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The analysis of scoring matrix in voting system with multiple criteria and multiple places

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Abstract. Decision groups usually employ voting systems that reflect the democratic content of that group. The issue of a group of decision-makers ranking candidates with multiple criteria is interesting. A group ranks the effects of those criteria by using a specific method. For each criterion, a group votes for candidates with multiple places. This study developed a procedure to determine the common set of weights of the places in those criteria. Candidates were subsequently ranked according to their sum of weighted votes that they earned in all the places of those criteria.

Keywords: Decision analysis; group decisions; voting system; data envelopment analysis

1. Introduction

Group decision is always a significant issue in daily life. In decision sciences, two distinct methodologies are basically present in group decision-making: multi-criteria decision-making (MCDM) and social choice (SC) theory [1].

If the MCDM methodology is used in group decision-making, the analytic hierarchy process (AHP) [2] is possibly one of the optimal choices. The method provides a comprehensive and rational framework to structure a decision problem, to represent and quantify its elements (criteria), to relate the criteria to overall goals, and to evaluate alternative solutions. Once the structure is constructed, decision makers use a system of pairwise comparisons to measure the component weights of the structure, and finally to rank the alternatives in the decision. The analytic network process (ANP) is a more general form of the AHP used in multi-criteria decision analysis [3,4].

In a group decision-making environment, aggregating the preference of individuals into a consensus rating is required. Combining the numerical pairwise comparison judgments of individuals is necessary to form a judgment for the group, of which two methods exist: consensus vote and geometrics mean [5]. Consensus vote requires the group to research an agreement on the value of each entry in a matrix of pairwise comparisons, though this is usually difficult to achieve. A superior approach is to regard the geometric mean. [5,6,7]. Furthermore, Leung and Cao [8] argued that the pairwise judgments in AHP are ambiguous, and that the determined weights may be meaningless.

However, Saaty [9] attributed the rank reversal of AHP to the assumption of hierarchic decomposition, which is judged to produce arbitrary results [10,11,12,13]. This phenomenon has not yet been fully resolved and may never be because the aggregation of preferences transposed from units of different scales cannot be easily interpreted and even questioned, according to the French school [14].

In addition, the scale of pairwise comparisons is supposedly ‘‘fundamental’’ in the mind, yet no rules exist for how a transformation to such a scale occurs. The transformation of an individual of a set of weights

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to the fundamental scale could change over time. This indicates to the decision maker their consistency, recommending revisions of comparisons that may be due to a manual error for setting the comparisons, occasionally forced due to the upper limitation of the comparison scale. However, Ishizaka and Labib [15] mentioned that the decision maker must answer a significantly larger number of questions, which may be relatively complex. For large problems, the ability to calculate the priority values from all the acceptable values, based on consistency level, renders combinations of pairwise comparisons unpractical due to the large number of possible different comparison matrices. Decision makers may not have formed a strong opinion on a particular judgment; thus the pairwise matrix might be incomplete. The approach is chiefly qualitative and difficult to judge and is arguably more burdensome to implement than the voting ranking approach [16] according to both the data requirement and validation viewpoint.

The Borda count [17] assigns points to candidates according to their position in the preferred order of the voters. Each voter selects and ranks p candidates from the set of n candidates (p is no more than n). The p candidates are placed as the first, the second, and so on to the pth place. In the simplest Borda count form, a vote of the first, the second, and so on, and the pth places are respectively counted as p, $(p - 1)$, and so on [18]. The Borda count can be restated as an arithmetic progress that starts with p points and with one point decreasing until the last place is assigned with one point. Finally, candidates are subsequently ranked according to their aggregated scores, which is the sum of the weighted votes.

Cook and Kress [19] proposed a procedure to rank candidates in a preferential election by employing data envelopment analysis (DEA) [20,21] to maximize the gap between consecutive weights of the scoring. For an important class of discrimination intensity functions, the procedure is equivalent to a scoring rule. In particular if discrimination intensity functions in their model are equal ε, a non-Archimedean infinitesimal positive constant, the procedure is equivalent to Borda' s rule. Liu and Tsai [22] considered that the sequences of weights are *arithmetic progression* (AP) and *geometric progression* (GP). Such as in Olympic Games, the judges grade the first three athletes from a large number of nations (candidates) in each competition event, and award medals of gold, silver, and bronze. The weights of gold, silver, and bronze are (3, 2, 1) in AP model. The AP model is also equivalent to Borda's rule.

This study developed a procedure to determine the common set of weights of the places in those criteria. According the Borda's rule and fixed order of weights property, we can obtain the weights of the place by simple formulations. Candidates are subsequently ranked according to the sum of the weighted votes they earned at all the places in those criteria.

The remainder of this paper is organized as follows: Section 2 introduces fixed order of weights property which is indicated the relationships between criteria and places; Section 3 introduces the applied procedure; Section 4 illustrates a numerical example; and lastly, Section 5 provides a conclusion.

