2011 3rd International Conference on Information and Financial Engineering
IPEDR vol.12 (2011) © (2011) IACSIT Press, Singapore

Multifractal Properties of the Industry Indices for Chinese and Japanese Stock Markets

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Abstract. Assessing the stock price indices is the foundation of forecasting the market risk. Multifractal has lots of advantage when explain the volatility of the stock prices. In order to analyze and compare the multifractal properties of Chinese and Japanese stock markets in the last decade, the empirical research is brought forward to daily industrial indices of the two stock markets via multifractal detrended fluctuation analysis (MF-DFA). After MF-DFA, this manuscript measures the market risk by calculating the average fractal dimension which can directly indicate roughness of the logarithmic return's scatter diagram on the 2-D plane. According to the results drawn, (i) all the two stock markets' industry indices have the multifractal features, but the multifractal spectrums differ greatly from each other, (ii) the largest fluctuations are more frequent in Chinese stock market, (iii) Chinese stock market volatility is larger than Japanese stock market volatility.

Keywords: Multifractal, MF-DFA, Average fractal dimension, Industry indices

1. Introduction

After the 1987-market crash and the 2008 financial crisis, many empirical studies have shown that applying the efficient market hypothesis (EMH) to describe the financial system can not explain when the large crash happens. Therefore, many researchers began to describe the dynamic characteristic of financial system via chaos and fractal [1][2][3][4].

The monofractal was firstly put into use to test on the variations of the return correlations for a large time period, while the famous Hurst exponent H was calculated to distinguish whether the correlations of in the stochastic time series are persistent or anti-persistent. Based on the self-similarity property of fractal, Tokinaga and Moriyasu [5] forecasted the time series by the fractal dimension which was obtained via the wavelet transform. Xiong [6] also applied the wavelet to measure the fractal dimension of Chinese stock market. But many studies have recently found that the multifractal is more reasonable to describe the financial system than the monofractal. Katsuragi [7] had found the evidence of multi-affinity in the Japanese stock market. 2002, Jan W. Kantelhardt et al. [8] developed the detrended fluctuation analysis (DFA) into the multifractal detrended fluctuation analysis (MF-DFA). Matia and Ashkenazy [9] found out the multifractal properties of price fluctuations of stocks and commodities by MF-DFA. Jiang and Zhou [10] found the evidence of multifractal long-range correlations in the stock market. Wang, Wei and Wu [11] found that, with regards to the WTI crude oil, the auto-correlations of volatilities were multifractal for small time scales while the auto-correlations were nearly monofractal for large time scales.

Compared to the previous work, the contributions of this paper are as follows: (i) via MF-DFA, the multifractal properties are found for all the industry indices of Chinese and Japanese stock markets, not only the market indices . (ii) It is also found that the average fractal dimention $\overline{f}$ can describe the volatility of the stock market. (iii) It offers a comprehensive comparison between Chinese and Japanese stock markets.

The paper is organized as the following lines. The data is described in Section 2. The applied methodology is commented in Section 3. The empirical results are described in Section 4. The discussion of the empirical results is in Section 5. Some concluding remarks are in Section 6. The acknowledgments are in Section 7. References are in Section 8.

2. Data

The data of twenty-two Chinese industry indices and Shanghai Composite Index (SHCOMP) were extracted from the Huatai Exchange System, while the data of thirty-six Tokyo Nikkei 500 industry indices and Tokyo Nikkei 225 Index (N225) were extracted from Bloomberg database. In this work, the logarithmic returns of closing daily indices between July 6th, 2001 and March 24th, 2011 are considered. The logarithmic returns were computed as $r_i = X_i = ln(P_{i+1}) - ln(P_i)$, with P_i being the index at time i. The twenty-two Chinese industry indices and thirty-six Japanese industry indices are given in Table 1.

Table 1. The list of Chinese and Japanese industry indices.

