligned} \text{s.t. } & \sum\nolimits_{j=1}^{n} \lambda_j y_{rk} \geq \bar{y}_{rk}, \\ & \sum\nolimits_{j=1}^{n} \lambda_j x_{ij} \leq x_{ik}, \; i = 1,\ldots,m, \\ & \sum\nolimits_{j=1}^{n} \lambda_j y_{r'k} \geq y_{r'k}, \; r' = 1,\ldots,r-1, r+1,\ldots,s, \\ & \sum\nolimits_{j=1}^{n} \lambda_j = 1, \; \lambda_j \geq 0, \; j = 1,\ldots,n. \end{aligned} \tag{2}$$

The ideal reference point for DMU_k (k = 1, …, n) is given by $(\underline{x}_{1k}^*,\ldots,\underline{x}_{mk}^*, \bar{y}_{1k}^*,\ldots,\bar{y}_{sk}^*)$. If $(\mathbf{x}^k, \mathbf{y}^k) = (\underline{x}_{1k}^*,\ldots,\underline{x}_{mk}^*, \bar{y}_{1k}^*,\ldots,\bar{y}_{sk}^*)$ we know that DMU_k is efficient and consequently $E^{PI}(\mathbf{x}^k, \mathbf{y}^k) = 0$. Now, assume that $(\mathbf{x}^k, \mathbf{y}^k) \neq (\underline{x}_{1k}^*,\ldots,\underline{x}_{mk}^*, \bar{y}_{1k}^*,\ldots,\bar{y}_{sk}^*)$ and then consider the following linear programming problem:

$$\max \beta_k \quad (k = 1,\ldots,n)$$

$$\begin{aligned} \text{s.t. } & \sum\nolimits_{j=1}^{n} \lambda_j x_{ij} \leq x_{ik} - \beta_k (x_{ik} - \underline{x}_{ik}^*), \; i = 1,\ldots,m, \\ & \sum\nolimits_{j=1}^{n} \lambda_j y_{rj} \geq y_{rk} + \beta_k (\bar{y}_{rk}^* - y_{rk}), \; r = 1,\ldots,s, \\ & \sum\nolimits_{j=1}^{n} \lambda_j = 1, \; \lambda_j \geq 0, \; j = 1,\ldots,n, \; \beta_k \geq 0. \end{aligned} \tag{3}$$

The solution $(\boldsymbol{\lambda}^*, \beta_k^*)$ of the above program can be used to determine the benchmark selection of DMU_k (k = 1, …, n) as follows:

$$x_{ik}^* = x_{ik} - \beta_k^* (x_{ik} - \underline{x}_{ik}^*), \; i = 1,\ldots,m, \tag{4}$$

$$y_{rk}^* = y_{rk} + \beta_k^* (\bar{y}_{rk}^* - y_{rk}), \; r = 1,\ldots,s. \tag{5}$$

As Asmild et al. [2] noted, MEA scores (i.e., the values of $E^{PI}(\mathbf{x}^k, \mathbf{y}^k)$ for DMU_k, k = 1, …, n) can be interpreted as the sum of input specific excesses and output specific shortages relative to the selected benchmark, i.e.,

$$\sum\nolimits_{i=1}^{m} (x_{ik} - x_{ik}^*) + \sum\nolimits_{r=1}^{s} (y_{rk}^* - y_{rk}) = \sum\nolimits_{i=1}^{m} \beta_k^* (x_{ik} - \underline{x}_{ik}^*) + \sum\nolimits_{r=1}^{s} \beta_k^* (\bar{y}_{rk}^* - y_{rk}). \tag{6}$$

Like most of DEA models, MEA model often identifies more than one candidate to be DEA efficient. For the inefficient units, since MEA scores are potential improvements inefficiency index, their MEA scores will be the less the better.

2. Worst-practice frontier MEA model

In the DEA tradition, plural DEA efficient candidates can be discriminated by cross-efficiency or super-efficiency evaluations. Cross-efficiency evaluation was first proposed by Sexton et al. [3], and later investigated by Doyle and Green [4]. The basic idea of cross-efficiency evaluation is to assess each unit with the weights of all the DMUs instead of with its own weights only. On the other hand, the idea of super-efficiency was first introduced by Andersen and Petersen [5]. The super-efficiency measure allows the analyst to rank units on the efficient frontier.

Unfortunately, we are unable to create such cross-efficiency and super-efficiency models of MEA due to absence of weights and infeasibility, respectively. However, we are able to discriminate the efficient units using other technique. According to the study of Wang and Chin [6], DEA efficient candidates can be discriminated by their worst relative total scores obtained by a DEA model with different frontier or production possibility set from the traditional DEA model.