2. Fixed order of weights property

This study introduced an interesting issue that a group of decision-makers rank candidates with the c criteria. For each criterion, the group votes for candidates at p places. The criteria can be classified into different classes according to their importance: $j = 1,\ldots, c$. Let the weights of places are $\mathbf{V} = (v_i^p \mid i = 1,\ldots,p)$ and the weights of criteria are $\mathbf{U} = (u_j^{pc} \mid j = 1,\ldots,c)$. Hence, the scoring matrix $\mathbf{W} = \mathbf{V}^{\mathrm{T}}\mathbf{U} = [w_{ij}^{pc} | i = 1,\ldots,p, j = 1,\ldots,c]$, the weights of the p places in those c criteria. Note, the superscripts p and c are the total numbers of places and criteria in the voting system. Candidates are subsequently ranked according to the sum of the weighted votes they earned at all the places in those criteria. Let b_{ij}^k denote the total number of votes that Candidate k received at the ith place on the criteria in Class j. The score obtained by the candidate k is Z_k:

$$Z_k = \sum_{j=1}^{c}\sum_{i=1}^{p} w_{ij}^{pc} b_{ij}^k \tag{1}$$

Liu and Tsai [22] consider the *fixed order of weights* (FOW) property that vote weights on the criteria at each place would have the orders as the criteria ranks and furthermore, the vote weights of a place should be higher than that vote weights on the lower places. In other words, the elements of the scoring matrix are in the orders such as $w_{11}^{pc} > w_{12}^{pc} > \ldots > w_{1c}^{pc} > \ldots > w_{i1}^{pc} > w_{i2}^{pc} > \ldots > w_{ic}^{pc} > \ldots > w_{p1}^{pc} > w_{p2}^{pc} > \ldots > w_{pc}^{pc}$, the scoring weights on each criterion j possess the relationship $w_{1j}^{pc} > w_{2j}^{pc} > \ldots > w_{pj}^{pc}$, and the scoring weights on each place i possess the relationship $w_{i1}^{pc} > w_{i2}^{pc} > \ldots > w_{ic}^{pc}$. The relationship between every two adjacent places and two adjacent classes is assumed simply as $w_{ic}^{pc} > w_{i+1,1}^{pc}$, $i = 1,\ldots, p - 1$. The weight of the ith place on the last class w_{ic}^{pc} is no less than the weight of the $(i + 1)$ place on Class 1 $w_{i+1,1}^{pc}$. The relationships between weights are shown in Table 1.

Table 1 The general scoring matrix

		Criteria				
		u_1^{pc}		u_j^{pc}		u_c^{pc}
	v_1^p	$w_{11}^{pc} >$	$\cdots$	$> w_{1j}^{pc} >$	$\cdots$	$> w_{1c}^{pc}$
		$\vee$ ⋮ $\vee$		$\vee$ ⋮ $\vee$		$\vee$ ⋮ $\vee$
Place	v_i^p	$w_{i1}^{pc} >$	$\cdots$	$> w_{ij}^{pc} >$	$\cdots$	$> w_{ic}^{pc}$
		$\vee$ ⋮ $\vee$		$\vee$ ⋮ $\vee$		$\vee$ ⋮ $\vee$
	v_p^p	$w_{p1}^{pc} >$	$\cdots$	$> w_{pj}^{pc} >$	$\cdots$	$> w_{pc}^{pc}$

Base on AP model and Borda count, the gaps of consecutive are 1. Let $w_{pc}^{pc} = x$. Then $w_{1c}^{pc} = px$, $w_{p1}^{pc} = x + (c - 1)$, and $w_{21}^{pc} = (p - 1)[x + (c - 1)]$. Because $w_{1c}^{pc} > w_{21}^{pc}$ we can obtain $x > (p-1)(c-1)$, $x = (p-1)(c-1)+1$, and $w_{p1}^{pc} = (p-1)(c-1)+c$. Hence, $v_1^p = p$, $v_i^p = p - i + 1$, $u_1^{pc} = (p - 1)(c - 1) + c$, $u_j^{pc} = u_1^{pc} - c + 1$. In general, the weights of votes $v_i^p = p - i + 1, i = 1,\ldots,p$, the weights of criteria $u_j^{pc} = pc - p - j + 2, j = 1,\ldots,c$, and $w_{ij}^{pc} = (p - i + 1)(pc - p - j + 2), i = 1,\ldots,p, j = 1,\ldots,c$. The results of weights are equal Liu and Tsai [22] when normalized the results of weights in their model.

Furthermore, the scoring matrix can be obtained. Finally, the aggregate score Z_k could be calculated by equation(1). The decision maker can use the maximum aggregate score to divide into each aggregate score of candidates for obtain the values of scores between zero to one.

3. The applied procedure

Firstly, the group votes for the c criteria at p places for their impact to evaluate the candidates. The criterion with a higher impact is classified into Class 1, Class 2 for a lower one, and so on. For example, five criteria are present: A_1, A_2, …, A_5 ranked in different classes according to their importances by the top decision makers/managers or other MCDM methodologies.

Thereafter, for each criterion, the group votes for the n candidates with p places. The values b_{ij}^k, $i = 1,\ldots, p$, $j = 1,\ldots, c$, $k = 1,\ldots, n$ could be collected. Finally, the proposed model could be used to obtain the weights of places and calculate the aggregate score of each candidate.