Country	Market Index	Industry Index
China	Shanghai Composite Index (SHCOMP)	Agroforestry, Communication, Conglomerate, Construction, Electronics, Finance, Foods, Hydropower, Information Technology, Lumbering, Machinery, Manufacture, Metal, Mining, Paper, Petrochemicals, Pharmaceuticals, Real Estate, Services, Textile, Transport, Wholesaling & Retailing
Japan	Tokyo Nikkei 225 Index (N225)	Air Transport, Autos, Bank, Chemicals, Communication, Construction, Electric Appl., Elec. Power, Fishery, Foods, Gas, Glass & Ceramics, Insurance, Iron, Land Transport, Machinery, Marine Transport, Mining, Nonfermetal, Oil, Other Finance, Other Products, Paper & Pulp, Pharmaceuticals, Prec. Instrument, Rail & Bus, Real Estate, Retailing, Rubber, Securities, Services, Shipbuilding, Textile, Trade Company, Trans. Equipment, Warehousing

3. Methodology

The methodology applied to process the data is the multifractal detrended fluctuation analysis (MF-DFA), which was first developed from the detrended fluctuation analysis (DFA) by Jan W. Kantelhardt et al. in 2002. MF-DFA is a fractal scaling method applied for detecting multifractal characterization of nonstationary time series and it can reliably determine the multifractal scaling behaviour of time series.

For a given stochastic time series X_i $(i=1,...,N)$, MF-DFA can be described as follows.

(i) Compute the profile

$$Y(i) = \sum_{j=1}^{i} \left(X_j - \langle X \rangle \right), i = 1, 2, \cdots, N \ , \tag{1}$$

for $\langle X \rangle$ is the mean of X_i.

(ii) Divide the Y_i into $N_s = \text{int}(N/s)$ nonoverlapping boxes of equal size s, and calculate the local trend for each box by a least-square fit. Since the length N of the series is not always a multiple of s, all the profile cannot be included. The same procedure is repeated starting from the opposite end. Thus, $2N_s$ boxes are obtained together. Then determine the variance

$$F^2(s,v) = \frac{1}{s} \sum_{i=1}^{s} \{ Y[(v-1)s + i] - y_v(i) \}^2, v = 1, 2, \cdots, N_s \ , \tag{2}$$

$$F^2(s,v) = \frac{1}{s} \sum_{i=1}^{s} \{ Y[N - (v - N_s)s + i] - y_v(i) \}^2, v = N_s + 1, \cdots, 2N_s \ . \tag{3}$$

Here, $y_v(i)$ is the fitting polynomial in box v. Because different order MF-DFA differ in the capability of eliminating trends in the series, linear (MF-DFA1), quadratic (MF-DFA2), cubic (MF-DFA3), or higher order polynomials can be considered in the fitting procedure.

(iii) Average over all boxes to obtain the qth order fluctuation function

$$F_q(s) = \left\{ \frac{1}{2N_s} \sum_{v=1}^{2N_s} \left[F^2(s,v)^{\frac{q}{2}} \right] \right\}^{\frac{1}{q}} , \tag{4}$$

where the index variable q can take any real value. When q=0, we calculate the fluctuation function as below:

$$F_q(s) = \exp \left\{ \frac{1}{4N_s} \sum_{v=1}^{2N_s} \ln \left[F^2(s,v) \right] \right\} . \tag{5}$$

For $q = 2$, the detrended fluctuation analysis (DFA) is retrieved. Repeat the above procedure (i) to (iii) for a broad range of box sizes s ($2m+2 \le s \le N/4$).

(iv) If the series X_i are long-rage power-law correlated, $F_q(s)$ increases as a power-law, $F_q(s) \sim s^{h(q)}$. Determine the scaling behaviour of the fluctuation function by analyzing the log-log plots $F_q(s)$ versus s for each value of q,

