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Optimization of Return under Risk constraint: An application on Indian Banks

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Abstract. Portfolio optimization, in case of finance, is the trade- off between risk and return to maximize profit or return from the portfolio. Financial regulations are country specific and it depends upon the economic conditions prevailing in the country. The portfolio of a commercial bank can be constrained by regulatory prescription of exposure limits, risk weights and returns from each category of assets. Hence, optimization of return, in case of the loan portfolio, presents a challenging problem due to its large set of local extremes. In this context, Genetic Algorithm is used as a possible solution to optimize the risk-return trade-off and give an ideal solution for portfolio optimization.

Keywords: Portfolio Management, Risk-Return Trade Off, Commercial Banking

1. Introduction

The main goal of investors is to achieve optimal allocation of funds among various financial assets. Searching for an optimal portfolio, characterized by random future returns, seems to be a difficult task and is usually formalized as a risk-minimization problem. However, many other risk-return criteria have been proposed in the financial literature which may significantly improve the portfolio selection criteria. A bank is a financial intermediary that accepts deposits and channels those deposits into lending activities. Banks are a fundamental component of the financial system, and are also active players in financial markets. The essential role of a bank is to connect those who have funds(such as investors or depositors), with those who seek funds. Banking is generally a highly regulated industry, and government restrictions on financial activities of banks have varied over time and location. The current set of global standards is called Basel II. Basel II is the second of the Basel Accords, which are recommendations on banking laws and regulations issued by the Basel Committee on Banking Supervision. The purpose of Basel II, which was initially published in June 2004, is to create an international standard that banking regulators can use when creating regulations about how much capital banks need to put aside to guard against the various types of financial and operational risks banks face. Bank earn through plethora of investments made in loans and equity investments. Each category of loans and investments has its own risk weight and return and it is necessary to combine various risk categories of assets with their returns in relation to the available capital so as to maximize the risk-weighted return and optimize the utilization of capital. A genetic algorithm (GA) is a search technique used in computing to find exact or approximate solutions to optimization and search problems. Genetic algorithms are categorized as global search heuristics. Genetic algorithms are a particular class of evolutionary algorithms (EA) that use techniques inspired by evolutionary biology such as inheritance, mutation, selection, and crossover.

2. Literature Review

Modern portfolio theory provides a well-developed paradigm to form a portfolio with the highest expected return for a given level of risk tolerance. Markowitz, 1952 and Markowitz, 1959, a creator of modern portfolio theory, originally formulated the fundamental theorem of mean–variance portfolio framework, which explains the trade-off between mean and variance each representing expected returns and risk of a portfolio, respectively. Although Markowitz's theory uses only mean and variance to describe the characteristics of return, his theory about the structures of a portfolio became a cornerstone of modern portfolio theory (Fama, 1970, Hakansson, 1970, Hakansson, 1974, Merton, 1990 and Mossin, 1969). Genetic algorithm is a stochastic optimization technique invented by Holland (1975) and a search algorithm based on survival of the fittest among string structures (Goldberg, 1989). They applied the idea from biology research to guide the search to an (near-) optimal solution (Wong & Tan, 1994). The general idea was to maintain an artificial ecosystem, consisting of a population of chromosomes. In this study, each chromosome represents the weight of individual stock of portfolio and is optimized to reach a possible solution. Attached to each chromosome is a fitness value, which defines how good a solution the chromosome represents. By using mutation, crossover values, and natural selection, the population will converge to only one chromosomes with good fitness (Adeli and Hung, 1995 H. Adeli and S. Hung, Machine learning: neural networks, genetic algorithms, and fuzzy systems, Wiley, New York (1995).Adeli & Hung, 1995). Recently, GA attracts much attention in portfolio formulations (Orito et al., 2003 and Xia et al., 2000). In the field of model solving, Arnone (Arnone et al., 1993) presented a Genetic Algorithm for an unconstrained portfolio optimization problem but the first use of genetic algorithm for Markowitz's model (without any extra constraints) was done by Shoaf (Shoaf, & Foster, 1996). Rolland utilized Tabu search (TS) to solve Markowitz's (Rolland, 1997). Later, to corroborate the necessity and desirability of heuristic algorithms, Mansini and Speranza proved that the portfolio selection problem with minimum transaction lots is an NP-complete problem. Subsequently, they proposed three heuristic algorithms to figure out the MAD model of Konno (Mansini, & Speranza, 1999). Afterwards, they (with Kellerer) extended their model in order to take fixed transaction costs into account (Kellerer, Mansini, & Speranza, 1999).

3. Data and Constraints

A typical Indian bank holds a portfolio of loans and equity investments. In India banks have an obligation to provide regulated sectors such as agriculture, housing, small and medium enterprises, commercial real estate etc.(Table:1). As per the concentration risk, the Banking sector regulator (RBI in India) has given different ceiling limit for each category of loans. These asset classes have different risk weights and returns. Each credit class is generally associated with a return.

