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Dynamic Multi-Attribute Decision Making Model with Grey Number Evaluations on Safety Management Performance of Subcontractors

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Abstract. In this study, both the Minkowski distance function and the grey number distance function are integrated into the TOPSIS to develop a dynamic multi-attribute decision making model for effectively handling the uncertain information. An illustration example of subcontractors' safety performance evaluation is adopted to demonstrate the practicability of the proposed model. Results show that the proposed model can effectively deal with the uncertain information risk and does not cause more computational burden.

Keywords: fuzzy theory, grey number, Minkowski distance function, TOPSIS

1. Introduction

The multi-attribute decision making (MADM) problem is to select an appropriate alternative from a finite number of feasible alternatives based on the features of each attribute with respect to every alternative. TOPSIS (technique for order preference by similarity to an ideal solution) is one of the widely used techniques to solve MADM problems [1]. TOPSIS is to rank a finite number of feasible alternatives in order of preference and then select a suitable alternative that conforms to the decision maker's ideal. The basic concept of TOPSIS technique is that the selected alternative will have the shortest Euclidean distance from the ideal solution and the farthest Euclidean distance from the anti-ideal solution. In recent years, TOPSIS has been successfully adopted to solve various MADM problems and demonstrated satisfactory results. Hence, this paper takes the TOPSIS technique as the main approach to solve MADM problems and extend it to the dynamic environment as a result of its effectiveness and practicability. Besides, this paper integrates the concepts of grey number and Minkowski distance function to TOSPSIS to effectively deal with the uncertain information and comprehensively synthesize the different evaluations among all periods.

2. The proposed dynamic multi-attribute decision making model

Before describing the detailed model, a positive grey number decision matrix, $\boldsymbol{D}^{\mathrm{t}}$, ought to be defined first as.

$$\boldsymbol{D}^{\mathrm{t}} = \begin{bmatrix} \otimes x_{11}^t & \otimes x_{12}^t & \cdots & \otimes x_{1m}^t \\ \otimes x_{21}^t & \otimes x_{22}^t & \cdots & \otimes x_{2m}^t \\ \cdots & \cdots & \cdots & \cdots \\ \otimes x_{n1}^t & \otimes x_{n2}^t & \cdots & \otimes x_{nm}^t \end{bmatrix},$$

where $\otimes x_{ij}^t$ denotes the grey number evaluations of the *i-th* alternative with respect to the *j-th* attribute at *t-th* period ($t = 1, \ldots, T$); $\otimes x_i^t = [\otimes x_{i1}^t, \otimes x_{i2}^t, \ldots, \otimes x_{im}^t]$ is the grey number evaluation series of the *i-th*

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alternative given at *t-th* period. It is noted that there should be T grey number decision matrices for the T evaluation periods. The procedure of the proposed model can be shown as the following five steps.

Step 1: Construct the normalized grey decision matrices

To avoid distort the linear ratio of difference between original and normalized data, this paper adopts a linear transformation function [2] to construct normalizing grey decision matrices. Generally, the property of attribute can be divided into two types: (1) the attribute is the larger-the-better; and (2) the attribute is the smaller-the-better. The transformation of the larger-the-better type attribute can be written as

$$\otimes r_{ij}^{t} = \frac{\otimes x_{ij}^{t}}{\max(\overline{x}_{ij}^{t})} = \left(\frac{\underline{x}_{ij}^{t}}{\max(\overline{x}_{ij}^{t})}, \frac{\overline{x}_{ij}^{t}}{\max(\overline{x}_{ij}^{t})}\right). \tag{1}$$

On the other hand, the normalization of the smaller-the-better type attribute can be calculated as

$$\otimes r_{ij}^{t} = -\frac{\otimes x_{ij}^{t}}{\min(\underline{x}_{ij}^{t})} + 2 = \left(\frac{-\overline{x}_{ij}^{t}}{\min(\underline{x}_{ij}^{t})} + 2, \frac{-\underline{x}_{ij}^{t}}{\min(\underline{x}_{ij}^{t})} + 2\right). \tag{2}$$

Step 2: Determine the positive and negative ideal alternatives for each period.

