2011 2nd International Conference on Construction and Project Management
IPEDR vol.15 (2011) © (2011) IACSIT Press, Singapore

Maximizing Net Present Value a review through literature

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Abstract. The vast majority of the project scheduling methodologies presented in the literature has been developed with the objective of minimizing the project duration. In doing so, the financial aspects of project management are largely ignored. If financial aspects taken into consideration of particular interest to project management, the maximization of project net present value (NPV) then decided as the more appropriate objective. Since the amount of literature related to the project scheduling problem is enormous, this paper addresses only the literature related to the project NPV maximization problem. In this paper we review the fundamental approaches for maximizing NPV project scheduling problem as well as to summarize and categorize the model that have been proposed and presented in the literature. We found out that metaheuristics strategy such as Genetic Algorithms and Tabu search have recently attracted more intention and been the most popular strategies for solving the project scheduling problem with objective of maximizing NPV.

Keywords: Net present value, NPV maximization, project scheduling

1. Introduction

Project scheduling is the process where the various activities that need to be undertaken during a projects lifetime should be scheduled. It is concerned with the techniques that can be employed to manage the activities that need to be undertaken during the development of a project. It is primarily concerned with attaching a timescale and sequence to the activities to be conducted within the project. This paper will focus on *project scheduling* that is the subset of project management. Project scheduling is the process where the various activities that need to be undertaken during a projects lifetime should be scheduled.

2. Literature Review

Since the amount of literature related to the project scheduling problem is enormous, this paper addresses only the literature related to the project NPV maximization problem. Because the most pragmatic project objective is, therefore, to maximize project NPV [1]. Scheduling problem subject NPV maximization can be distinguished into two categories; the unconstrained project scheduling problem, which occurs when no constraints on resource usage are imposed such that the activities are only subject to precedence constraints, and the resource-constrained project scheduling problem (RCPSP).

2.1. Unconstrained resource

Russell [2] was the first to introduce the objective of maximizing the NPV of cash flows in a network with the unconstrained problem. Russell's objective function is to

$$\text{maximize NPV} = \sum_{i=1}^{n} CF_i \exp(-\alpha T_i) \quad (1)$$

where $\exp(\alpha Ti)$ = β (the discount factor)
CF = Cash Flow

For uniformity of expression, the criterion Eq. (1) is sometimes rewritten as:

maximize NPV = $\sum_{i=1}^{n} CF_i \beta^{-T_i}$ (2)

Initially, the nonlinear objective function Eq. (2) is approximated by considering only the first (linear) term of the associated Taylor series expansion.

Assuming a current non-optimum but feasible solution given by the event times Ti°, we have, for Ti, close to Ti°,

$$\sum_{i=1}^{n} CF_i \exp(\alpha T_i) = \sum_{i=1}^{n} CF_i \exp(-\alpha T_i^0) - \sum_{i=1}^{n} (T_i - T_i^0) CF_i \alpha \exp(-\alpha T_i^0)$$

$$= \sum_{i=1}^{n} CF_i \exp(-\alpha T_i^0) + \sum_{i=1}^{n} T_i^0 CF_i \exp(-\alpha T_i^0)$$

$$- \sum_{i=1}^{n} T_i CF_i \exp(-\alpha T_i^0)$$

And the original objective Eq. (1) is replaced by the maximisation of the linear objective:

$$-\sum_{i=1}^{n} T_i \mathrm{C_i} \exp(-\alpha T_i^0) = -\sum_{i=1}^{n} T_i C_i \beta^{\mathrm{T_i^0}}$$

