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Optimal Scheduling in a Milk Production Line Based on Mixed Integer Linear Programming

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Abstract - A Mixed Integer Linear Programming (MILP) model is proposed in this paper that targets the optimal production scheduling in a single milk production line. The model takes into account all the standard constraints encountered in production scheduling (material balances, inventory limitations, machinery capacity, labor shifts and manpower restrictions). Furthermore, it considers special features that characterize milk production, which are limitations in production sequencing mainly due to different fat contents and flavors of various products and sequence-dependent setup times and costs. The objective function that is minimized considers all major sources of variable cost that depend on the production schedule, i.e. changeover cost, inventory cost and labor cost. The model is applied to a milk production line of a Sala industry in Iran and the results are presented and discussed.

Keywords: Production scheduling; Mixed integer linear programming; Sequence-dependent setups.

1. Introduction

Several restrictions encountered in everyday production complicate the scheduling problem. The available machine time and man hours constitute a significant restriction to the problem. Another issue in the scheduling process is accomplishment of the production targets. There are several possible modes of operation and the choice among them depends on the goal sought, which could be the optimization of production earliness or tardiness or the optimization of the incurring production profit or cost.. There could also be limitations in the production sequence. Technical issues can arise due to configuration modifications in the machinery during transitions or due to changes in the packaging materials. Both require a changeover time,during which the production is seized. In some cases the equipment has to be cleaned during transitions,which additionally creates a changeover costdue to losses of product quantities and consumption of utilities. Much of the scheduling research is directed towards problems with sequence-independent transitions.A scheduling methodology that incorporates sequence-dependent changeovers would be closer to reality and increase its effectiveness to a great extent.This can be justified as follows:If a sequence of operations requires excessive changeover time,it should not be preferred,despite the fact that its setup cost may be small. Setup cost in this situation would be high; setup time would vary depending on the cleaning procedure. A comprehensive review of scheduling problems that consider sequence-dependent transitions between products can be found in Reklaitis(2000) and Allahverdi et al. (1999).Another important issue that must be considered in milk production scheduling is the rather short life-cycle of the products that must be consumed in a matter of weeks or even days. Furthermore, customers prefer that the milk they buy is as fresh as possible. This pushes towards a just-in-time mode of operation,which makes the supply chain more susceptible to fluctuations in demand. All the above factors need to be taken into consideration in the design and implementation of the scheduling process. (Schuermann &Kannan, 1977; Sullivan & Secrest, 1985)

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1.1. Nomenclature

Indices

- i days
- j, k, l products

1.2. Parameters

- N scheduling horizon (days)
- P number of products
- demand(i, j) demand for product j on day i (ton)
- csetup(j,k) changeover cost from product j to productk (€)
- tsetup(j,k) changeover time from product j to productk (h)
- cstorage storage cost (€/1000 cups/day)
- costs for the three shifts (€/h)
- u(j) machine speed for product j (ton/h)
- openinv(i, j), tarinv(i, j) opening and target inventory level
- of product j at the end of day i (ton)
- M(j), l(j) maximum and minimum production lots(ton)

1.3. Decision variables

- prod(i,j) produced quantity of product j on day i (ton)
- inv(i, j) inventory level of product j at the end of day i(ton)
- Time(i) total utilization of machine, including changeover
- times on day i (h)
- BIN(i, j) production of product j on day i (1/0)
- BINSETUP(i,j,k) changeover from product j to product k on day i (1/0)The rest of the paper is structured as follows: In the next section the motivating example that led to the development of the model is briefly described.

2. Problem definition – model formulation

The problem that is examined in this paper has the following structure: Given

- the daily demand of each product,
- the starting inventory,
- setup costs and times for the transitions between products,
- the production speed of each product,
- the inventory holding cost,
- the labor cost for the three working shifts,
- the sequencing limitations

3. objective of the model is to decide and calculate

- the products to be manufactured
- in each day and their respective quantities,
- the machine time (starting time and ending time) utilized by each product
- the inventory quantities of each product at the
- end of each day. At the same time, the constraints of the problem should be met.