The positive ideal alternative, A^{t+}, and the negative ideal alternative, A^{t-}, at the *t-th* period can be defined as

$$A^{t+} = \left\{ (\max_i \overline{r}_{ij}^{t} \,|\, j \in J), (\min_i \underline{r}_{ij}^{t} \,|\, j \in J') \,|\, i \in n \right\} = \left[r_1^{t+}, r_2^{t+}, \ldots, r_m^{t+} \right],$$

And

$$A^{t-} = \left\{ (\min_i \underline{r}_{ij}^{t} \,|\, j \in J), (\max_i \overline{r}_{ij}^{t} \,|\, j \in J') \,|\, i \in n \right\} = \left[r_1^{t-}, r_2^{t-}, \ldots, r_m^{t-} \right].$$

Step 3: Calculate the measures from the positive and negative ideal alternatives for each period.

At the *t-th* period, the separation measures from the positive ideal alternative, d_i^{t+}, and from the negative ideal alternative, d_i^{t-}, are computed through the weighted grey number Minkowski distance function as

$$d_i^{t+} = \left\{ \frac{1}{2} \sum_{j=1}^{m} w_j \left[\left| r_j^{t+} - \underline{r}_{ij}^{t} \right|^p + \left| r_j^{t+} - \overline{r}_{ij}^{t} \right|^p \right] \right\}^{1/p}, \tag{3}$$

and

$$d_i^{t-} = \left\{ \frac{1}{2} \sum_{j=1}^{m} w_j \left[\left| r_j^{t-} - \underline{r}_{ij}^{t} \right|^p + \left| r_j^{t-} - \overline{r}_{ij}^{t} \right|^p \right] \right\}^{1/p}, \text{ respectively.} \tag{4}$$

In Eqs. (3) and (4), $p \geq 1$ and integer, w_j is the weight for the attribute j.

Step 4: Synthesize the measures of all evaluated periods.

In this step, this paper adopts the concepts of fuzzy membership grade and clustering algorithm to aggregate the evaluations from each period. Let the ideal alternative and anti-ideal alternative be two categories; u_i and $(1-u_i)$ are the aggregation membership grades of the *i-th* alternative with respect to the ideal alternative and anti-ideal alternative, respectively. $v(t)$ is the period weight of the *t-th* period, $v(t) > 0$, $v = (v(1), v(2), \ldots, v(T))$, $\sum_{t=1}^{T} v(t) = 1$. $v(t)d_i^{t+}$ and $v(t)d_i^{t-}$ is the period-weighted distance of A_i with respect to the ideal alternative and anti-ideal alternative, respectively. Then, the objective function of clustering algorithm can be constructed as

$$F(u) = \sum_{t=1}^{T} \{ [u_i v(t) d_i^{t+}]^2 + [(1-u_i) v(t) d_i^{t-}]^2 \}. \tag{5}$$

To obtain a stable result of Eq. (5), we calculate the optimum solution vector, $u = (u_1, u_2, \ldots, u_n)$, based on the minimum of the objective function . Let $\frac{\partial F}{\partial u_i} = 0$,

$$\frac{\partial F}{\partial u_i} = 2\sum_{t=1}^{T}[u_i v(t) d_i^{t+}][v(t) d_i^{t+}] + 2\sum_{t=1}^{T}[(1-u_i)v(t) d_i^{t-}][-v(t) d_i^{t-}] = 0\,, \tag{6}$$

then

$$u_i = [\sum_{t=1}^{T}(v(t) d_i^{t-})^2]/[\sum_{t=1}^{T}(v(t) d_i^{t+})^2 + \sum_{t=1}^{T}(v(t) d_i^{t-})^2]\,, \tag{7}$$

where $0 \le u_i \le 1$. Larger u_i means better alternative. When only single period is considered, $T = 1$ and $v(1) = 1$, Eq. (7) will be

$$u_i = \frac{(d_i^{t-})^2}{(d_i^{t+})^2 + (d_i^{t-})^2}\,. \tag{8}$$

Step 5: Rank the preference order.

A set of alternatives now can be preference ranked by the descending order of the value of u_i.

3. Illustrative example

In this section, this paper adopts an evaluation example on safety management performance of subcontractors of an engineering corporation to demonstrate the feasibility and practicability of the proposed model. The safety performance of each subcontractor is evaluated by the following attributes: (1) general safety strategy (SP), (2) safety organization and resources (SR), (3) safety planning and implementation (SI), and (4) safety improvement of system evaluation (SE). A real case is adopted to demonstrate the applicability of the proposed model. In this case, four subcontractors are considered, namely S1, S2, S3 and S4, and three-period evaluations, i.e., T1, T2 and T3, are considered. The evaluation results (including the lower limit, LL, and the upper limit, UL, of each grey number evaluation) are summarized in Table 1. The interval range of grey number depends on the uncertainty of the obtained information from each subcontractor.

