2011 International Conference on Innovation, Management and Service
IPEDR vol.14(2011) © (2011) IACSIT Press, Singapore

A Linear Programming Approach for Different Serial Machines Scheduling with Optimizing Batch Size in a Flow Oriented Synchronized Production

Imran Aslan [1]

Research Assistant & PhD Candidate

The University of Erzincan, Faculty of Economy and Administrative

Abstract. Linear Programming (LP) models are widely used by companies to meet the demand for a variety of products efficiently. In a flow-shop, each product has to be processed by a number of machines in synchronized lines. This work presents a novel interactive linear programming approach for solving the multiproduct aggregate production planning problem with precise forecast demand, related operating costs, capacity and production-inventory model with constant/stock dependent demand. The proposed approach attempts to minimize total costs with reference to inventory levels, overtime, subcontracting and backordering levels, and machine and warehouse capacity. In this study, the problem of scheduling a set of parts with sequence-dependent setup times is considered. The production smoothing problem under presence of setup and processing times vary among the products. The master production-inventory problem of the firm was divided into two sub-problems which were concerned with determining the batch sizes and production sequences of products, respectively.

Keywords: Linear Programming, Batch Size, Optimization

1. Introduction

A Linear Programming procedure was developed to solve the batching problem for the current problem. LP is an important field of optimization for several reasons. Many practical problems in operations research can be expressed as linear programming problems. A number of algorithms for other types of optimization problems work by solving LP problems as sub-problems.

Long a household name in Europe and worldwide, the firm is exploding onto the scene in the United States with its sleek design, superior engineering and unparalleled service. Founded in 1899 on a promise of “immer besser”, a German phrase meaning Forever Better, the firm has distinguished itself in the appliance market with a century-long resume of major innovations. The firm’s innovative heritage, state-of-the-art design and engineering aesthetic have inspired comparison to other German brands synonymous with innovation. The firm sells a wide range of exceptional consumer appliances including vacuum cleaners, laundry systems, rotary irons, dishwashers, built-in convection ovens, cook tops, vent hoods, built-in coffee systems and steam ovens. The firm manufactures its appliances and parts in nine factories in Germany and one in Austria. The production and inventory activities of the firm were expected to be reorganized due to high competition and new global players coming from China and other low labour countries. The firm has lost some of its market share. Thus, the firm wants to decrease the costs to enlarge their segment without decreasing the quality of its products. It is aimed to develop a production system similar to the one of Toyota derived from lean management with small stocks, one-piece-flow and quick flow through the plants. Moreover, to reduce the average number of parts in a production order and costs (Inventory, setup, staff etc.) are reduced by using full capacity and calculating the optimal, better minimal and maximal order-amount regarding the hole production system. This model is developed as basis before implementation. Better decisions can be made based on data on hand. Furthermore, wastes are to be eliminated.

[+] Corresponding author: Imran Aslan Tel.: + 905315151428.

E-mail address: imranaslan@gmail.com, iaslan@erzincan.edu.tr

.

2. Theoretical Background

There are many models for production and inventory problems. Each model has some advantages and disadvantages. The selection of optimization model depends on decisions requirements, system variables, environment, system design etc. Moreover, hybrid models combining linear programming and various models to get optimal result can be used. [1] Aggregate production planning deals with decreasing uncertainties to meet the forecasted demand and to make policies and decisions related hiring, layoffs, overtime, subcontracting, backorders, inventory level and investments. [2] This approach aims to minimize indefinite total costs and the risk of getting high costs and maximize the possibility of getting lower costs. [3]

Scheduling with setup times or setup costs is very important in today's modern manufacturing. This increase the complexity of problem and a suitable model should be selected to get optimal solution. Optimization can give companies competitive advantages. The optimal production-work load allocation brings strategic advantages for firms. The optimal management of Supply Chain Activities which are now global decreases the costs. [4, 5]

Items consist of different parts. The divergent or convergent of this part affects the production scheduling. The convergent parts are expected to be produced together or in series. For divergent parts, some changes may be required. The convergent parts can be produced with same the dies and there is no need to change the dies again. [6]