$$\ln(F_q(s)) = h(q)\ln s + \ln A \,. \tag{6}$$

$h(q)$ is generalized Hurst exponent, while $h(2)$ is identical to the well-known Hurst exponent H for stationary time series. In general, if the time series are monofractal series, $h(q)$ is independent of q. If the time series are multifractal series, $h(q)$ depends on q. For positive values of q, $h(q)$ describes the behaviour of the boxes with large fluctuations, while for negative values of q, $h(q)$ describes the behaviour of the boxes with small fluctuations. It is often that $h(q)$ of large fluctuations is smaller than $h(q)$ of small fluctuations. When $h(q) > 0.5$, the fluctuations related to q are persistently auto-correlated. When $h(q) < 0.5$, the fluctuations related to q are anti-persistently auto-correlated. If $h(q) = 0.5$, the fluctuations related to q display a random walk behaviour. Here we define $\Delta h(q) = h(q_{\min}) - h(q_{\max})$, which measures the degree of multifractal properties. Larger $\Delta h(q)$ means the stronger multifractal properties.

(v) If the time series is a stationary, positive and normalized sequence, DFA can be replaced by the standard fluctuation analysis (FA). So MF-DFA can be related to the standard textbook box counting formalism by Eq. (7).

$$\tau(q) = qh(q) - 1 \,. \tag{7}$$

Thus, MF-DFA has been related with the classic multifractal scaling exponents $\tau(q)$. Via a Legendre transform, we can get

$$\begin{cases} \alpha = d\tau(q)/dq = h(q) + q \cdot dh(q)/dq \\ \Delta\alpha = \alpha_{\max} - \alpha_{\min} \end{cases} , \tag{8}$$

$$\begin{cases} f(\alpha) = \alpha q - \tau(q) = q(\alpha - h(q)) + 1 \\ \Delta f = f(\alpha_{\min}) - f(\alpha_{\max}) \end{cases} . \tag{9}$$

Here, α is the singularity strength or Hölder exponent, while $f(\alpha)$ is dimention of the subset of series characterized by α. In this paper, we also calculate the average fractal dimention as below:

$$\overline{f} = \frac{1}{\Delta\alpha}\int_{\alpha_{\min}}^{\alpha_{\max}} f(\alpha)d\alpha \,. \tag{10}$$

Beacause $\overline{f}$ is the average fractal dimention of the all subsets, larger $\overline{f}$ means greater roughness of the series scatter diagram on the 2-D plane.

4. Empirical results

In MF-DFA, the logarithmic returns are employed and the index variable q takes the integral numbers between -10 and 10. It is found that MF-DFA3 is suitable for Chinese stock market while MF-DFA2 is suitable for Japanese stock market. Table 2 and Table3 present respectively the MF-DFA empirical results of Chinese and Japanese stock markets.

Table 2. Values of parameters of multifractal spectrums for Chinese stock market

Index	$h(2)$	$\Delta h(q)$	$\alpha_{\min}$	$\alpha_{\max}$	$\Delta\alpha$	$f(\alpha_{\min})$	$f(\alpha_{\max})$	Δf	$\overline{f}$
SHCOMP	0.4944	0.3554	0.3174	0.8221	0.5048	0.3498	0.1569	0.1929	0.7817
Agroforestry	0.5003	0.3574	0.3274	0.8498	0.5224	0.3473	0.0031	0.3443	0.7386
Communication	0.4918	0.2095	0.3175	0.6712	0.3537	0.2190	0.3388	-0.1197	0.7542
Conglomerate	0.5334	0.3275	0.3528	0.8395	0.4867	0.2640	0.1441	0.1199	0.7572
Construction	0.5204	0.2773	0.3510	0.7826	0.4317	0.2873	0.1697	0.1177	0.7591
Electronics	0.5274	0.2385	0.4395	0.8008	0.3614	0.6467	0.1250	0.5217	0.7837
Finance	0.4821	0.2796	0.3036	0.7088	0.4052	0.3105	0.4346	-0.1241	0.8177
Foods	0.5254	0.3332	0.3495	0.8281	0.4786	0.3231	0.2231	0.1001	0.7891
Hydropower	0.5140	0.3853	0.3201	0.8699	0.5498	0.2824	0.0730	0.2093	0.7510
Information Technology	0.5220	0.1539	0.4509	0.7126	0.2617	0.6228	0.2992	0.3236	0.8046
Lumbering	0.5554	0.2151	0.4338	0.7725	0.3387	0.4643	0.3008	0.1635	0.8053
Machinery	0.5273	0.3248	0.3889	0.8698	0.4809	0.4447	0.0007	0.4440	0.7479

