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Two Results about Ruin Probability in Discrete Time Risk Model with Constant Interest Rate

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Abstract. In this we consider the ruin problems under the discrete time insurance risk model with interest rate, and proof the surplus is Markov chain. The series expansion and the integral equation of ruin probability and the surplus distribution at ruin moment.

Keywords: discrete time insurance risk model; Markov chain; Ruin probability; Integral equation.

1. Introduction

Ruin probability problem is the insurance company business and risk management will face important problem, not only such, this problem is also risk theory research priorities. Risk model the way according to charge the premium can be divided into continuous model and discrete model two kinds. Because insurance companies operating process and the time closely related, in recent years about risk model research has concentrated continuous model, and got a series of conclusion. Discrete model studies have relatively lags behind some. Discrete model is divided insurance company business into uniform period, the amount of the claim for each time period as independent and identically distributed random variables. Comparative continuous model to discrete the model considering the problems with insurance company more integrity. In this paper, we consider a class of introducing interest rate risk model, the introduction of the interest rate of reality, to strengthen in the current model under the condition of the central bank set interest rates more practical significance.

2. Model description

Consider premium income as random variables and join the discrete risk model interest rates [6]。Hypothesis in time to consider with constant interest rate δ , u is initial reserve for insurance company, then in the moment n , the insurance company for the accumulation of surplus:

$$U_n = u(1+\delta)^n + \sum_{i=1}^{n} X_i (1+\delta)^{n-i+1} - \sum_{i=1}^{n} Y_i (1+\delta)^{n-i} , \tag{1}$$

Among them, $\{X_i, i \ge 1\}$ and $\{Y_i, i \ge 1\}$ are independent and identically distributed sequence of the nonnegative random variables, They mean respectively $[i-1, i)$ time interval insurance company premium income and claims spending。

We assume X_i and Y_i expectations are limited, make

$$Z_i = Y_i - (1+\delta) X_i$$

Then Z_i is also a net loss of independent and identically distributed random variable sequence。

So the surplus process can be rewritten for:

$$U_n = u(1+\delta)^n - \sum_{i=1}^{n}(1+\delta)^{n-i} Z_i \text{。} \quad (2)$$

type arrangement(2)

$$U_n = u(1+\delta)^n - \sum_{i=1}^{n}(1+\delta)^{n-i} Z_i$$

$$= (u(1+\delta)^{n-1} - \sum_{i=1}^{n-1}(1+\delta)^{n-1-i} Z_i)(1+\delta) - Z_n$$

$$= U_{n-1}(1+\delta) - Z_n \text{。}$$

In order to guarantee the normal operation of insurance companies, it must add certain risk load $E[Z_i] < 0$。Set respectively $F_X(x), F_Y(y)$ are distribution function of X_1, Y_1,then the then distribution function of Z_1:

$$H(z) = P\{Z_1 \le z\} = \int_0^{+\infty} P\{Y_1 - (1+\delta)X_1 \le z \mid X_1 = x\} dF_X(x)$$

$$= \int_0^{+\infty} F_Y((1+\delta)x + z) dF_X(x) \text{。}$$

Can be seen from type , the distribution of net loss determine jointly by the distribution of premium and the distribution of claims.

Define $\Psi_\delta(u)$ is the ruin probability of risk process U_n , i.e.

$$\Psi_\delta(u) = P\{\bigcup_{n\ge 0}(U_n < 0) \mid U_0 = u\} \text{。}$$

Let $T_\delta = \inf\{n \ge 0 \mid U_n < 0\}$ be the time of ruin, obviously T_δ is stopping time, then

$$\Psi_\delta(u) = P\{T_\delta < \infty \mid U_0 = u\} \text{。}$$

Because of difficulty, the risk model with interest rate risk analysis model is currently one of the hot researches; the class of problems research is still not fully developed. Yang (1998) [3] said to bring interest rate risk model discrete time were studied by martingale method have the ruin probability index is given. [5] [6] for a class of discrete time constant interest rate risk model was studied, and the obtained several important risk issues, using mainly update distribution equation method. This paper discussed more system of discrete constant interest rate risk model, using the markov chain, with state transition probability giving ruins probability, etc, and then gives the show the surplus distribution at ruin moment and the surplus distribution at ruin of the instantaneous

3. Homogeneous markov chain

The following assumption $\delta > 0$. First proof $\{U_n\}_{n\ge 0}$ is a homogeneous markov chain。

