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# Stochastic Dominance Analysis on Investor Preferences in Brent Crude Oil Markets

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**Abstract.** This paper examines investor preferences for oil spot and futures based on mean-variance (MV) and stochastic dominance (SD) methods. The MV criterion cannot distinct the preferences of both assets whereas SD test concludes that risk-averse investors prefer investing in the spot index, while risk seekers are attracted to the futures index to maximize their expected utilities.

**Keywords:** Stochastic dominance, risk averter, risk seeker, futures market, spot market.

## 1. Introduction

Extensive research has been undertaken to study the relationship between oil spot and futures prices and their associated returns. The literature on the relationships between oil spot and futures prices has examined issues such as market efficiency and price discovery (Bopp and Sitzer, 1987; Crowder and Hamid, 1993). Silvapulle and Moosa (1999) find that futures prices lead spot prices. Bekiros and Diks (2008) on the other hand report that the pattern of leads and lags changes over time. Empirical studies also indicate that commodity prices can be extremely volatile at times, and that sudden changes in volatility are quite common in commodity markets (Wilson et al., 1996).

Many empirical studies in the literature have employed conventional parametric tests, such as the mean-variance (MV) criterion and CAPM statistics for their analyses. These approaches rely on the normality assumption and the first two moments. However, the presence of non-normality in portfolio returns distributions has been well documented. Stochastic dominance (SD) approach differs from the conventional parametric approaches. It endorses the minimum assumptions of investor utility functions and analyzes the entire returns distributions directly. In addition, the SD approach is widely regarded as one of the most useful tools to rank investment prospects as the ranking of assets has been shown to be equivalent to utility maximization for investors (Tesfatsion, 1976; Stoyan, 1983). In this paper, we employ the SD test proposed by Davidson and Duclos (2000) (hereafter DD) to examine the behavior of risk averters and risk seekers with regard to Brent crude oil spot and futures indices.

## 2. Data and Methodology

We use daily Brent crude oil spot and futures indices collected from *Datastream*. The sample period is from January 1, 1989 to June 30, 2008. We further examine the effects of two major oil crises: OPEC's decision on reduction of capacity in 1999 and the 2003 Iraq War. The full sample period is divided into sub-periods based on these two events dates. The first pair of sub-periods are pre-OPEC sub-period (Pre-OPEC) and the sub-period thereafter (OPEC) with October 29, 1999 as the cut-off point. The second pair of sub-periods are pre-Iraq-War sub-period (pre-Iraq) and the sub-period thereafter (Iraq War) using March 20, 2003 as the cut-off point. For computing the CAPM statistics, we use the 3-month U.S. T-bill rate

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and the Morgan Stanley Capital International index returns (MSCI) as proxies for the risk-free rate and the global market index, respectively.

For purposes of comparison, we also calculate the MV and CAPM statistics[1]. The MV model developed by Markowitz (1952) and Tobin (1958), and the CAPM statistics developed by Sharpe (1964), Treynor (1965) and Jensen (1969), are commonly used to compare investment prospects. CAPM statistics include the beta, Sharpe ratio, Treynor's index and Jensen (alpha) index to measure performance[2].

SD theory is developed by Hadar and Russell (1969), Hanoch and Levy (1969) and Rothschild and Stiglitz (1970). Let *F* and *G* be the cumulative distribution functions (CDFs), and *f* and *g* be the corresponding probability density functions (PDFs) of two investments, *X* and *Y* respectively with common support of [a, b], where $a < b$. Define

$$H_0^A = H_0^D = h\,,\ \ H_j^A(x) = \int_a^x H_{j-1}^A(t)\,dt \ \text{ and } \ H_j^D(x) = \int_x^b H_{j-1}^D(t)\,dt \qquad (1)$$

for $h = f, g$ ; $H = F, G$ ; and $j = 1, 2, 3$ .

We call the integral $H_j^A$ the $j^{th}$ order ascending cumulative distribution function (ACDF) and the integral $H_j^D$ the $j^{th}$ order descending cumulative distribution function (DCDF), for *j* = 1, 2, 3 and for H = F and G.