Manufacture	0.5210	0.3625	0.3504	0.8713	0.5208	0.3545	0.0622	0.2923	0.7632
Metal	0.5256	0.3702	0.3693	0.8877	0.5184	0.4331	0.0858	0.3472	0.7723
Mining	0.5529	0.2008	0.4798	0.7953	0.3155	0.6141	0.2397	0.3744	0.7963
Paper	0.5062	0.2728	0.3978	0.7984	0.4006	0.5490	0.1738	0.3751	0.7857
Petrochemical	0.5166	0.4223	0.2966	0.8851	0.5886	0.2022	0.1353	0.0669	0.7574
Pharmaceuticals	0.5226	0.3593	0.3110	0.8456	0.5345	0.1519	0.0964	0.0555	0.7300
Real Estate	0.4900	0.2965	0.3272	0.7524	0.4252	0.3882	0.3257	0.0625	0.8155
Services	0.4867	0.3594	0.3395	0.8535	0.5140	0.4390	0.0157	0.4234	0.7495
Textile	0.5421	0.2645	0.3986	0.8030	0.4043	0.3780	0.2240	0.1539	0.7805
Transport	0.5014	0.3772	0.3048	0.8473	0.5425	0.2710	0.0765	0.1944	0.7535
Wholesaling & Retailing	0.5089	0.3620	0.3519	0.8729	0.5211	0.4167	0.0007	0.4160	0.7498

Table 3. Values of parameters of multifractal spectrums for Japanese stock market