Lemma 1 $\{U_n\}_{n\ge 0}$ is a homogeneous markov chain and state transition probability is $K(x,\Gamma) = P\{x(1+\delta) - Z_1 \in \Gamma\}$. Γ is any interval on real sets。

Proof Let $\mathcal{F}_n$ be the σ-algebra generated by $\{U_k, k \le n\}$.

$$P\{U_n \in \Gamma \mid \mathcal{F}_{n-1}\} = P\{U_{n-1}(1+\delta) - Z_n \in \Gamma \mid \mathcal{F}_{n-1}\}$$

Obviously $U_{n-1} \in \mathcal{F}_{n-1}, Z_n$ and $\mathcal{F}_{n-1}$ are independent, thus

$$P\{U_n \in \Gamma \mid \mathcal{F}_{n-1}\} = P\{U_{n-1}(1+\delta) - Z_n \in \Gamma \mid U_{n-1}\}$$

then

$$P\{U_n \in \Gamma \mid \mathcal{F}_{n-1}\} = P\{U_n \in \Gamma \mid U_{n-1}\}$$

state transition probability $K(n-1, x; n, \Gamma)$ is

$$P\{U_{n-1}(1+\delta)-Z_n \in \Gamma \mid U_{n-1}=x\} = P\{x(1+\delta)-Z_1 \in \Gamma\} 。$$

Can be seen from the type of state transition probability $K(n-1,x;n,\Gamma)$ Has nothing to do with the time n, i.e. $K(x,\Gamma)$.

4. Ruin probability

Ruin probability is insurance company most concern, is also the most concern of the key research risk model.。Now we use the properties of $\{U_n\}_{n\geq 0}$ give the analytical expressions of ruin probability $\Psi_\delta(u)$.

Theorem 1 ruin probability $\Psi_\delta(u)$ have the following expansion

$$\Psi_\delta(u)=\sum_{n=1}^{\infty}\int_0^{\infty}K(u,dx_1)\int_0^{\infty}K(x_1,dx_2)\cdots\int_0^{\infty}K(x_{n-2},dx_{n-1})\int_{-\infty}^{0}K(x_{n-1},dx_n)$$

Proof

$$\Psi_\delta(u)=P\{T_\delta<\infty \mid U_0=u\}=\sum_{n=1}^{\infty}P\{T_\delta=n \mid U_0=u\}$$

$$=\sum_{n=1}^{\infty}P\{U_1>0,U_2>0,\cdots,U_{n-1}>0,U_n<0 \mid U_0=u\}$$

$$=\sum_{n=1}^{\infty}P\{U_1>0 \mid U_0=u\}P\{U_2>0 \mid U_1>0\}\cdots P\{U_n<0 \mid U_{n-1}>0\}$$

$$=\sum_{n=1}^{\infty}\int_0^{\infty}K(u,dx_1)\int_0^{\infty}K(x_1,dx_2)\cdots\int_0^{\infty}K(x_{n-2},dx_{n-1})\int_{-\infty}^{0}K(x_{n-1},dx_n) 。$$

Theorem 2 Ruin probability $\Psi_\delta(u)$ satisfy the following integral equation

$$\Psi_\delta(u)=\int_{-\infty}^{u(1+\delta)}\Psi_\delta(u(1+\delta)-z)dH(z)$$

Proof When $n\geq 2$,

$$P\{T_\delta=n \mid U_0=u\}=P\{T_\delta=n,U_1>0 \mid U_0=u\}=\int_0^{\infty}P\{T_\delta=n-1 \mid U_0=x\}K(u,dx)$$

Sum to type on both sides by n, we get

$$\sum_{n=2}^{\infty}P\{T_\delta=n \mid U_0=u\}=\int_0^{\infty}\sum_{n=2}^{\infty}P\{T_\delta=n-1 \mid U_0=x\}K(u,dx) .$$

$$\Psi_\delta(u)-P\{T_\delta=1 \mid U_0=u\}=\int_0^{\infty}\Psi_\delta(x)K(u,dx)$$

$$\Psi_\delta(u)-P\{U_1<0 \mid U_0=u\}=\int_0^{\infty}\Psi_\delta(x)K(u,dx)$$

When $x<0$ we have $\Psi_\delta(x)=1$, from **lemma 1 that**

$$\Psi_\delta(u)=E[\Psi_\delta(u(1+\delta)-Z_1)]=\int_{-\infty}^{u(1+\delta)}\Psi_\delta(u(1+\delta)-z)dH(z) 。$$

5. The surplus distribution at ruin moment

Insurance company is risk investment institution; its business situation is underwriter and policy-holder issue of concern. Ruin probability of only describes the insurance company may, in order to further bankruptcy described the severity of insurance company go ruin, and then discuss the surplus distribution at ruin moment.

