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Economic Design of the VSI Charts for Cumulative Counting of Conforming Items

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Abstract. Using the CCC-chart with the variable sampling interval (VSI) scheme has been proposed recently to enhance the efficiency of monitoring a high-quality process by the traditional CCC-chart. This paper presents an economic model for designing the VSI CCC-chart. In the economic design, a cost function is constructed, involving the cost of sampling and testing, the cost of false alarms, the cost to detect and remove the assignable cause, and the cost when the process is operating out-of-control. A heuristic method of finding optimal values of adaptive design parameters for minimizing the cost function is also presented. The VSI and FSI CCC-charts are compared with respect to the expected loss per unit time.

Keywords: CCC control charts, economic design, genetic algorithms, variable sampling interval.

1. Introduction

The control chart is a primary statistical process control tool to detect the occurrence of assignable causes so that a remedial action can be taken before a large number of defective products are manufactured in a process. The traditional approach to monitoring the defect rate in attribute data is to use a p-chart, which is based on the number of defects in a series of sampling intervals. However, with the rapid increase of automated technologies, it is possible for enterprises to pursue the goal of a high-quality process with almost zero defects. For such cases, it is better to use the cumulative conformance count (CCC) control chart, which is based on the time-between-defects [1].

The CCC-chart was first introduced by Calvin [2] and further developed by Goh [3]. The strategy of using 100% inspection is typically suggested for the implementation of the CCC-chart. However, considering practical factors such as the inspection time and cost, this strategy would lead to a cost for inspection and thus limit the application of CCC-charts. Instead of inspecting the items individually, Liu *et al*. [4] used sampling to reduce the inspection cost and time. They considered every individual item inspected as a sample (i.e., sample size is one), and called the time between two successive samples the sampling interval. In order to improve the detection speed of control charts for process changes, they further employed a variable sampling interval (VSI) scheme that allows an adaptive length of sampling interval, depending on the indication of last point on the chart. If the CCC statistic (i.e., time-between-defects) shows some indication of a process change, then the control is tightened by shortening the sampling interval. Otherwise, the control is loosened by lengthening the sampling interval. The results of the study showed that the efficiency of a CCC chart can be improved by using the VSI scheme without increasing the rate of inspected items and false alarm occurrences. However, they did not consider the economic aspect. Therefore, we extend Liu *et al*.'s work by presenting a methodology for designing the VSI CCC-chart based on the economic criterion. The rationale of VSI CCC-chart is first reviewed. Then an economic design for the VSI CCC-chart in accordance with Markov chain approach is presented. The optimal design parameters can be found through an optimization tool so that the cost function is minimized. An industrial example is used to provide numerical illustrations.

2. The VSI CCC-chart and Markov chain

2.1. The CCC-chart

Let X be the cumulative counts of items inspected until a defective item is observed. Given the defect rate of p_0, X can be modeled by a geometric distribution with the probability mass function $f(x)$ and cumulative distribution function $F(x)$ as follows.

$$f(x) = (1 - p_0)^{x-1} p_0 ,\ F(x) = 1 - (1 - p_0)^x ,\ x = 1,2,\cdots \tag{1}$$

When a CCC-chart is applied to monitor the process parameter p_0, the values of X (denoted by $X(i)$, $i = 1,2,\cdots$) are calculated and plotted over time on the chart with the upper control limit (UCL) and lower control limit (LCL). It implies that the process is better and the fraction of nonconformance is smaller when $X(i)$ goes beyond the UCL, and so the low-sided CCC chart, which signals only when $X(i) \le$ LCL, is often considered for simplicity and practicality.

Letting α be an acceptable rate of false alarm, according to the probability control limits the LCL can be determined by $P\{X(i) \le \text{LCL}\} = \alpha$ (or $1 - (1 - p_0)^{\text{LCL}} = \alpha$). Since the geometric distribution is discrete, the control limit can be rounded to integers and calculated by

$$\text{LCL} = [\ln(1-\alpha)/\ln(1-p_0)] , \tag{2}$$

where [y] stands for the largest integer not greater than y. The true false alarm rate α' obtained from the rounded control limit may be not exactly equal to α, but it is extremely close when p_0 is very small.