3.1 There are restrictions in:

- production demands,
- due dates of orders,
- sequencing of operations,
- available machine time and man hours.
- More precisely, the basic characteristics of the proposed

3.2 scheduling tool are the following:

- Model formulation and time representation: The formulation that is presented in this paper uses binary variables to indicate whether setup between two products takes place or not. The total scheduling horizon is separated into discrete-time periods, whose length isequal to one day. Within each time period, continuous- time formulation was preferred in order to reduce the size of the model and the required solution time and increase the accuracy of the model regarding time representation.
- Food industry-specific: The methodology that is presented in this work is oriented towards the food industry, as it takes into consideration a set of production restrictions that are frequent in food and especially milk production practice. Moreover, transitions between products are sequence dependent, meaning that both changeover times and costs are considered as sequence dependent. One more industry-specific requirement is the fact that at the endof the day all the equipment must be cleaned. So, production is seized at the end of each day. (P. Doganis, H. Sarimveis / Journal of Food Engineering 80 (2007) 445–453 447)
- Demand satisfaction: Demand is product-specific and is considered daily, that is, there are product orders and due dates within the scheduling horizon, often multiple within the week. Early production is possible but tardiness is not allowed.
- Decision variables: A number of model characteristics are not fixed or pre-determined in order to allow the solution algorithm to search for a combination of their values that would optimize the objective function.
- Objective function: The objective function not only takes makespan into consideration, but also includes setup costs, standard and overtime labor costs and inventory costs.The model is formulated as a Mixed Integer Linear Programming (MILP) problem, which is explained in details in the sequel.

3.3 Parameters

- scheduling horizon,
- number and sequencing of products,
- demand of each product for each day,
- setup time and cost for each possible transition,
- storage cost of a unit of product for a day,
- labor cost for each shift,
- machine speed for each product,
- opening inventories and target inventories at the end of the scheduling horizon.

3.4 Decision variables

The optimal values of the decision variables are provided by the solution of the optimization problem and can be grouped into continuous variables and binary variables. For each day in the scheduling horizon, the optimal values of the following variables are obtained:

3.5 Continuous variables

- The produced quantity of each product.
- The inventory level of each product at the end of the day.
- The total utilization of the machine including the setup times.

3.6 Binary variables

- Binary variables (one for each product) indicating whether the respective product is to be produced in the particular day.
- Binary variables (one for each possible transition) indicating whether the respective changeover will take place or not.

4. Objective function – minimization of variable cost

$$\sum\sum\sum csetup(j,l).BINSETUP(i,j,l)+\sum\sum inv(i,j).\ cstorage +\sum cost\ .\ Time(i)$$

The objective function represents the production cost,

which is comprised of the setup costs, the inventory holding costs and the labor costs for all days of the scheduling horizon. Raw material and utility costs do not depend on any particular schedule and are not included in the objectivefunction.

4.1 Constraints

The constraints that must be satisfied are expressed by

the following set of equations. The names of the parameters and variables are explained in the nomenclature.

4.1.1 Relationship between continuous variables and binary

Variables

Prod(i, j)≤m(j).bin(i,,j)

Prod(i,,j)≥μ(j).bin(i,,j)

where m(j) and μ (j) indicate the maximum and the smallest lot sizes allowed. The above inequalities express the restriction that production of product j in day i is allowed

(prod(i, j) > 0) if and only if the binary variable BIN(i, j)

takes the value of 1. Similarly, product j is not manufactured

in day i (prod(i, j) = 0), if and only if the binary variable

BIN(i, j) takes the value of 0.

4.1.2 Total material balance for each product throughout the

scheduling horizon

Openinv(j)+∑prod(i,,j)=∑demand(i,,j)+inv(N,,j)

The summation of produced quantities of product j throughout the production horizon plus the initial inventory

must equal the sum of demand of all days plus the inventory of product j at the end of the last day.

• Earliness is possible, but no tardiness is allowed

Inv(1,,j)=openinv(j)+prod(1,,j)-demand(1,,j)

Inv(1,j)≥0

Inv(i,j)=inv(i-1,,j)+prod(i,,j)-demand(i,,j)   i>1

Constraints (5 and (6 calculate the daily inventory levels

for each product j, while at the same time require that daily demands are satisfied and no tardiness is allowed. At the end of the first day, the inventory must equal the initial inventory plus any produced quantity, reduced by the demand of that day (Eq. 5 On any other day, the inventory must be equal to the inventory level of the previous day plus any produced quantity, reduced by that day's demand.

• Satisfaction of the target inventory level at the end of the

scheduling horizon

Inv(N,,j)=tarinv(j)

The inventory levels at the end of the scheduling horizon must meet the targets.