Russell [2] transforms the nonlinear objective function into a linear one by approximation using the first term of the associated Taylor series expansion. He does not report computational results with his procedure apart from two small example problems, although reference is made to a computer program being developed to solve this problem. Because of the development of fast and efficient network computer codes since the publication of this paper, there do not seem to be any theoretical obstacles to implementing his approach. His research showed that the cost-critical path is quite different from the time-critical path when monetary objectives are considered. Grinold [3] transforms the unconstrained problem formulated by Russell [1] into an equivalent linear programming problem. This problem is exploited by the solution procedure that determines the optimal solution by exploring the set of feasible trees on the project network such that all activities have zero slack. This procedure is also used to illustrate, with an example, the trade-off between NPV and project duration He does not provide extensive computational results for his procedure. Elmaghraby and Herroelen [4] critique both Russell's [1] and Grinold's [3] formulations to develop a simplified algorithm that gives the optimal schedule for the project scheduling problem with NPV objective. They show that, in general, it is optimal to schedule events with associated positive cash flows as early as possible, and events with net negative cash flows as late as possible subject to restrictions imposed by network structure. They also illustrate that net cash flows are dependent on the time of realization of cash flow nodes and in the absence of a project deadline, if the NPV is less than zero, the project will be delayed indefinitely. Demeulemeester [5] have proposed a new optimal algorithm that performs a recursive search on partial tree structures that utilize the concept of scheduling activities early if they bring in payments and delaying those activities that incur expenses. Computational tests report encouraging results in comparison to the Grinold [3] procedure.

2.2. Resource constrained

To schedule project with max NPV objective, previous research is categorized the procedures as either an optimization or heuristic solution method. Optimal procedures have been termed *exact* or *analytical* procedures because they usually involve some form of mathematical programming or other rigorous analytical procedure. Heuristic procedures involve the use of some rule-of-thumb or *heuristic* in determining priorities among jobs competing for available resources. However, these two methods consist of procedures which aim at producing the best possible schedule.

2.2.1 Optimization procedures

With optimal procedures, an initial sub-categorization can be made according to the type of mathematical technique employed in the search for the best possible solution. Existing procedures are divided according to whether they utilize some form of integer linear programming, a variation of some enumerative or other technique. Examples are *branch and bound* [6].

Doersch and Patterson [7] were the first to study in the context of the resource-constrained max-npv problem. They introduced a binary integer programming approach to the NPV project scheduling problem. This model included a constraint on capital for expenditure on activities in the project such that the available capital increased as progress payments were made. The objective function also included the cash flows associated with the completion of activities and any penalties incurred for late completion. The model was solved to optimality for projects involving 15–25 activities. The results indicated that at high cost of capital or long project duration, it is important to evaluate bonus/penalty and capital constraints while scheduling activities. However, detailed computational results are not provided. Smith-Daniels and Smith-Daniels [8] extend the Doersch and Patterson [7] zero-one formulation to accommodate material management costs. The NPV of the project was maximized subject to material and capital constraints and solved to optimality on small problems. They concluded that not only do ordering and holding cost force activities with common requirements to start at the same time or close to each other, the additional constraints also result in lowering overall project cost even though they may cause activities, and hence the project, to be delayed. Tavares proposed a new dynamic programming formulation and solution method, where the optimality conditions were derived using calculus of variations for a set of interconnected projects [9]. The objective function to be maximized included a net of the discounted sum of the benefits generated along the program, the discounted sum of the cost of project expenditures, and a term to penalize the variation in expenses over time. This program was applied successfully to a large railway construction project in Portugal. Patterson [10] presented a zero–one programming model and a backtracking algorithm to maximize the NPV of the constrained project scheduling problem. It is unique in that it can also be used to minimize project duration. The solution methodology utilized the fact that the minimum duration problem is easier to solve than the max NPV problem and used it as a heuristic to generate starting solutions on which right-shifting of cash flows was applied to improve NPV. 91 problems, ranging from 10 to 500 activities, were tested on both objectives using MINSLK and random rules, with optimal solutions found only for the smaller problems. The MINSLK rule generated higher NPV than the random rule. Baroum and Patterson [11] proposed a branch and bound procedure directly designed to solve the project scheduling problem with NPV objective. Icmeli and Erenguc [12] also developed a branch-and-bound algorithm for the RCPSP with cash flows which used the minimal delaying alternatives concept for branching.

Since optimization techniques have not been successful in solving this limited resource problem and are impractical to compute for large-scale projects, another method is proposed.