The usual method of using the CCC-chart to monitor a process is to obtain samples of fixed size ($n_0 = 1$) at fixed sampling interval (saying h_0) between two successive samples; and this is referred to as fixed sampling interval (FSI) CCC-chart.

2.2. The VSI CCC-charts

The VSI CCC-chart is a modification of the FSI CCC-chart by using the VSI scheme to increase the detection speed of process changes. In this scheme, the area of the CCC-chart is divided into the safety, warning and action regions. Before the value of X goes beyond the LCL (i.e., falling into the action region), the VSI CCC-chart allows the sampling interval to switch between the maximum interval h_1 ($\ge h_0$) and minimum interval h_2 ($\le h_0$), depending on the position the current point falls on the chart. If current point $X(i)$ falls into the safety region, the maximum sampling interval h_1 will be used afterward for the inspection until the occurrence of next defect; whereas if $X(i)$ falls in the warning region, the minimum sampling interval h_2 will be used subsequently for the inspection until the next defect appears.

The safety, warning, and action regions are given by the warning limit (WL) and the lower control limit (LCL) as follows. The safety region is given by (WL, ∞), the warning region is given by (LCL, WL], and the action region is given by (0, LCL].

2.3. Application of the Markov Chain Approach

In the literature, the most widely used statistical measure for comparing the efficiencies of different adaptive control chart is called the adjusted average time to signal ($AATS$). The memoryless property of the exponential distribution allows the computation of $AATS$ using the Markov Chain approach. The fundamental concepts used in the following paragraphs can be found in Çinlar [5].

Let ATC (average time of the cycle) be the average time from the cycle start to the time the chart signals after the process change. Then,

$$AATS = ATC - 1/\lambda \tag{3}$$

At each sampling time during the period ATC, one of the nine transient states is reached according to the status of the process (in or out-of-control), the control mode (loosened or tightened control), and inspection result: *State 1*: the process is in-control and the item inspected with loosened control is conforming; *State 2*: the process is in-control and the item inspected with tightened control is conforming; *State 3*: the process is in-control, the item inspected with loosened/tightened control is defective, and X falls into the safety region; *State 4*: the process is in-control, the item inspected with loosened/tightened control is defective, and X falls into the warning region; *State 5*: the process is in-control, the item inspected with loosened/tightened control is defective, and X falls into the action region; *State 6*: the process is out-of-control and the item inspected with loosened control is conforming; *State 7*: the process is out-of-control and the item inspected with tightened control is conforming; *State 8*: the process is out-of-control, the item inspected with loosened/tightened control is defective, and X falls into the safety region; *State 9*: the process is out-of-control, the item inspected with loosened/tightened control is defective, and X falls into the warning region; When *State 5* is reached, the signal the chart produces is a false alarm. If the item inspected is defective and X falls into the action region when the process status is out-of-control, then the signal is a true alarm and the absorbing state, *State 10*, is reached.

Accordingly, the transition probability matrix can be obtained. Then by the elementary properties of Markov Chains [5], $r'(I-Q)^{-1}$ provides the mean number of transitions in each transient state before the true alarm signals, in which $r' = (r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8, r_9)$ is the vector of starting probability such that $\sum r_i = 1$; I is the identity matrix of order 9; and Q is the transition matrix where the elements associated with the absorbing state have been deleted. The product of the average number of visiting the transient state and the corresponding sampling interval determines the period ATC. Thus,

$$ATC = r'(I-Q)^{-1}t \tag{4}$$

where t is the vector of the sampling time intervals corresponding to the nine transient states for the next sampling. Here we set the vectors $r' = (0,0,0,0,1,0,0,0,0)$ and $t' = (h_1, h_2, h_1, h_2, h_2, h_1, h_2, h_1, h_2)$. The fifth element in t' placed by h_2 aims to provide an additional protection to prevent problems that arise during start-up.