The most commonly-used SD rules corresponding to three broadly defined utility functions are the first-, second- and third-order Ascending SD (ASD) for risk averters, denoted as FASD, SASD and TASD, respectively. All investors are assumed to have non-satiation (more is preferred to less) under FASD, non-satiation and risk aversion under SASD; and non-satiation, risk aversion and decreasing absolute risk aversion (DARA) under TASD. The ASD rules are defined as follows (see Hanoch and Levy, 1969):

*X dominates Y by FASD (SASD, TASD) denoted* $X \succ_1 Y$ *(* $X \succ_2 Y$ *,* $X \succ_3 Y$ *) if and only if* $F_1^A(x) \le G_1^A(x)$ *(* $F_2^A(x) \le G_2^A(x)$*,* $F_3^A(x) \le G_3^A(x)$*) for all possible returns x, and the strict inequality holds for at least one value of x.*

SD for risk seekers is called Descending SD (DSD) and defined as follows (see Wong and Ma, 2008):

*X dominates Y by FDSD (SDSD, TDSD)) denoted by* $X \succ^1 Y$ *(* $X \succ^2 Y$ *,* $X \succ^3 Y$ *) if and only if* $F_1^D(x) \ge G_1^D(x)$ *(* $F_2^D(x) \ge G_2^D(x)$*,* $F_3^D(x) \ge G_3^D(x)$*) for all possible returns* $x$*, the strict inequality holds for at least one value of* $x$*; where FDSD (SDSD, TDSD) stands for first-order (second-order, third-order) Descending SD.*

In the finance literature, when two prospects have been compared. If the perceived distribution of return on prospect *X* stochastically dominates that of prospect *Y* in a particular manner then we can conclude that the agent has a preference for prospect *X*. There are two broad classes of SD tests. One is the minimum/maximum statistic, while the other is based on distribution values computed on a set of grid points. The SD test developed by DD has been examined to be one of the most powerful approaches and yet less conservative in size (see Lean et al., 2008).

## 2.1. Davidson and Duclos (DD) Test

Let $\{(f_i, s_i)\}$ $(i = 1, ..., n)$[3] be pairs of observations drawn from the random variables *X* and *Y*, with distribution functions *F* and *G* respectively and with their integrals $F_j^A(x)$ and $G_j^A(x)$ defined in (1) for *j* = 1, 2, 3. For a grid of pre-selected points $x_1$, $x_2$, ..., $x_k$, the $j^{th}$ order Ascending DD test statistic for risk averters, $T_j^A$ is:

$$T_j^A(x) = \frac{\hat{F}_j^A(x) - \hat{G}_j^A(x)}{\sqrt{\hat{V}_j^A(x)}} \quad \text{where} \quad \hat{V}_j^A(x) = \hat{V}_{F_j}^A(x) + \hat{V}_{G_j}^A(x) - 2\hat{V}_{FG_j}^A(x);$$

[1] Due to space constraint, we do not report the results but it is available upon request.
[2] Refer to Sharpe (1964), Treynor (1965) and Jensen (1969) for details regarding the definitions of these indices and statistics.
[3] In the context of this paper, *f* denotes the returns of futures prices, while *s* denotes the returns of spot prices.

$$\hat{H}_j^A(x) = \frac{1}{N(j-1)!}\sum_{i=1}^{N}(x-z_i)_+^{j-1},$$

$$V_{H_j}^A(x) = \frac{1}{N}\left[\frac{1}{N((j-1)!)^2}\sum_{i=1}^{N}(x-z_i)_+^{2(j-1)} - H_j^A(x)^2\right], H = F, G; z = f, s;$$

$$V_{FG_j}^A(x) = \frac{1}{N}\left[\frac{1}{N((j-1)!)^2}\sum_{i=1}^{N}(x-f_i)_+^{j-1}\left(x-s_i\right)_+^{j-1} - F_j^A(x)\hat{G}_j^A(x)\right].$$

In order to test SD for risk seekers, the DD statistic for risk averters is modified to be the Descending DD test statistic, $T_j^D$, such that:

$$T_j^D(x) = \frac{\hat{F}_j^D(x) - \hat{G}_j^D(x)}{\sqrt{\hat{V}_j^D(x)}} \quad \text{where} \quad \hat{V}_j^D(x) = \hat{V}_{F_j}^D(x) + \hat{V}_{G_j}^D(x) - 2\hat{V}_{FG_j}^D(x);$$

$$\hat{H}_j^D(x) = \frac{1}{N(j-1)!}\sum_{i=1}^{N}(z_i-x)_+^{j-1},$$

$$V_{H_j}^D(x) = \frac{1}{N}\left[\frac{1}{N((j-1)!)^2}\sum_{i=1}^{N}(z_i-x)_+^{2(j-1)} - H_j^D(x)^2\right], H = F, G; z = f, s;$$

$$V_{FG_j}^D(x) = \frac{1}{N}\left[\frac{1}{N((j-1)!)^2}\sum_{i=1}^{N}(f_i-x)_+^{j-1}\left(s_i-x\right)_+^{j-1} - F_j^D(x)\hat{G}_j^D(x)\right];$$

where the integrals $F_j^D\left(x\right)$ and $G_j^D\left(x\right)$ are defined in (1) for $j = 1, 2, 3$.