Table: 1

Investment Types	Risk Weight Max(%)	Risk-Weight-Ideal (%)	Return (%)	Book-Value (%)	Regulatory Loan Requirement
SME	20	20	12.00	W1	Minimum 12%
Commercial Real Estate	20	50	14.50	W2	No limit
Large Corporation	20	20	11.50	W3	No Limit
Residential Property	20	50	14.00	W4	Minimum 10%
Consumer Credit	20	50	12.00	W5	No Limit
Regulatory Retail	20	50	12.50	W6	Minimum 18%
Equity Investment	20	60	20.00	W7	Maximum 5%
Sovereign	20	0	8.25	W8	Minimum 25%
Banks	20	20	10.25	W9	No Limit
PSE	20	20	9.75	W10	No Limit

Assets are divided into different credit classes as defined above. The returns in the table are for AAA credit class which is the best credit class for each segment. The portfolio allocation is to be restrained for the first two credit class in each segment i.e. AAA and AA bonds-loans. The mutation depends on the encoding as well as the crossover.

4. Mathematical Model

For adjusting risk of each asset class the formulation used is:

$$AR_i = R_i - CC * RW_i \text{ --------------------------(1)}$$

Where:

AR_i = Adjusted Return

R_i = Return on i-th asset class

CC = Cost of capital

RW_i =Risk Weight

The paper has used AR_i in place of expected return to account for the special case of Banks. Risk of the combined portfolio is calculated as per the following

$$E(R_i) = \alpha_i + \beta_i E(R_m) \text{ ----------------------------(2)}$$

$$Var(R_i) = \beta_i^2 \sigma_m^2 + \sigma_{e_i}^2 \text{ ----------------------------(3)}$$

$$Cov(R_i, R_j) = \beta_i \beta_j \sigma_m^2 \text{ ----------------------------(4)}$$

Betas required in the above equation have been calculated through regression from historical data. The return on market has been replaced by Prime Lending Rate taking into account the special case of bank portfolio. Outputs from equations (2), (3) and (4) are used to calculate Portfolio risk according to the equation:

$$\sigma_p^2 = \sum_i \sum_j w_i w_j \sigma_i \sigma_j \rho_{ij} \text{ --------------------(5)}$$

From the values of expected portfolio return (Adjusted return) as calculated from equation (1) and Portfolio risk calculated from equation (5) we can create our fitness function and constraints needed in Genetic optimization model as follows:

Fitness function:

$$F(x) = \frac{AR_i - \text{int} - opt}{\sigma} \text{ -------------------(6)}$$

Where: int = Interest cost to bank on deposits

opt =Operating cost of bank

Constraints are formulated as:

$$\sum_i w_i = 1$$

$$w_1 + w_3 \geq 0.12$$

$$w_6 \geq 0.18$$

$$0 \leq w_i \leq 1$$

5. Genetic Algorithm Specifications

Population size of 30 chromosomes was taken. Each chromosome was binary encoded with string length equaling 10 to cover the range of weights from 0-100%.Elitism was set at top 3 fittest chromosome. Elitism is a method, where the best chromosomes (or a few best chromosomes) are copied to new population. The rest is done in classical way. Elitism can very rapidly increase performance of GA, because it prevents losing the best found solution. Crossover probability is set to 0.4 as crossover as it is the main criterion for the genetic algorithm to evolve. Mutation probability is kept low with lets us not to destroy better chromosomes already found. Mutation method used here is adaptive. Stopping criterion is either 100 generation reached or the best chromosome fitness – worst chromosome fitness is less than 10^{-6} , whichever criterion is reached first.

6. Results and Discussions

Above Methodology was applied on both Ideal and worst case scenarios given in Table 1. Ideal case being portfolio invested in AAA credit class where as worst case being AA. Efficient frontier was created in both cases and genetic algorithm was applied on both the efficient frontier to find the optimal portfolio weights. Optimal Portfolio according to genetic algorithm is:

Asset Class	Weight for Investment (%)
SME	14.95
Commercial Real Estate	4.85
Large Corporation	6.00
Residential Property	5.10
Consumer Credit	10.80
Regulatory Retail	18.00
Equity Investment	8.22
Sovereign Loans	9.05
Banks	9.78
PSE	13.25

Optimal Portfolio Risk = 13.41%

Optimal Portfolio Return = 11.89%

As portfolio risk increases from 0%, when all the asset value is invested in sovereign bonds, to 60%, when whole portfolio is invested in Equity investments, the Optimal Portfolio according to genetic algorithm is:

Asset Class	Weight for Investment (%)
SME	13.61
Commercial Real Estate	4.18
Large Corporation	6.00
Residential Property	5.21
Consumer Credit	9.82
Regulatory Retail	18.00
Equity Investment	7.71
Sovereign Loans	21.56
Banks	4.51
PSE	9.40

Optimal Portfolio Risk = 14.61%

Optimal Portfolio Return = 9.12%

7. Conclusions

The portfolio built by the optimization model was mean-variance dominating for both worst and ideal cases than that of the current market portfolio. Portfolio was built under Indian Banking Regulations and managed to outperform the current portfolio of these banks. This model can be further improved if optimization is also done inside each asset class taking into account all the credit class of each asset.

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