In our model, we provide a property to find the normalized scoring matrix in general voting systems. Decision maker can use the equation $v_i^p = p - i + 1, i = 1,\ldots,p$ and $u_j^{pc} = pc - p - j + 2, j = 1,\ldots,c$ to produce the scoring matrix $w_{ij}^{pc} = (p - i + 1)(pc - p - j + 2), i = 1,\ldots,p, j = 1,\ldots,c$ easily. The equation $u_j^{pc} = pc - p - j + 2$ is the minimum multiple value of scoring matrix to satisfy the constraints of *fixed order of weights*. Decision maker also can adjust the multiple such as $u_j^{pc} = pc - p - j + 2 + a$ to find the scoring matrix for satisfying the property of *fixed order of weights*.

Employ the *fixed order of weights* property, the vote weights of frequent voting cases are arranged in Table 2.

Table 2 Vote weights w_{ij}^{pc} of several voting cases

No. of place	Number of criteria														
	c=2		c=3			c=4				c=5					
	u_1^{p2}	u_2^{p2}	u_1^{p3}	u_2^{p3}	u_3^{p3}	u_1^{p4}	u_2^{p4}	u_3^{p4}	u_4^{p4}	u_1^{p5}	u_2^{p5}	u_3^{p5}	u_4^{p5}	u_5^{p5}	

			3	**2**	**5**	**4**	**3**	**7**	**6**	**5**	**4**	**9**	**8**	**7**	**6**	**5**
***p*=2**	v_1^2	**2**	6	4	10	8	6	14	12	10	8	18	16	14	12	10
	v_2^2	**1**	3	6	5	4	3	7	6	5	4	9	8	7	6	5
			4	**3**	**7**	**6**	**5**	**10**	**9**	**8**	**7**	**13**	**12**	**11**	**10**	**9**
***p*=3**	v_1^3	**3**	12	9	21	18	15	30	27	24	21	39	36	33	30	27
	v_2^3	**2**	8	6	14	12	10	20	18	16	14	26	24	22	20	18
	v_3^3	**1**	4	3	7	6	5	10	9	8	7	13	12	11	10	9
			5	**4**	**9**	**8**	**7**	**13**	**12**	**11**	**10**	**17**	**16**	**15**	**14**	**13**
***p*=4**	v_1^4	**4**	20	16	36	32	28	52	48	44	40	68	64	60	56	52
	v_2^4	**3**	15	12	27	24	21	39	36	33	30	51	48	45	42	39
	v_3^4	**2**	10	8	18	16	14	26	24	22	20	34	32	30	28	26
	v_4^4	**1**	5	4	9	8	7	13	12	11	10	17	16	15	14	13

4. Numerical example

One company wants to select three supply chain members from ten candidates. Six decision makers are present in the selection committee. The director managers choose the importance of criteria before the review meeting. Their orders of criteria are cost (Class 1), delivery (Class 2), quality (Class 3), and flexibility (Class 4). In the review meeting, the decision makers vote the three places for candidates according to their preferences for each criterion. The collected data are shown in Table 3.

Table 3 Votes table b_{ij}^k

		Criterion											
		Cost (*j* = 1)			**Delivery (*j* = 2)**			**Quality (*j* = 3)**			**Flexibility (*j* = 4)**		
Place *i*		1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd
Candidate *k*	**A**	1	1	1	0	1	1	2	2	0	0	2	0
	B	2	0	1	1	0	1	0	1	2	2	0	1
	C	1	1	0	1	1	0	1	1	1	1	0	1
	D	0	2	1	1	1	2	0	0	1	2	0	1
	E	1	1	0	1	0	0	2	0	0	0	1	1
	F	0	0	2	1	1	0	0	1	2	1	1	0
	G	0	1	0	1	0	1	1	1	0	0	1	0
	H	1	0	1	0	0	0	0	0	0	0	1	0
	I	0	0	0	0	2	1	0	0	0	0	0	1
	J	0	0	0	0	0	0	0	0	0	0	0	1

Using the scoring matrix in Table 2, the aggregate scores of candidates are shown in Table 4. The normalized scores of candidates are as same as the results of aggregates score in Liu and Tsai [22]. Finally, the decision maker can choose Candidate A, B, and C to be their supply chain members.

Table 4 Aggregate scores and ranks

		Score Z_k	**Normalized score**	**Rank**
Candidate *k*	**A**	195	1	1
	B	187	0.9590	2
	C	171	0.8769	3
	D	170	0.8718	4
	E	146	0.7487	5
	F	132	0.6769	6
	G	110	0.5641	7

H	54	0.2769	8
I	52	0.2667	9
J	7	0.0359	10

5. Conclusion and discussion

This study obtains the weight relationships between the numbers of criteria and the numbers of places when the evaluated criteria are ordered according to their level of importance by using a MCDM method or the preference of a priority committee/manager. The procedure avoids the complex comparison problems in traditional AHP and makes the evaluated process easily in decision making problems.

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