Gerber and Kass(1987) defined the function of the surplus distribution at ruin moment was

$$G_\delta(u,x)=P\{T_\delta<\infty,-x\le U_{T_\delta}<0\mid U_0=u\}\text{ 。}$$

Theorem 4 The surplus distribution at ruin moment $G_\delta(u,x)$ have the following expansion

$$G_\delta(u,x)=\sum_{n=1}^{\infty}\int_0^\infty K(u,dx_1)\int_0^\infty K(x_1,dx_2)\cdots\int_0^\infty K(x_{n-2},dx_{n-1})\int_{-x}^0 K(x_{n-1},dx_n)$$

Proof

$$G_\delta(u,x)=P\{T_\delta<\infty,-x\le U_{T_\delta}<0\mid U_0=u\}=\sum_{n=1}^{\infty}P\{T_\delta=n,-x\le U_n<0\mid U_0=u\}$$

$$=\sum_{n=1}^{\infty}P\{U_1>0,U_2>0,\cdots,U_{n-1}>0,-x\le U_n<0\mid U_0=u\}$$

$$=\sum_{n=1}^{\infty}P\{U_1>0\mid U_0=u\}P\{U_2>0\mid U_1>0\}\cdots P\{-x\le U_n<0\mid U_{n-1}>0\}$$

$$=\sum_{n=1}^{\infty}\int_0^\infty K(u,dx_1)\int_0^\infty K(x_1,dx_2)\cdots\int_0^\infty K(x_{n-2},dx_{n-1})\int_{-x}^0 K(x_{n-1},dx_n)\text{ 。}$$

Theorem 5 The surplus distribution at ruin moment $G_\delta(u,x)$ satisfy the following integral equation

$$G_\delta(u,x)=\int_{-\infty}^{u(1+\delta)+x}G_\delta(u(1+\delta)-z,x)dH(z)$$

Proof When $n\ge 2$,we have

$$P\{T_\delta=n,-x\le U_n<0\mid U_0=u\}=P\{T_\delta=n,-x\le U_n<0,U_1>0\mid U_0=u\}$$

$$=\int_0^\infty P\{T_\delta=n-1,-x\le U_{n-1}<0\mid U_0=y\}K(u,dy)\text{ .}$$

Sum to type on both sides by n, we get following integral equation

$$G_\delta(u,x)=\int_{-x}^0 K(u,dy)+\int_0^\infty G_\delta(y,x)K(u,dy)$$

When $-x\le y\le 0$ we have $G_\delta(y,x)=1$. Similarly when $y<-x$ we have $G_\delta(y,x)=0$ .From lemma 1, we get

$$G_\delta(u,x)=E[G_\delta(u(1+\delta)-Z_1,x)]=\int_{-\infty}^{u(1+\delta)+x}G_\delta(u(1+\delta)-z,x)dH(z)\text{ 。}$$

6. References

[1] N.L. Bower, H.U. Gerber, J.C. Hickman, D.A. Jones, C.J. Nes2bitt. Actuarial Mathematics [M]. Society of Actuaries, Itasca, IL, 1986.

[2] Gerber, Goovaerts, Kass. On the probability and serverity of ruin [J]. ASTIN Bulletin 1987;17: 151-163

[3] H. Yang, Non-exponential bounds for ruin probability with interest Effect included, Scandinavian Actuarial Journal,1(1998),66-79.

[4] T. Rolski, H. Schmidli, V. Schmidt and J. Teugels, Stochastic Processes for Insurance and Finance, Wiley and Sons, New York,1999.

[5] Sun Li Juan, Gu Lan, Liu Li Xin. Ruin problems for the discrete time risk model [J]. Mathematics in Economics, 2001, (4): 1~3.

[6] Sun Li Juan, Gu Lan. Ruin problems for the discrete time insurance risk mode [J]. Chinese Journal of Applied Probability and Statistics, 2002, (3): 293~294.