Index	$h(2)$	$\triangle h(q)$	α_{min}	α_{max}	$\triangle\alpha$	$f(\alpha_{min})$	$f(\alpha_{max})$	$\triangle f$	$\overline{f}$
N225	0.5108	0.3897	0.2367	0.8012	0.5645	0.0582	0.1939	-0.1357	0.7053
Air Transport	0.4953	0.3206	0.2928	0.7555	0.4627	0.2882	0.2908	-0.0026	0.7775
Autos	0.5351	0.2235	0.3454	0.7235	0.3781	0.2205	0.2338	-0.0133	0.7101
Bank	0.4307	0.4166	0.1333	0.7439	0.6106	0.0001	0.1238	-0.1237	0.6960
Chemicals	0.5117	0.2647	0.2557	0.6720	0.4163	0.1265	0.3573	-0.2308	0.7096
Communication	0.4954	0.3132	0.3144	0.7763	0.4619	0.3133	0.1993	0.1140	0.7708
Construction	0.4392	0.4324	0.1142	0.7531	0.6389	0.0006	0.1427	-0.1421	0.6903
Electric Appl.	0.5066	0.3423	0.2682	0.7776	0.5094	0.0835	0.2453	-0.1618	0.7127
Elec. Power	0.4130	0.3460	0.1482	0.6406	0.4924	0.1034	0.4324	-0.3290	0.7562
Fishery	0.4950	0.2947	0.3210	0.7738	0.4528	0.2701	0.1494	0.1207	0.7381
Foods	0.4702	0.3292	0.2053	0.6907	0.4854	0.1301	0.3087	-0.1786	0.7294
Gas	0.4291	0.2986	0.1756	0.6197	0.4441	0.0835	0.4612	-0.3777	0.7456
Glass & Ceramics	0.5274	0.3174	0.3168	0.8022	0.4854	0.1844	0.1354	0.0490	0.7136
Insurance	0.4188	0.4761	0.0932	0.7666	0.6734	0.0001	0.0808	-0.0807	0.6859
Iron	0.5412	0.2385	0.3188	0.7071	0.3883	0.1214	0.3818	-0.2604	0.7227
Land Transport	0.5168	0.2790	0.2360	0.6733	0.4373	0.0006	0.4258	-0.4252	0.6890
Machinery	0.5266	0.3134	0.2707	0.7556	0.4849	0.0547	0.2305	-0.1758	0.6991
Marine Transport	0.4990	0.2387	0.3039	0.7025	0.3986	0.1524	0.2490	-0.0966	0.7213
Mining	0.5097	0.2210	0.3031	0.6638	0.3607	0.1915	0.4114	-0.2199	0.7470
Nonfermetal	0.5158	0.3561	0.2827	0.8085	0.5258	0.1699	0.1336	0.0363	0.7215
Oil	0.4889	0.2372	0.2621	0.6469	0.3848	0.1160	0.4086	-0.2926	0.7305
Other finance	0.5036	0.3895	0.2767	0.8433	0.5666	0.1239	0.1049	0.0190	0.7206
Other products	0.5538	0.3126	0.3444	0.8288	0.4844	0.2016	0.0810	0.1206	0.7032
Paper & Pulp	0.4573	0.4146	0.1636	0.7582	0.5946	0.0009	0.2068	-0.2059	0.7088
Pharmaceuticals	0.4185	0.3578	0.1235	0.6679	0.5444	0.0007	0.1384	-0.1377	0.6691
Prec. Instrument	0.5312	0.2011	0.2810	0.6215	0.3405	0.0352	0.5716	-0.5364	0.6812
Rail & Bus	0.4838	0.2607	0.3128	0.7053	0.3925	0.3482	0.3343	0.0139	0.7872
Real Estate	0.5114	0.3796	0.3134	0.8739	0.5605	0.2170	0.0007	0.2163	0.7140
Retailing	0.5118	0.4184	0.2400	0.8531	0.6131	0.0001	0.0839	-0.0838	0.7010
Rubber	0.4876	0.4016	0.2316	0.8024	0.5708	0.0886	0.2202	-0.1316	0.7404
Securities	0.5110	0.3224	0.3106	0.7914	0.4808	0.1736	0.2426	-0.0690	0.7543
Services	0.4999	0.3924	0.2190	0.7951	0.5761	0.0002	0.1701	-0.1699	0.7009
Shipbuilding	0.5222	0.4113	0.2613	0.8592	0.5979	0.0237	0.1118	-0.0881	0.7080
Textile	0.4887	0.4339	0.1549	0.7690	0.6141	0.0002	0.2507	-0.2505	0.6974
Trade Company	0.5397	0.3252	0.2837	0.7909	0.5072	0.0007	0.1977	-0.1970	0.6923
Trans. Equipment	0.5461	0.3202	0.3198	0.8267	0.5069	0.0973	0.0371	0.0602	0.6814
Warehousing	0.4397	0.4286	0.1160	0.7558	0.6398	0.0005	0.1001	-0.0996	0.6895

Firstly, $h(q)$ of all the indices is depend on q and $h(q)$ is a decreasing founction of q, which means all the daily indies have the multifractal properties. So compared with the monofractal, the multifractal is more reasonable to characterize the volatility of stock market. As we all know: if H (Hurst exponent) > 0.5, the correlations in the signal are persistent; if H < 0.5, the correlations in the signal are anti-persistent. Because $h(2)$ can be considered as the Hurst exponent, it is found that about 82% (=18/22) Chinese industry indices are persistent series while only half (=18/36) of Japanese industry indices are persistent series. SHCOMP is anti-persistent (0.4944) while N225 is persistent (0.5108). In addition, for all the indices, $h(q)$ of large fluctuations is small than 0.5 while $h(q)$ of small fluctuations is greater than 0.5, which means the small fluctuations are persistently auto-correlated while the great fluctuations are anti-persistently auto-correlated. Therefore, different fluctuations of the two stock markets have different mechanisms.

Secondly, $\Delta h(q)$ is different for each index, which means the multifractal properties degrees are different among all the industry indices. In Chinese stock market, $\Delta h(q)$ (0.1539) of information technology (IT) industry is smallest while $\Delta h(q)$ (0.4223) of petrochemical industry is largest. In Japanese stock market, $\Delta h(q)$ (0.2011) of prec. instrument industry is smallest while $\Delta h(q)$ (0.4761) of insurance industry is largest. Overall, $\Delta h(q)$ (0.3554) of SHCOMP is smaller than $\Delta h(q)$ (0.3897) of N225 in the last decade.