To minimize the stock dependence of firms, different warehouses can be used. Small warehouses can be used inside of firm for semi-finished products. Due to limited storage location and limited inventory budget, the balance of production-warehouses should be managed optimally in order satisfy dynamic demand. Different heuristic methods or linear programs can used to get optimal solution for the trade-off between setup and inventory holding costs. Used methods can be compared to find which gives the best solution in a period of time. Later, the suitable method can be used for similar problems or cases. The scheduling of warehouse is another important point to minimize the costs. Finished products in inventory are to be replenished otherwise; the demand cannot be met on time. If the demand is dynamic, warehouse can be scheduled in a more precision manner, but, the stochastic demand decreases the precision of warehouse. [7, 8] An optimal safety stock level for each product can be determined based on the analysis of historic data consisting of a collection of spare part orders. This stock is beneficial to determine the reordering time or production of parts. The right amount of safety stock increases the flexibility of firm and satisfaction level of costumers, and decreases the costs. Moreover, the combination of inventory, production lines and distribution raises the flexibility of the firm and decreases the costs. Fast moving products can be transported directly to customers without waiting in inventory. [9, 10]

3. Case Study

The case study is developed to make a basis for the model. Parts and machines in one production line were selected to create the model. This model is used as model for other lines. The model is flexible and the variables can be changed.

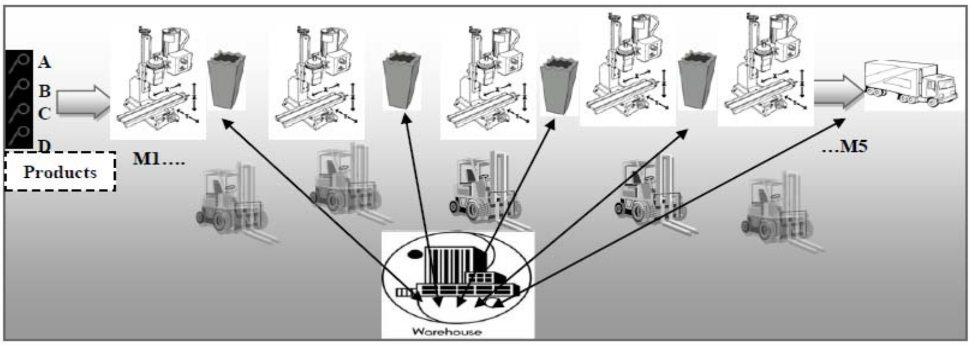

Fig. 1: Modelling System

Parameters:
Machines: Number of different machines (5)

Others:

- Setup time for each machine different
- Cycle time for each A, B, C and D parts Capacity of each machine
- Waiting times between machines(can be assumed zero If there is some stock among machines, they will work all full capacity)
- Storage between machines is limitless
- Batch size for each product
- Machine cost(€/h)(can be added to the whole cost)
- Stock cost and stock out cost resulted in penalty cost(It can be added linear equations if needed)
- Staff cost (can be considered in production cost) and Average setup cost for each machine

Decisions can be static or one-period models. In a static model, it is assumed that all decisions are made at a single point of time. In our case, it will be shown how to use linear programming to determine optimal decisions in multi-period or dynamic models. Dynamic models arise when the decision maker makes decisions at more than one point in time. In a dynamic model, decisions made during the current period influence decisions made during future periods. For example, the firm wants to learn how many part for each product to produce during each month. If it produces a large number of units during the current month, this would reduce the number of units that should be produced during future months.

In first situation: In the first situation, they can produce the whole amounts of parts for each product demanded during planning period in one setup. In this case, it is expected that the inventory costs will increase while the setup costs will decrease.

Period/Time	08:00	09:00	10:00	11:00	12:00	13:00	14:00	15:00	16:00	17:00	18:00
Period1	Setup for A + Production of A parts										
Period 2	Setup for B + Production of B parts										
Period 2	Setup for C + Production of C parts										
Period 2	Setup for D + Production of D parts										

Table 1: One product production per day and one setup case

In second situation: In this case, four products are produced daily for a period of time as shown below but as seen that there are so many setups of machines leading to high setup costs. However, it is expected that the inventory cost will be less as compared with the first situation. The batch size for each case will be found and costs for each case will be compared. The differences between first situation and second situation are production costs, time spent for setup of machines, inventory costs, amount produced daily, machine working hours etc.