The efficient units obtained from the traditional DEA construct an efficient (best-practice) frontier. The efficiency scores from the traditional DEA can be called the best relative efficiency. The worst relative efficiency is contrary to the best relative efficiency and represents the efficiency score each candidate receives in the worst favorable situation. The DEA models that can result in the worst relative efficiency are called worst-practice frontier DEA (WPF-DEA) models. These models are created based on a worst-practice frontier or production possibility set. For detail of the concept of WPF-DEA models please see Wang and Chin [6], and Liu and Chen [7]. With the similar concept we are able to create a WPF-MEA model in the subsequent.

In contrast of the potential improvements inefficiency index, to calculate the value of the potential deteriorations efficiency index $E^{PD}(\mathbf{x}^k, \mathbf{y}^k)$ for DMU$_k$, the anti-ideal input reference point $\mathbf{x}^{**}(\mathbf{x}^k)$ is found by solving m linear programming problems (one for each input dimension):

$$\begin{aligned} &\overline{x}_{ik}^{*} = \max \overline{x}_{ik} \quad (i = 1,\ldots,m) \\ &\text{s.t. } \textstyle\sum_{j=1}^{n} \lambda_j x_{ij} \geq \overline{x}_{ik}, \\ &\textstyle\sum_{j=1}^{n} \lambda_j x_{i'j} \geq x_{i'k}, \ \ i' = 1,\ldots,i-1,i+1,\ldots,m, \\ &\textstyle\sum_{j=1}^{n} \lambda_j y_{rj} \leq y_{rk}, \ \ r = 1,\ldots,s, \\ &\textstyle\sum_{j=1}^{n} \lambda_j = 1, \ \ \lambda_j \geq 0, \ \ j = 1,\ldots,n. \end{aligned} \tag{7}$$

The anti-ideal output reference point $\mathbf{y}^{**}(\mathbf{y}^k)$ of DMU$_k$ (k = 1, ..., n) is found by solving s linear programming problems (one for each output dimension):

$$\begin{aligned} &\underline{y}_{rk}^{*} = \min \underline{y}_{rk} \quad (i = 1,\ldots,m) \\ &\text{s.t. } \textstyle\sum_{j=1}^{n} \lambda_j y_{rk} \leq \underline{y}_{rk}, \\ &\textstyle\sum_{j=1}^{n} \lambda_j x_{ij} \geq x_{ik}, \ \ i = 1,\ldots,m, \\ &\textstyle\sum_{j=1}^{n} \lambda_j y_{r'j} \leq y_{r'k}, \ \ r' = 1,\ldots,r-1,r+1,\ldots,s, \\ &\textstyle\sum_{j=1}^{n} \lambda_j = 1, \ \ \lambda_j \geq 0, \ \ j = 1,\ldots,n. \end{aligned} \tag{8}$$

The anti-ideal reference point for DMU$_k$ (k = 1, …, n) is given by $(\overline{x}_{1k}^{*},\ldots,\overline{x}_{mk}^{*}, \underline{y}_{1k}^{*},\ldots,\underline{y}_{sk}^{*})$. If $(\mathbf{x}^k, \mathbf{y}^k) = (\overline{x}_{1k}^{*},\ldots,\overline{x}_{mk}^{*}, \underline{y}_{1k}^{*},\ldots,\underline{y}_{sk}^{*})$ we know that DMU$_k$ is worst efficient and consequently $E^{PD}(\mathbf{x}^k, \mathbf{y}^k) = 0$. Now, assume that $(\mathbf{x}^k, \mathbf{y}^k) \neq (\overline{x}_{1k}^{*},\ldots,\overline{x}_{mk}^{*}, \underline{y}_{1k}^{*},\ldots,\underline{y}_{sk}^{*})$ and then consider the following linear programming problem:

$$\begin{aligned} &\max \alpha_k \quad (k = 1,\ldots,n) \\ &\text{s.t. } \textstyle\sum_{j=1}^{n} \lambda_j x_{ij} \geq x_{ik} + \alpha_k (\overline{x}_{ik}^{*} - x_{ik}), \ \ i = 1,\ldots,m, \\ &\textstyle\sum_{j=1}^{n} \lambda_j y_{rj} \leq y_{rk} - \alpha_k (y_{rk} - \underline{y}_{rk}^{*}), \ \ r = 1,\ldots,s, \\ &\textstyle\sum_{j=1}^{n} \lambda_j = 1, \ \ \lambda_j \geq 0, \ \ j = 1,\ldots,n, \ \ \alpha_k \geq 0. \end{aligned} \tag{9}$$

