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A Multiple Attribute Decision Making Model in the Presence of Grey Numbers

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Abstract. The multiple attribute decision making (MADM) is an important research field in decision science and operations research. Several commonly used method such as the TOPSIS (Technique for order performance by similarity to ideal solution) and RR (Relative Ratio) were proposed to solve the MADM problems. In some cases the data set is incomplete and uncertain and using the classical MADM models is impossible. In these cases employing the grey numbers has been recommended in the literature. In this study we propose RR model based on grey numbers. A real case study is used to show advantages of the proposed model and the results are compared with grey TOPSIS model.

Keywords: MADM, TOPSIS, RR, Grey Number, Decision Making

1. Introduction

In real-life management decision situations, multiple attribute decision making (MADM) models and methods have been proposed as aids. MADM problems become an important type of multiple criteria decision making (MCDM) problems. The MADM is an active research field in Operations Research, Decision Science and Management Science.

TOPSIS method is a technique for order preference by similarity to ideal solution and proposed by Hwang and Yoon (1981). The ideal solution (also called positive ideal solution) is a solution that maximizes the benefit criteria/attributes and minimizes the cost criteria/attributes, whereas the negative ideal solution (also called anti-ideal solution) maximizes the cost criteria/attributes and minimizes the benefit criteria/ attributes. The so-called benefit criteria/attributes are those for maximization, while the cost criteria/attributes are those for minimization. The best alternative is the one, which is closest to the ideal solution and farthest from the negative ideal solution. [1]

The closest option to the positive ideal in TOPSIS method with Euclidean distance is not necessarily the farthest option from negative ideal. Relative Ratio (RR) method was represented by Li (2009) to eliminate the existing fault in TOPSIS method. This method relies on ranking and selecting a set of alternatives in the presence of interfering indexes. The agreed optimum answer in this method (with ranking list) is determined based on the pattern that the selected alternative must close to positive ideal as much as possible and simultaneously is far from the negative option. [2]

The above decision making models should be modified and necessary changes are remodeled to use imprecise information under conditions that decision information is uncertain. Grey numbers are among the numbers that are used to state uncertain information. These numbers will be mentioned in the next sections.

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Objective of this survey is to represent a new RR model based on grey numbers. Then superiority of this model will be illustrated in comparison with GREY TOPSIS model in an applied example.

2. Grey Systems Theory

White number, grey number and black number are three classifications of describing information uncertainty. Let $\otimes x = [\underline{x}, \overline{x}] = \{x | \underline{x} \le x \le \overline{x}, \underline{x} \;\&\; \overline{x} \in R\}$. Then, Ä x which has two real numbers $\underline{x}$ (the lower limit of Ä x) and $\overline{x}$ (the higher limit of Ä x) is defined as follows.

1. If $\underline{x} \to -\infty$ and $\overline{x} \to +\infty$, then Ä x is called the black number which means without any meaningful information.

2. Else if $\underline{x} = \overline{x}$, then Ä x is called the white number which means with complete information.

3. Otherwise, $\otimes x = [\underline{x}, \overline{x}]$ is called the grey number which means insufficient and uncertain information.

In practice, the concept of a grey number is often appeared in the engineering practice for construction activities. For example, we may not be able to precisely appraise the budget of a new project, but we can provide a possible range of budget by our experience and knowledge. This range (interval number) is a grey number. Grey number is a concept from grey theory, proposed by Deng [3–5], to deal with the insufficient and incomplete information. Since grey theory proposed, however, the applications are mostly based on the white numbers. Nevertheless, the obtained information from real world is always uncertain or incomplete. Hence, extending the applications of grey numbers is necessary.