Period/ Time	08:00	09:00	10:00	11:00	12:00	13:00	14:00	15:00	16:00	17:00	18:00
Period1	Production of A parts with setup time			Production of B parts with setup time			Production of C parts with setup time			Production of D parts with setup time	
Period 2	Production of A parts with setup time			Production of B parts with setup time			Production of C parts with setup time			Production of D parts with setup time	
Period 2	Production of A parts with setup time			Production of B parts with setup time			Production of C parts with setup time			Production of D parts with setup time	
Period 2	Production of A parts with setup time			Production of B parts with setup time			Production of C parts with setup time			Production of D parts with setup time	

Table 2: Multi products manufactured per day with many setups

Minimum cost found by each case will be base for decisions. The firm must determine how many parts of A, B, C and D products should be produced during each of the next four quarters (Periods). The demand during each of the next four quarters is shown below for each product. The firm must meet demands on time. At the beginning of the first period, the firm has an inventory of 100 parts for each product. At the beginning of each period, the firm must decide how many parts should be produced during that period. For simplicity, it is assumed that parts manufactured during a period can be used to meet demand for that period. During each period, the firm can produce up to a quantity of each product with a regular labor at a total cost € 40, 50, 60 and 45 per part respectively. By overtime during a period, the firm can produce each part by 5 € more for each part. At the end of each period, that is after production and demand is satisfied, a carrying or holding cost of 5 € per part is incurred. It is assumed that the cost of production for holding, setup, regular and overtime is constant during each period.

Solution: For each period, the firm must determine the number of products that should be produced by regular time or overtime.
X_{it}= Number of i products produced by regular time during period t. t=1, 2, 3, and 4 and i=A, B, C, and D
y_{it}= Number of i products produced by overtime during period t.
l_{it}= number of parts on hand at end of period t for each product.
S_t = Setup Cost during t period and d is the demand of product during t period.
S_k is total setup cost for each machine during period t(k=1(M_1), 2(M_2), 3(M_3), 4(M_4), and 5(M_5)) It is convenient to define decision variables for the inventory at the end of period.
Objective Function: Total Cost= Cost of production regular-time + cost of production overtime + inventory cost + Setup Cost = $\Sigma_{kj=0}$ [40($x_{A1} + x_{A2} + x_{A3} + x_{A4}$) + 50($x_{B1} + x_{B2} + x_{B3} + x_{B4}$) + 60 ($x_{C1} + x_{C2} + x_{C3} + x_{C4}$) + 45($X_{D1} + X_{D2} + X_{D3} + X_{D4}$) + 45($y_{A1} + y_{A2} + y_{A3} + y_{A4}$) + 55($y_{B1} + y_{B2} + y_{B3} + y_{B4}$) + 65 ($y_{C1} + y_{C2} + y_{C3} + y_{C4}$) + 50($y_{D1} + y_{D2} + y_{D3} + y_{D4}$) + 5($l_{A1} + l_{A2} + l_{A3} + l_{A4} + l_{B1} + l_{B2} + l_{B3} + l_{B4} + l_{C1} + l_{C2} + l_{C3} + l_{C4} + l_{D1} + l_{D2} + l_{D3} + l_{D4}$) + ($S_1 + S_2 + S_3 + S_4 + S_5$)]
j= 0, 1…k = Number of cycle times
Constraints: Before determining the firm's constraints, two observations are made that will aid in formulating multi period-scheduling models. Inventory at the end of period t= inventory at the end of period (t-1) + period t production – period t demand
Constraints related ending inventory:
$l_{it} = l_{it-1} + (x_{it} + y_{it}) - d_{it}$ (t=1, 2, 3, and 4)
l_{i0} = Inventory at the end of period 0 = Inventory at the beginning of period 1
For product i during t period:
l_{A0} = Inventory at beginning of period 1 for product A
Main Equation: $l_{At} = l_{At-1} + (x_{At} + y_{At}) - d_{At}$
Sub equations:
Constraints related meeting demand during t period:
t's demand will be met on time on time if $l_{it} \geq 0$
$l_{i(t-1)} + (x_{it} + y_{it}) \geq d_{it}$ (i= A, B, C and D) or $d_{it} = l_{i(t-1)} + (x_{it} + y_{it}) \geq 0$
The sign restrictions $l_{it} \geq 0$ will ensure that demand will be meet on time.
Supplying product A's demand during t period:
Budget Constraint(s):
You define a budget as a target not to be excesses for yourself
$C_iX_{it} + C_iy_{it} + (S_1 + S_2 + S_3 + S_4 + S_5) + 5(l_{A1} + l_{A2} + l_{A3} + l_{A4} + l_{B1} + l_{B2} + l_{B3} + l_{B4} + l_{C1} + l_{C2} + l_{C3} + l_{C4} + l_{D1} + l_{D2} + l_{D3} + l_{D4}) \leq$ €b_t (t=1, 2, 3, and 4) and i=A, B, C, and D
Capacity Constraints:
The constraint(s) for the whole system consisting of 5 machines as one system and one inventory:
Time Constraint is limited for production. The cycle time for each product is considered as one. There are two types of time constraints: one is regular time during t period (R_t) and the other one is total over time during t period (O_t). The type of constraint can be changed upon request.
$5X_{it} + 4X_{it} + 5X_{it} + 4X_{it} \leq R_t$ and O_t
In this case, it is considered that the machines have limited capacity; sperate systems: C_{rkt} is capacity of machine k during period t period at regular-time in minutes
C_{okt} is capacity of machine k during t period at overtime in minutes
T_{ik} is the time needed to produce one part of product i on machine k = M_1, M_2, M_3, M_4, and M_5
For Regular Time:
$T_{ik}X_{it} + T_{ik}X_{it} + T_{ik}X_{it} + T_{ik}X_{it} \leq C_{rkt}$
t=1,2,3, and 4
i=A, B, C, and D
k=1, 2, 3, 4, and 5(Machines)
Resources constraints:
We can assume the resources used for production are limited.
R_t in gram (gr) is the max. Amount of raw material available at t period .It will limit production.
U_i is unit consumption rate in gram for product i.
$U_iX_{it} + U_iX_{it} + U_iX_{it} + U_iX_{it} \leq R_t$
The firm's problem has several limitations: These limitations will change objective function and constraints if considered.