2.2.2 Heuristic procedures

Heuristic procedures can be broadly classified into Priority-rule-based-scheduling and metaheuristic approaches; Genetic Algorithms; Tabu search; Simulated Annealing. The idea behind heuristic algorithms for resource constrained project scheduling is to rank the activities by some rule, this may be managerial priority, earliest start times, and to schedule the activities in that ranking order ensuring that the resource limits on the project are never exceeded. Thus activities considered to be important in some sense are scheduled as soon as possible. Smith-Daniels and Aquilano [13] who compared the duration and NPV of a late-start critical path schedule to that of an early-start critical path schedule. It was assumed that cash outflows occurred at the beginning of the period and a single project payment was received on completion of the project. Their assumptions were tested using the 110 Patterson problems. An improved average NPV and lower average duration can be found for late-start schedules than early-start schedules. Smith-Daniels and Aquilano [13] considered the resource constrained max-npv problem. They concluded that a heuristically determined right shifted schedule yields a higher NPV and lower average duration than schedules derived with heuristics that schedule each activity as early as possible. Ulusoy and Özdamar [14] presented an iterative scheduling algorithm with the objective of improving both the project duration and NPV. The consecutive forward/backward scheduling passes made by the iterative algorithm result in a smoother resource profile, which, along with right-shifting of activities, improves both the project duration and NPV. In the cash flow model assumed here, activity expenditures occur at their starting times and payment is made on completion of the project. The algorithm was tested on two sets of problems from the literature. The results demonstrated that under the assumed cash flow model, the iterative scheduling algorithm improved both criteria. Baroum and Patterson [15] evaluated several heuristic approaches used by project and contract managers involving single- and multi-pass procedures. Their single-pass procedures used priority weights based upon cumulative future cash flows for all successor activities. Multi-pass procedures were enhancements to improve upon the single-pass solution obtained. A full factorial experimental design was used to assess the performance of the heuristic procedures. The computational results demonstrated the efficiency of the discounted cash flow, positional weight heuristics over more traditional methods. Icmeli and Erenguc [16] applied a tabu search procedure to a starting feasible solution generated using a simple single-pass algorithm. The initial solution was improved over several iterations by moving each activity one time unit early or late from its current completion time, with the restriction that the resulting completion time should not violate earliest and latest completion times for the activity. They also investigated the use of long-term memory within tabu search to further improve the results. Computational results on 50 problems from the Patterson set indicated that these procedures were both efficient and close to optimal. Zhu and Padman [17] applied distributed computing concepts to the RCPSP through the use of an Asynchronous Team (A-Team) approach. An A-team is a software organization that facilitates cooperation amongst multiple heuristic algorithms so that together they produce better solutions than if they were acting alone. They embedded several simple heuristics for solving the RCPSP within the iterative, parallel structure of A-Team which provides a natural framework for distributed problem solving.

3. Discussion

Many researches focus during the past years has been on metaheuristics. These approaches have recently attracted more intention than priority rule-based methods. Genetic Algorithms and Tabu search have been the most popular strategies. The activity list has been the most widely used representation. It has usually been employed in its classical form, while a few researchers have extended it. Past research [2][11][15] has developed many different deterministic, single-pass heuristic decision rules for maximizing project NPV. A limitation of these single-pass rules is that they only generate a single solution or schedule for a problem. A general observation is that the new propose techniques contain more components than earlier procedures [9]. Many methods consider both scheduling directions instead of only forward scheduling, more than one type of local search operator, or even more than one type of metaheuristic strategy.

4. Conclusion

Considering the development during the past years, metaheuristic approaches have recently attracted more intention than other methods. New techniques tend to more components than earlier one. The use of existing ideas occasionally seems to be less creative than developing new ideas. Some of the integration efforts have put well-known techniques into a new and promising context, and the results have often been encouraging [9]. Hence, even if a good solution has been found, an investigating of an improved solution procedure for the project scheduling problem is, therefore, a worth while endeavor.

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