Thirdly, as $\Delta\alpha$ stands for the difference of singularity strength, $\Delta\alpha$ can also measure the degree of multifractal properties. According to the $\Delta\alpha$, we can obtain nearly the same conclusions about the multifractal properties strength as $\Delta h(q)$ does.

Fourthly, based on the standard textbook box counting formalism, $\alpha_{\min}$ stands for the singularity strength of the largest fluctuation, while $\alpha_{\max}$ stands for the singularity strength of the smallest fluctuation, so Δf stands for frequency ratio of the largest fluctuation and the smallest fluctuation. As we can see, Δf (0.1929) of SHCOMP is positive while Δf (-0.1357) of N225 is negative. Additionally, for the industry indices, nearly 91% (=20/22) Δf of Chinese industry indices is positive, but only 25% (=9/36) Δf of Japanese industry indices is positive. Therefore, the largest fluctuations of Chinese stock market are obviously more frequent than that of Japanese stock market. In Chinese stock market, Δf (-0.1241) of finance industry is smallest while Δf (0.5217) of electronics industry is largest. In Japanese stock market, Δf (-0.5364) of prec. instrument industry is smallest while Δf (0.2163) of real estate industry is largest.

At last, combined with the logarithmic return's scatter diagram of index on the 2-D plane, we can compare all the industry indices of the two stock markets via calculating the average fractal dimension $\overline{f}$ by Eq.(10) and finally find that there are more frequent largest fluctuations in Chinese stock market. Here we take SHCOMP and N225 for example. The results are present in Fig.1 and Fig.2, showing that the roughness of scatter diagram on the 2-D plane is accordance with the average fractal dimension $\overline{f}$. The average fractal dimension $\overline{f}$ of SHCOMP is 0.7817 which is greater than 0.7053 of N225, at the meantime, the scatter diagram of SHCOMP is more rough-and-tumble for more plots scattering in the area of $r > \left|0.02\right|$ and its multifractal spectrum is a little fatter than that of N225. As Δf of SHCOMP is positive while Δf of N225 is negative, it is clear that the left part of multifractal spectrum is higher for SHCOMP while the right part of multifractal spectrum is higher for N225.