# 3. Empirical Results and Discussion

We employ the first three orders of Ascending DD statistics for the two series with the results reported in Table 1. Table 1 shows that 5% of $T_1^A$ is significantly positive whereas 6% of it is significantly negative. Thus, the hypotheses that futures stochastically dominate spot or vice-versa at first-order are rejected, implying that no arbitrage opportunity exists between these two series. Table 1 also shows that 7% of $T_2^A$ (5% of $T_3^A$) being significantly positive and no $T_2^A$ ($T_3^A$) being significantly negative. This implies that oil spot marginally SASD (TASD) dominates futures, and hence risk-averse investors would prefer investing in oil spot than futures.

[Table 1 here]

It is well known that investors could be risk-seeking (Tversky and Kahneman, 1992). In order to examine the risk-seeking behavior, we compute the Descending DD statistics of first three orders. Table 2 shows that 6% (5%) of the positive (negative) values of $T_1^D$ is significant. This indicates that there is no FDSD relationship between the two series. For the second and third orders, results in Table 2 show that 7% (9%) of $T_2^D$ ($T_3^D$) are significantly positive while no $T_2^D$ ($T_3^D$) is significantly negative. This concludes that the oil futures SDSD and TDSD the oil spot and consequently, risk-seeking investors prefer oil futures to spot to maximize their utility.

[Table 2 here]

In addition, neither FASD nor FDSD leads us to conclude that market efficiency or market rationality could hold in the oil spot and futures markets. The preferences of risk-averse and risk-seeking investors towards spot and futures do not violate market inefficiency, unless the oil market has only one type of investors.

The oil market is very sensitive, not only to news, but also to the expectation of news (Maslyuk and Smyth, 2008). From the DD test statistics, we find that all values of $T_j^A$ and $T_j^D$ ($j$ = 1, 2 and 3) for both risk averters and risk seekers are not significant at the 5% level for the first three orders in the pre-OPEC sub-period. However, in the OPEC sub-period, Table 1 shows that 17% (16%) of $T_2^A$ ($T_3^A$) are significantly positive and none of the $T_2^A$ ($T_3^A$) is significantly negative, while Table 2 reveals that 22% (30%) of $T_2^D$ ($T_3^D$) are significantly positive and none of the $T_2^D$ ($T_3^D$) is significantly negative at the 5% level. Similar inferences can be drawn for the Iraq War sub-period. Hence, we conclude that, compared with the full sample period, the risk-averse investors prefer the spot index more, and risk seekers are attracted to the futures index more to maximize their expected utility in both the OPEC and Iraq War sub-periods.

# 4. Conclusions

This paper offers a robust decision tool for investment decisions with uncertainty to the Brent crude oil markets. Our results show that there is neither arbitrage opportunity nor preference being prevalent between these two indices for both risk-averse and risk-seeking investors in the pre-OPEC and pre-Iraq War sub-periods. However, risk-averse investors prefer the oil spot, while risk seekers are attracted to the oil future in order to maximize their expected utility in the OPEC and Iraq War sub-periods.

Table 1: Results of DD Test for Risk Averters

| Sample | FASD | | SASD | | TASD | |
|---|---|---|---|---|---|---|
| | $\%T_1^A > 0$ | $\%T_1^A < 0$ | $\%T_2^A > 0$ | $\%T_2^A < 0$ | $\%T_3^A > 0$ | $\%T_3^A < 0$ |
| Whole Period | 5 | 6 | 7 | 0 | 5 | 0 |
| Pre-OPEC | 0 | 0 | 0 | 0 | 0 | 0 |
| OPEC | 14 | 14 | 17 | 0 | 16 | 0 |
| Pre-Iraq | 4 | 4 | 2 | 0 | 0 | 0 |
| Iraq War | 3 | 16 | 7 | 0 | 0 | 0 |

Table 2: Results of DD Test for Risk Seekers

| Sample | FDSD | | SDSD | | TDSD | |
|---|---|---|---|---|---|---|
| | $\%T_1^D > 0$ | $\%T_1^D < 0$ | $\%T_2^D > 0$ | $\%T_2^D < 0$ | $\%T_3^D > 0$ | $\%T_3^D < 0$ |
| Whole Period | 6 | 5 | 7 | 0 | 9 | 0 |
| Pre-OPEC | 0 | 0 | 0 | 0 | 0 | 0 |
| OPEC | 14 | 14 | 22 | 0 | 30 | 0 |
| Pre-Iraq | 4 | 4 | 5 | 0 | 0 | 0 |
| Iraq War | 16 | 3 | 21 | 0 | 26 | 0 |

# 5. Acknowledgements

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4 Due to the space constraint, we cannot list all the references in the paper.