The solution ($\boldsymbol{\lambda}^*$, α_k^*) of the above program can be used to determine the anti-benchmark selection of DMU$_k$ (k = 1, …, n) as follows:

$$x_{ik}^{**} = x_{ik} + \alpha_k^{*} (\overline{x}_{ik}^{*} - x_{ik}), \ \ i = 1,\ldots,m, \tag{10}$$

$$y_{rk}^{**} = y_{rk} - \alpha_k^{*} (y_{rk} - \underline{y}_{rk}^{*}), \ \ r = 1,\ldots,s. \tag{11}$$

WPF-MEA scores (i.e., the values of $E^{PD}(\mathbf{x}^k, \mathbf{y}^k)$ for DMU$_k$, k = 1, …, n) can be interpreted as the sum of input specific shortages and output specific excesses relative to the selected anti-benchmark, i.e.,

$$\textstyle\sum_{i=1}^{m} (x_{ik}^{**} - x_{ik}) + \sum_{r=1}^{s} (y_{rk} - y_{rk}^{**}) = \sum_{i=1}^{m} \alpha_k^{*} (\overline{x}_{ik}^{*} - x_{ik}) + \sum_{r=1}^{s} \alpha_k^{*} (y_{rk} - \underline{y}_{rk}^{*}). \tag{12}$$

Like MEA model, WPF-MEA model also often identifies more than one candidate to be DEA efficient. Our purpose, however, is to choose the winner from among those DEA efficient candidates. So, we solve the model only for DEA efficient candidates. Those worst efficient candidates obtained from WPF-MEA model are not our concern. We aim to compare the WPF-MEA scores for the efficient candidates obtained from MEA model to determine the best winner who should be the DEA efficient candidate with the greatest worst relative total score. For the other efficient candidates, since WPF-MEA scores are potential deteriorations efficiency index, their WPF-MEA scores will be the less the better.

3. Empirical example

To illustrate the models and measures developed in the previous, reconsider the OECD healthcare resources data set of Retzlaff-Roberts et al. [8]. The data set contains observations for 27 countries in 1998. We include four healthcare resource inputs (practicing physicians per 1000 population, inpatient beds per 1000 population, Magnetic Resonance Imager (MRI) units per one million population, share of GDP allocated to healthcare) and two health status outputs (life expected years from birth, and infant survival rate per 1000 live births). The descriptive statistics of the data set are presented in Table 1.

Table 1. Descriptive statistics of OECD data for 27 countries in 1998.

	Inputs				Outputs	
	Physicians	Beds	MRI	Expenditure	Life expectancy	Infant survival rate
Average	2.8	7.4	4.5	7.96	76.6	0.9927
Minimum	1.2	1.1	0.1	3.94	68.7	0.9621
Maximum	5.9	18.1	18.8	13.64	80.6	0.9965
St. Dev.	1.0	4.1	4.3	2.00	3.03	0.0066

The results of MEA model are presented in Table 2.

Table 2. Results of MEA model.

Country	β_k^*	Ideal inputs				Ideal outputs	
		Physicians	Beds	MRI	Expenditure	Life expectancy	Infant survival rate
Australia	0.222	2.453	7.530	4.353	8.462	78.725	0.952
Austria	0.385	2.541	6.819	5.615	7.692	78.469	0.956
Belgium	0.351	2.873	6.006	2.462	8.322	78.271	0.946
Canada	0.000	2.100	4.700	1.800	9.490	78.600	0.945
Czech republic	0.000	3.000	8.900	1.400	7.150	74.600	0.948
Denmark	0.000	3.300	4.500	2.500	8.300	76.200	0.953
Finland	0.000	3.000	7.800	8.700	6.910	77.300	0.958
France	0.313	2.766	7.975	2.275	9.170	78.496	0.954
Germany	0.474	2.791	6.570	3.828	8.842	78.407	0.958
Greece	0.000	4.100	5.000	1.200	8.280	77.900	0.933
Hungary	0.556	2.326	5.149	0.793	5.991	74.212	0.926
Ireland	0.000	2.200	3.700	0.300	6.360	75.900	0.938
Italy	0.380	4.433	4.904	5.119	7.907	78.810	0.948
Japan	0.000	1.900	16.500	18.800	7.630	80.600	0.964
Korea	0.000	1.300	5.100	4.000	5.040	74.400	0.923
Mexico	0.000	1.600	1.100	0.100	4.540	74.700	0.842
Netherlands	0.338	2.371	8.754	2.840	8.190	78.279	0.953
New Zealand	0.266	2.116	5.565	2.399	7.989	77.857	0.937
Norway	0.000	2.400	14.500	0.700	8.920	78.400	0.960
Poland	0.514	2.172	4.009	0.313	6.053	74.697	0.922
Portugal	0.446	2.541	3.781	1.790	7.200	76.513	0.945
Spain	0.000	4.400	3.900	3.800	7.060	78.100	0.950
Sweden	0.000	3.100	3.800	6.800	8.370	79.400	0.965
Switzerland	0.333	1.887	15.930	12.867	9.445	79.556	0.954
Turkey	0.000	1.200	2.500	0.600	3.930	68.700	0.621
UK	0.000	1.700	4.200	3.400	6.740	77.200	0.943
USA	0.339	2.373	3.458	5.310	11.147	77.373	0.936

Efficient candidates from MEA model include 14 countries with under-bar in the first column of the above table. These efficient candidates will be discriminated using WPF-MEA model. The results are presented in the following table.