Grey number, actually, can be considered as a special case of fuzzy number. Assume two fuzzy numbers $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ then the Euclidean distance between ã and b̃ can be calculated by [5]

$$d(\tilde{a}, \tilde{b}) = \sqrt{\tfrac{1}{3}\left[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2\right]} \tag{1}$$

When fuzzy numbers ã and b̃ are transformed into grey numbers $\otimes a = [\underline{a}, \overline{a}]$ and $\otimes b = [\underline{b}, \overline{b}]$ respectively, the Euclidean distance between Äa and Äb can be easily obtained by

$$d(\otimes\tilde{a}, \otimes\tilde{b}) = \sqrt{\tfrac{1}{2}\left[(\underline{a} - \underline{b})^2 + (\overline{a} - \overline{b})^2\right]} \tag{2}$$

By integrating the concept of Eq. (2) and weighted Minkowski distance function, the weighted grey number Minkowski distance function can be further established. Assume a two m-dimension grey number series, then the weighted grey number Minkowski distance between Äx and Äy can be calculated by

$$d(\otimes x, \otimes y) = \sqrt[p]{\frac{1}{2}\sum_{j=1}^{m} w_j\left[|\underline{x}_j - \underline{y}_j|^p + |\overline{x}_j - \overline{y}_j|^p\right]} \tag{3}$$

Euclidean distance is a special case of Eq. (3) at p=2; the well-known city-block distance function is another special case of Eq. (3) at p=1. By using the concept of Eq. (3), TOPSIS can be further applied on the decision making with uncertain information.

Current applications of grey systems theory could be classified into five scopes of assessment, modeling, prediction, decision-making and control. One of the prominent methods of this theory is in decision making scope.

3. The proposed multi-attribute decision making model

This study represents a multi attribute decision making model under uncertain conditions based on action of the grey numbers and Minkowski distance function. Before the details of the model are explained, consider the following decision matrix of grey numbers. In this matrix x_{ij} is evaluation of grey related to ith alternative with orders of the index. Procedure of the model could be showed through the following steps.

$$D = \begin{bmatrix} x_{11} & x_{12} & . & . & . & x_{1m} \\ x_{21} & x_{22} & . & . & . & x_{2m} \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ x_{n1} & x_{n2} & . & . & . & x_{nm} \end{bmatrix}$$

Step 1: Multiple attributes are usually incommensurable. Therefore, the decision matrix has to be normalized so that the units and dimensions of attribute functions (or values) are eliminated. In this paper we convert decision matrix into an un -scale matrix by means of the following relation:

$$r_{ij} = \begin{cases} \left(\frac{\underline{x}_{ij}}{\max \overline{x}_{ij}}, \frac{\overline{x}_{ij}}{\max \overline{x}_{ij}} \right) & j \in J \\ \left(\frac{\min \underline{x}_{ij}}{\overline{x}_{ij}}, \frac{\min \underline{x}_{ij}}{\underline{x}_{ij}} \right) & j \in J' \end{cases}, \qquad \begin{cases} i = 1, \dots, n \\ j = 1, \dots, m \end{cases} \tag{4}$$

Step 2: Assume that A^+ is positive ideal and A^- is negative ideal solution, respectively . In this case we have

$$\left\{ (\max_i \overline{r}_{ij} \mid j \in J), (\min_i \underline{r}_{ij} \mid j \in J') \mid i \in n \right\} = \left\{ r_1^+, r_2^+, ..., r_m^+ \right\} \tag{5}$$

$$\left\{ (\min_i \underline{r}_{ij} \mid j \in J), (\max_i \overline{r}_{ij} \mid j \in J') \mid i \in n \right\} = \left\{ r_1^-, r_2^-, ..., r_m^- \right\} \tag{6}$$

Step 3: The distance of the alternative A_i to A^+ and A^- is measured as below with weighted Minkowski distance respectively:

$$d_p{}^+(A_i) = \left\{ \frac{1}{2} \sum_{j=1}^{m} w_j \left[| r_j^+ - \underline{r}_{ij} |^p + | r_j^+ - \overline{r}_{ij} |^p \right] \right\}^{\frac{1}{p}} \tag{7}$$

$$d_p{}^-(A_i) = \left\{ \frac{1}{2} \sum_{j=1}^{m} w_j \left[| r_j^- - \underline{r}_{ij} |^p + | r_j^- - \overline{r}_{ij} |^p \right] \right\}^{\frac{1}{p}} \tag{8}$$

If p=2, the above function will be converted into weighted Euclidean distance.