4.2 Case study

The case study presented here concerns a specific milk production line, where 4products are produced. The products are indicated in Table 1 along with the production speeds. The problem for a six-day schedulinghorizon. The proposed tool was utilized to calculate the optimal production schedule for a week, for which the complete list of data are shown in Tables 1–5 Both opening and target inventories are set to zero for all 4different products in this case study.

• Setup constraints

We need the binary variable BINSETUP(i, j,l) to take the value of 1 if and only if there is a changeover from product j to product l on day i. This is achieved by the following set of inequalities:where k is a sufficiently small number.It is shown that for each case there is only one possiblevalue of BINSETUP (i, j, l), which is becoming equal to 1, only if BIN(i, j) is 1 and BIN(i,l) is 1 .The MILPoptimization problem that was formulated was solved using the LINDO 6.1The optimal production schedule, daily machine utilization and daily inventory levels are depicted respectively in Tables 6-8e production schedule contains the decisions concerning the products to be manufactured every day and therespective quantities. As can be observed, production is accommodated towards the minimization of cost.Table 8 gives the daily machine utilization time allocated to each product. Additionally, the total time that the machine is utilized every day (including the changeover times) is shown in the last row of the same table

Table 1 Production sequence and machine speed Priority Product Machine speed (in ton)

machine speed	PRODUCTION	
	P	U(j)
1	Pasteurized milk	36
2	Sterile milk	30
3	Homogenized milk	24
4	Coco milk	18

Table 2 Changeover costs (in €)

Changeover costs	PRODUCTION			
	P	Sterile	Homogenized	Coco
1	Pasteurized milk	130000	520000	650000
2	Sterile milk		520000	650000
3	Homogenized milk			790000

Table 3 Change over times (in h)

Changeover times	PRODUCTION			
	P	Sterile	Homogenized	Coco
1	Pasteurized milk	0.8	0.3	0.4
2	Sterile milk		0.3	0.4
2	Homogenized milk			0.5

Tale4 Maximum & minimum production on day(in ton)

Capacity of production	PRODUCTION		
	p	µ (j)	m(j)
1	Pasteurized milk	18	288
2	Sterile milk	15	240
3	Homogenized milk	12	192
4	Coco milk	9	144

Table 5 Production demand during the scheduling horizon (in ton)

Production	Daily demand						
P	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday	sum
Pasteurized milk	50		200		150	150	550
Sterile milk		27	150	83	100	60	420
Homogenized milk	20	25	53	43		19	160
Coco milk	42		20	30	28	15	135

Table 6 The calculated production schedule (in ton)

Production	Daily production						
P	Saturday	Sunday	monday	tuesday	Wednesday	Thursday	sum
Pasteurized milk	288	94			150	18	550
Sterile milk	72		150	83	100	15	420
Homogenized milk	20	12	12	12		14	160
Coco milk	42	56			28		135

Table7 Daily production time including setup times (in h)

Production	Daily production time					
P	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday
Pasteurized milk	8	2.6			4.2	0.5
Sterile milk	2.4		5	2.8	3.3	0.5
Homogenized milk	0.8	0.5	0.5	0.5		4.3
Coco milk	2.3	3.1		0.5	1.6	

Table 8 Inventory levels beyond the safety limits at the end of each day (in ton)

inventory	Daily inventory					
P	saturday	sunday	monday	tuesday	Wednesday	Thursday
Pasteurized milk	238	94				
Sterile milk	72					
Homogenized milk						104
Coco milk		56				

In these charts both production times for the different products and changeover times between products can be observed. Finally, Table 8 provides full knowledge of the inventory profile throughout the scheduling horizon.

5. Conclusions

The problem of production scheduling for a milk productionline of a dairy industry was studied in this work.The specific restrictions of the milk production process were taken into consideration in the formulation of a production scheduling optimization problem. The problem contains only linear equalities and inequalities, so that the global optimum solution is reached in a very short time. Another important aspect is that the optimization criterion is not restricted to time consideration (i.e. production time or earliness/tardiness), but includes production sequence dependent costs, labor costs and inventory holding costs, in order to represent more realistically the production cost while achieving production goals. The model produces the complete production schedule for a selected future horizon, including the sequence of products that should be produced every day and the respective quantities and the inventory levels at the endof each day. Ultimately, a production planning system that would include all production lines in a plant, including raw material and inventory capacity considerations, would offer a complete scheduling tool for the dairy industry.

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1) 46920000

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