Table 1. Evaluations of subcontractors at each period

Period	Subcontractor	SP		SR		SI		SE	
		LL	UL	LL	UL	LL	UL	LL	UL
T1	S1	65	70	75	80	70	75	75	80
	S2	70	75	75	80	75	85	70	80
	S3	75	80	75	85	75	80	75	85
	S4	70	80	70	75	75	80	70	75
T2	S1	65	70	70	75	65	70	70	80
	S2	80	85	60	70	80	85	75	80
	S3	70	75	70	80	75	80	70	75
	S4	70	80	65	70	70	75	75	85
T3	S1	70	75	60	70	75	85	65	70
	S2	75	80	80	85	75	85	80	85
	S3	75	85	75	80	70	75	70	80
	S4	70	80	75	85	65	70	75	85

First, the raw decision matrix is normalized into comparable decision matrices. Because these four attributes are all the larger-the-better type, the raw decision matrices can be normalized by Eq. (1). Table 2 shows the normalized decision matrices.

Table 2. Normalized decision matrices at each period

Period	Subcontractor	SP	SR	SI	SE

		LL	UL	LL	UL	LL	UL	LL	UL
T1	S1	0.81	0.88	0.88	0.94	0.82	0.88	0.88	0.94
	S2	0.88	0.94	0.88	0.94	0.88	1.00	0.82	0.94
	S3	0.94	1.00	0.88	1.00	0.88	0.94	0.88	1.00
	S4	0.88	1.00	0.82	0.88	0.88	0.94	0.82	0.88
T2	S1	0.76	0.82	0.88	0.94	0.76	0.82	0.82	0.94
	S2	0.94	1.00	0.75	0.88	0.94	1.00	0.88	0.94
	S3	0.82	0.88	0.88	1.00	0.88	0.94	0.82	0.88
	S4	0.82	0.94	0.81	0.88	0.82	0.88	0.88	1.00
T3	S1	0.82	0.88	0.71	0.82	0.88	1.00	0.76	0.82
	S2	0.88	0.94	0.94	1.00	0.88	1.00	0.94	1.00
	S3	0.88	1.00	0.88	0.94	0.82	0.88	0.82	0.94
	S4	0.82	0.94	0.88	1.00	0.76	0.82	0.88	1.00

Then, the separation measures of every subcontractor, d_i^{t+} and d_i^{t-}, at each period can be further calculated. Take period T1 for example, we can easily find $A^{1+} = [1, 1, 1, 1]$ and $A^{1-} = [0.81, 0.82, 0.82, 0.82]$ from Table 2. Then, d_i^{1+} and d_i^{1-} of each subcontractor can be calculated. In the same way, d_i^{t+} and d_i^{t-} of other periods can be further obtained. After that, aggregate the evaluations of each d_i^{t+} and d_i^{t-} for periods T1, T2 and T3 to calculate u_i. Finally, we summarize the calculations of d_i^{t+}, d_i^{t-} and u_i of each subcontractor as Table 3.

Table 3. Separation measures and membership grades of each subcontractor

Subcontractor	T1		T2		T3		u_i
	d_i^{1+}	d_i^{1-}	d_i^{2+}	d_i^{2-}	d_i^{3+}	d_i^{3-}	
S1	0.12	0.08	0.16	0.10	0.19	0.10	0.27
S2	0.11	0.10	0.12	0.15	0.06	0.20	0.59
S3	0.08	0.13	0.13	0.13	0.12	0.15	0.65
S4	0.13	0.08	0.13	0.12	0.13	0.17	0.39

According to the results of Table 3, we can find the priority is $S3 \succ S2 \succ S4 \succ S1$.

4. Conclusions

In this study, the Minkowski distance function is adopted to overcome the over effects of weighting in the original TOPSIS technique. Besides, the concept of fuzzy number distance function is used to construct the grey number distance function, and also integrated with Minkowski distance function to propose the weighted grey number Minkowski distance function to effectively handle the uncertain information. Finally, we have integrated the above-mentioned concepts, TOPSIS technique and aggregation approach to establish an effective dynamic decision making model.

This paper also has taken a real case of subcontractors' safety performance evaluation from an engineering corporation to demonstrate the practicability of the proposed model. Results show that the proposed dynamic decision making model can effectively deal with the uncertain information risk and does not cause more computational burden. The proposed dynamic decision model is not only efficient and robust, but also quite good for real-world applications.

5. References

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