- Production cost may not be a linear function of quantity produced.
- Future Demand may not be known with certainty.
- Penalty cost is not considered and all demands are met on time.
- Costs are assumed to be constant during all periods.
- Salvage value of products is not considered at the end of periods.

4. Conclusion

Increasing global competition forces firms to decrease costs. New competitors from low labor countries produce the same products of global firms with lower costs. Customers from middle or lower segments even from high segments have started to buy these products. Thus, many big firms lose some part of their markets' shares. Even they find new markets, these new competitors catch them in these new segments and start to compete with them again. It is clear that competition is unavoidable and firms have to find new ways to stay competitive in the market. One way is to decrease internal costs by optimization. The firm has produced products for upper segments for years with high quality and with very well after sales services. However, it has lost some of its market share and wants to gain new segments in middle segments and it can be done by decreasing costs. Hence, the optimization processes have been started.

In the same line, four different products can be produced but it is not seen as profitable. If the number of machines decreases, then in one period, four different products can be profitable with low setup costs. There is a high demand for each product when compared with full capacity. But when the amount of demand for each product decreases, the multi-product production can be more profitable. The firm decided to develop a model based on that model by using Java Programming. The number of constraints, products and other variables can be increased. Sensitivity analysis can be used to change the values and see what the results are. Moreover, the range of resources to be used in production can be found. When the huge production potential of the firm is considered, this model provides very beneficial results. A simulation program could be a helpful tool to illustrate the model. There are other models such as dynamic programming, heuristic methods that can be used as decision support models.

5. References

[1] J. Mula, D. Peidro, M. Diaz-Madronero, E. Vicens. Mathematical programming models for supply chain production and transport planning, European Journal of Operational Research. 2010, 204: 377-390.

[2] R. Wang, T. Liang. Application of fuzzy multi-objective linear programming to aggregate production planning, Computers & Industrial Engineering.2004, 46: 17-41.

[3] R. Wang, T. Liang. Applying possibilistic linear programming to aggregate production planning, Int. J. Production Economics. 2005, 98: 328-341.

[4] R. Pastor, J. Altimiras, M. Mateo. Planning production using mathematical programming: The case of a woodturning company, Computers & Operations Research. 2009, 36: 2173 -2178.

[5] P. Tsiakis , L. G. Papageorgiou. Optimal production allocation and distribution supply chain Networks, Int. J. Production Economics. 2008, 111: 468-483.

[6] J.M. Spitter, C.A.J. Hurkens, A.G. de Kok, J.K. Lenstra, E.G. Negenman. Linear programming models with planned lead times for supply chain operations planning, European Journal of Operational Research. 2005, 163: 706-720.

[7] B. Das, K. Maity, M. Maiti. A two warehouse supply-chain model under possibility/ necessity/credibility measures, Mathematical and Computer Modelling. 20007, 46: 398-409.

[8] S. Minner. A comparison of simple heuristics for multi-product dynamic demand lot-sizing with limited warehouse capacity, Int. J. Production Economics.2009, 118: 305-310.

[9] P. Lonardo, D. Anghinolfi, M. Paolucci, F. Tonelli. A stochastic linear programming approach for service parts optimization, CIRP Annals - Manufacturing Technology. 2008, 57: 441-444.

[10] B. Bilgen. Application of fuzzy mathematical programming approach to the production allocation and distribution supply chain network problem, Expert Systems with Applications. 2010, 37: 4488-4495.