Table 3. Results of WPF-MEA model.

Efficient candidates	Anti-ideal inputs				Anti-ideal outputs		α_k^*	WPF-MEA scores
	Physicians	Beds	MRI	Expenditure	Life expectancy	Infant survival rate		
Canada	5.220	16.811	13.942	13.417	73.060	0.812	0.287	5.366
Czech republic	3.873	11.404	7.446	9.409	71.390	0.853	0.313	2.403
Denmark	4.896	11.699	8.612	11.820	72.655	0.826	0.288	3.109

Finland	4.159	13.354	11.402	11.441	75.456	0.847	0.275	2.194
Greece	5.651	10.426	9.382	11.232	74.412	0.860	0.259	2.838
Ireland	3.100	8.3	1.5	6.800	70.700	0.903	0.322	5.566
Japan	1.900	16.5	18.8	7.630	80.600	0.964	0.000	0.000
Korea	4.353	12.413	9.318	11.018	71.105	0.694	0.348	4.079
Mexico	4.194	11.656	9.720	10.920	69.125	0.654	0.364	6.111
Norway	2.878	15.871	12.73	10.494	76.502	0.911	0.333	4.122
Spain	5.756	9.9152	9.310	10.840	74.338	0.839	0.349	3.438
Sweden	5.758	14.44	13.908	12.981	74.756	0.846	0.307	4.492
Turkey	1.200	2.5	0.6	3.930	68.700	0.621	0.000	0.000
UK	5.433	15.539	13.479	13.529	71.488	0.725	0.322	5.794

Since WPF-MEA scores for efficient candidates are potential deteriorations efficiency index, greater score means farther from the worst efficient frontier. Their WPF-MEA scores will be the greater the better. As a result, Mexico has the greatest WPF-MEA scores which can be considered the best winner. UK can be ranked the second place among the efficient candidates. We can further rank Ireland the third and Canada the fourth, etc. Japan and Turkey will be ranked the last place among these efficient candidates.

4. Conclusion

Although there are no proper cross-efficiency and super-efficiency evaluations of MEA model, we can rank the efficient units from MEA model using the suggested technique and the proposed WPF-MEA model in the previous. However, there is still one drawback of the suggested technique. There is no guarantee that full ranking can be achieved. A full ranking technique for MEA model could be the direction of future research. Another direction could be finding a better efficiency index. For example, we may consider other efficiency indices such as the following L_2-norm distance function.

$$\alpha_k^* \sqrt{\sum_{i=1}^{m} (\overline{x}_{ik}^* - x_{ik})^2 + \sum_{r=1}^{s} (y_{rk} - \overline{y}_{rk}^*)^2}$$

5. References

[1] P. Bogetoft and J. L. Hougaard. Efficiency evaluations based on potential (non-proportional) improvements. *Journal of Productivity Analysis*. 1999, **12** (3): 233-247.

[2] M. Asmild, J. L. Hougaard, D. Kronborg, and H. K. Kvist. Measuring inefficiency via potential improvements. *Journal of Productivity Analysis*. 2003, **19** (1): 59-76.

[3] T. R. Sexton, R. H. Silkman, and A. J. Hogan. Data envelopment analysis: Critique and extensions. In: Silkman, R.H. (Ed.), Measuring Efficiency: An Assessment of Data Envelopment Analysis. Jossey-Bass, San Francisco, CA., 1986, pp. 73-105.

[4] J. Doyle and R. Green. Efficiency and cross-efficiency in DEA: derivation, meanings and uses. Journal of the Operational Research Society. 1994, 567-578.

[5] P. Andersen and N. C. Petersen. A procedure for ranking efficient units in data envelopment analysis. Management Science. 1993, 39: 1261-1264.

[6] Y.-M. Wang, and K.-S. Chin. Discriminating DEA efficient candidates by considering their least relative total scores. Journal of Computational and Applied Mathematics. 2007, 206: 209-215.

[7] F.-H. Liu and C.-L. Chen. The worst-practice DEA model with slack-based measurement. *Computers & Industrial Engineering*. 2009, **57**: 496-505.

[8] D. Retzlaff-Roberts, C. F. Chang, and R. M. Rubin. Technical efficiency in the use of health care resources: a comparison of OECD countries. *Health Policy*. 2004, **9** (1): 55-72.