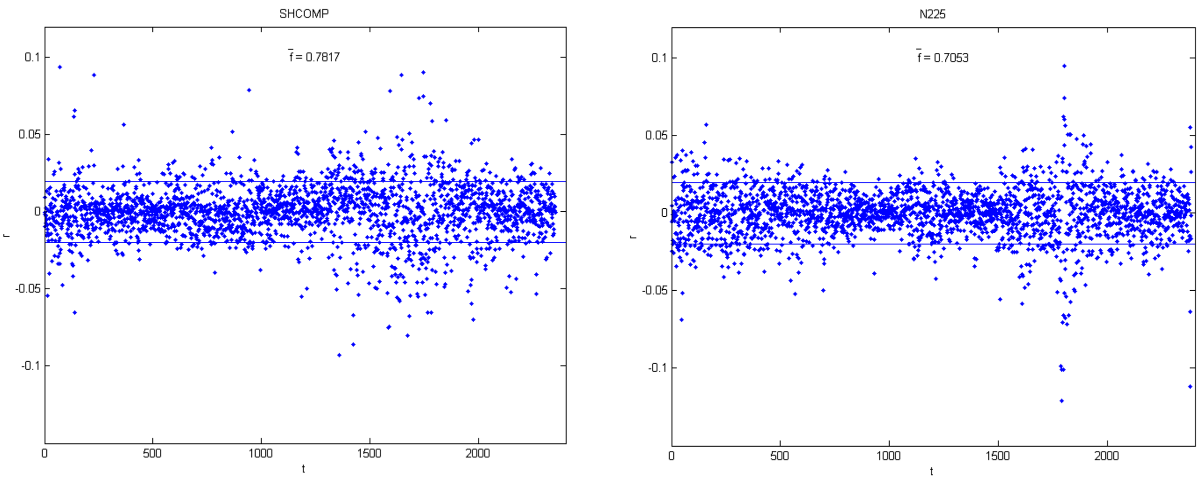

Fig. 1. The scatter diagrams of logarithmic returns for SHCOMP and N225

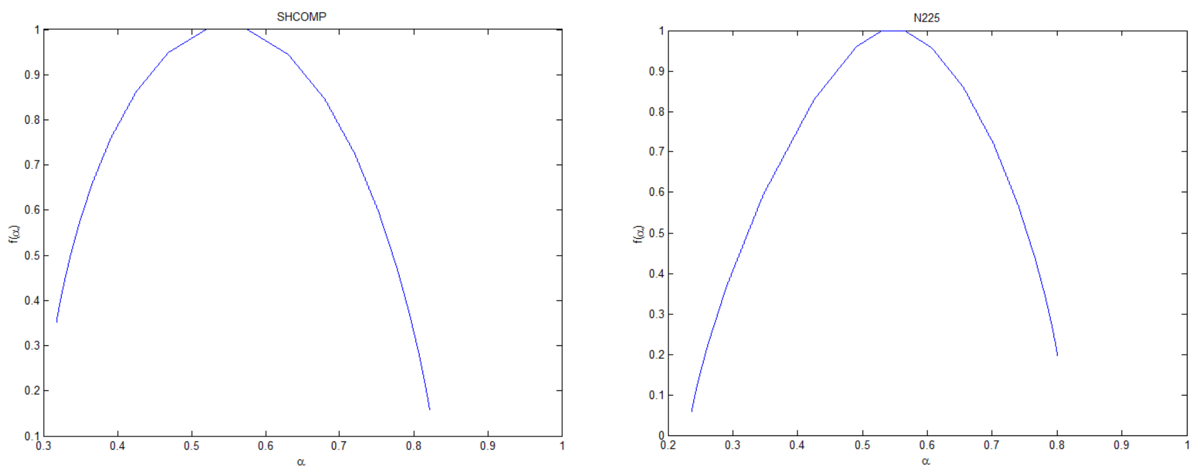

Fig. 2. The multifractal spectrums of logarithmic returns for SHCOMP and N225

5. Discussion

This year, being a more and more important role, China has replaced Japan to be the 2nd largest GDP country in the world for its economy vitality. Although Japanese economy was badly impacted by the financial crisis in 2008, compared with Chinese economy, it is still relatively mature and steady. According to Δf and $\overline{f}$ of all the indices, Chinese stock market has more large fluctuations and bigger volatility than Japanese stock market, which means China had a higher market risk than Japan during the last decade.

Especially for Japanese industry indices, it's interesting to find tha Δf and $\Delta\alpha$ of prec. instrument industry index is the smallest while Δf of real estate industry index is the largest. As we all know, Japan is very famous for the products of precise instrument. In some way, the Japanese companies monopolize the prec. instrument market. So prec. instrument industry index is the steadiest among all the industry indices. At the beginning of 1990s, the bubbles of Japanese economy broke accompanied with a more than twenty years' drop of the real estate price. That's why the largest fluctuations of the real estate industry index are more frequent than those of the other industry indices.

6. Conclusion

For the industry indices of Chinese and Japanese stock markets, the application of MF-DFA showed that the multifractal is more appropriate for describing the dynamic characteristics of stock market than the monofractal. Based on empirical results, not only SHCOMP and N225 have the multifractal properties, but also do the twenty-two Chinese industry indices and thirty-six Japanese industry indices have the multifractal properties. Although all the indices have the multifractal features, different indices differ from each other for their own multifractal spectrums. As Chinese stock market is an emerging market, the largest fluctuations are more frequent and a larger volatility exits, which reflects that Chinese stock market has more market risk than Japanese stock market. As a developed market, Japanese stock market is relatively steady.

7. Acknowledgements

The author is thankful for the financial support offered by State Scholarship Fund of China Scholarship Council (CSC) and Mr. Zhenlian Xie of Guangfa Futrures Co., LTD. for the data collection.

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