Step 4: Whatever $d_p^+(A_i)$ is smaller, distance of alternative A_i to the positive ideal will be lower, therefore the alternative with the following distance is an agreed answer that has the least distance with positive ideal:

$$d_p{}^+ = \min_{1 \le i \le n} d_p{}^+(A_i) \tag{9}$$

Anyway, this issue doesn't represent a guarantee about that such answer has the highest distance from negative ideal. In a similar way whatever $d_p^-(A_i)$ is larger, distance of alternative A_i to negative ideal will be more, thus the alternative with the following distance is an agreed answer that has the largest distance with negative ideal:

$$d_p{}^- = \max_{1 \le i \le n} d_p{}^-(A_i) \tag{10}$$

But it doesn't provide a guarantee on existence of an answer with the least distance from positive ideal.

Step 5: Relative proportion of alternative A_i is defined as

$$x_p(A_i) = \frac{d_p^-(A_i)}{d_p^-} - \frac{d_p^+(A_i)}{d_p^+}, \qquad i = 1, \mathrm{K}, n \tag{11}$$

$x_p(A_i)$ measures the amount that alternative A_i is simultaneously close to positive ideal and far from the negative ideal.

Step 6: Organize alternatives in descending order. Whatever $x_p(A_i)$ is larger, alternative A_i is more desirable, thus alternative A^* that is true in

$$\xi_p(A^*) = \max_{1 \le i \le n} \xi_p(A_i) \tag{12}$$

is the optimum answer.

4. Example

In this section, we adopt a subcontractor selection example from an engineering corporation to demonstrate the feasibility and practicability of the proposed model. Selection of subcontractor is an important issue in the field of construction management, for the success or failure of a project is usually influenced by the quality of subcontractor. Practically, the company under study invites internal experts to form a decision group to select an appropriate subcontractor from a list of qualified candidates. Every subcontractor is evaluated by the following attributes:

1. Reliability (RA). The reliability of subcontractors is evaluated by their reputation, records and financial condition.

2. Schedule-control Ability (SA). The schedule-control ability is measured by subcontractors' mobilization and efficiency.

3. Management Ability (MA). This attribute is to assess the quality, safety, and environmental management level of each subcontractor.

4. Labor Quality (LQ). This attribute is to evaluate the workers' skill level, coordination and cooperation of subcontractors.[6]

Table 1

Decision Matrix

	RA		SA		MA		LQ	
	L	U	L	U	L	U	L	U
A1	75	85	75	85	70	80	75	80
A2	65	70	70	80	65	70	70	75
A3	75	80	75	80	80	85	75	80
A4	70	75	65	70	70	75	75	80

If we normalize table 1 by means of the relations (4), the following table is obtained.

Table 2

Normalized Matrix

	RA		SA		MA		LQ	
	L	u	l	u	L	u	l	u
A1	0.8824	1.0000	0.8824	1.0000	0.8235	0.9412	0.9375	1
A2	0.7647	0.8235	0.8235	0.9412	0.7647	0.8235	0.8750	0.9375
A3	0.8824	0.9412	0.8824	0.9412	0.9412	1.0000	0.9375	1
A4	0.8235	0.8824	0.7647	0.8235	0.8235	0.8824	0.9375	1
A+	1.0000		1.0000		1.0000		1 .0000	
A-	0.7647		0.7647		0.7647		0.8750	

Table 3 shows positive and negative ideals for each alternative and relative ratio of each alternative.

Table3

	d_i^+	d_i^-	c_i (Grey Topsis)	RR_i (Grey Relative Ratio)
A1	0.085194	0.14983	0.63751	-0.21429
A2	0.158254	0.075504	0.323002	-1.7517
A3	0.070159	0.148962	0.679814	-0.0058
A4	0.143155	0.085503	0.373933	-1.46976

According to results of table 3, we can obtain the priority A3>A1>A4>A2 for the alternatives. The last column of table that is obtained through RR model has reached to similar results too.

5. Conclusions

Today, many decision indexes in most decision-making subjects are qualitative or the existing information about them is uncertain. Thus, necessity of using models by which we can analyze conditions of decision and achieve an accurate decision becomes obvious. This paper represents a new model for the above conditions by considering such conditions at the time of applying one of the MADM techniques and through grey numbers that is called RR model. The represented model is easily usable and implemented in most decision-making conditions with uncertain inputs.

An applied example has been used in this paper to show the possibility and workability of the recommended model. Results reveal that not only the recommended model eliminates some limitations of TOPSIS model but also it could be practically used for decision-making in various and especially uncertain conditions to choose and rank alternatives.

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