3. The cost model

In developing the cost model of a process controlled by the VSI CCC-chart, we make the following assumptions: (a). Process starts with an in-control state ($p = p_0$), but after a random time of in-control operation it will be disturbed by a single assignable cause that causes a change in the defect rate of the process ($p = p_1$); (b) After the change, the process remains out-of-control until the assignable cause is eliminated; (c) The inter-arrival time of the assignable cause disturbing the process is assumed to follow an exponential distribution with a mean of $1/\lambda$ hours; (d) To detect the change in process defect rate, an item is inspected at each sampling time; the number of inspected samples until a defect occurs is recorded and plotted on the chart in sequence; (e) Before the number goes beyond the LCL, the length of subsequent sampling intervals will vary with their position on the chart. Otherwise, the process will be stopped and a search will start to find the assignable cause and adjust the process.

The production cycle, which is divided into four time intervals of in-control period, out-of-control period, searching period due to false alarm, and the time period for identifying and correcting the assignable cause.

Expected length of in-control period is $1/\lambda$. Expected length of out-of-control is the average time needed for the control chart to produce a signal after the change in process defect rate and is given by $AATS$.

Let t_0 be the average amount of time wasted searching for the assignable cause when the process is in-control, and $E(FA)$ be the expected number of false alarms per cycle given by

$$E(FA) = r'(I-Q)^{-1}f \tag{5}$$

where $f' = (0,0,0,0,1,0,0,0,0)$. Then the expected length of searching period due to false alarm is given by $t_0 E(FA)$. The time to identify and correct the assignable cause following an action signal is a constant t_1. Therefore, the expected length of a production cycle can be represented in aggregate as

$$E(T) = ATC + t_0 E(FA) + t_1 \tag{6}$$

If one defines V_0 = the hourly profit earned when the process is operating in control state; V_1 = the hourly profit earned when the process is operating in out-of-control state; C_0 = the average search cost if the given signal is false; C_1 = the average cost to discover the assignable cause and adjust the process to in-control state; s = the cost for each inspected item. Then the expected net profit from a production cycle is given by

$$E(C) = V_0(1/\lambda) + V_1(M - 1/\lambda) - C_0 E(FA) - C_1 - sE(N) \tag{7}$$

where $E(N)$ is the average number of inspected items during a production cycle, as given by

$$E(N) = r'(I-Q)^{-1}\eta \tag{8}$$

In which η' is the vector of sample sizes corresponding to the nine transient states for next sampling. Note that individual observations are used when implementing the VSI CCC-chart, and so η' is a vector of 1's. Finally, based on the renewal reward process assumption, the expected loss per hour $E(L)$ is given by

$$E(L) = V_0 - E(C)/E(T) \quad (9)$$

4. Solution Procedure

The cost function $E(L)$ is a function of the process parameters, the cost parameters, and the design parameters. The economic design of a control chart for a particular application is to derive the design parameters that minimize $E(L)$ for the given process and cost parameters. This minimization problem can be regarded as a decision problem with mixed continuous-discrete decision variables and a discontinuous and non-convex solution space. Because it maybe inefficient and time-consuming to apply typical non-linear programming techniques to a search for the optimal solution, we employed genetic algorithms (GAs) due to the less chance of converging to local optima in a multimodal space than do the typical techniques.

GAs are search algorithms that were developed based on an analogy with natural selection and population genetics in biological system [6]. They have been commonly used or modified for solving many kinds of optimization problems. Recently, several extensive applications to the design optimization problem of quality control charts have been presented [7]–[9]. The operations of GAs include four steps: 1) randomly generate an initial solution population of candidate solutions (i.e., design parameters), each represented as a string of bits; 2) assign each bit string a value according to a fitness function (i.e., the objective function that minimizes the $E(L)$) and select strings from the old population randomly but biased by their fitness; 3) recombine these strings by using the crossover and mutation operators; 4) produce a new generation of strings that are more fit than the previous one. The termination condition is achieved when the number of generations is large enough or a satisfied fitness value is obtained. The following setting of control parameters for the GAs manipulation has been used: population size was set to 75; crossover probability was set up to 0.3; mutation rate was set up to 0.25; and the number of generations was set to at least 30,000 times.

5. Industrial example

5.1. Industrial Example

Consider an injection molding process that produces a micro-prism array of optical elements in the three stages of filling, packing, packing and cooling. The surface roughness of the mould directly impacts the surface quality of the product. Thus, the mould's should be polished to ensure it reach the optical class level. This so called optical class level is defined as the Average Roughness (Ra) of 100 nm. If Ra is less than 100 nm, it is in the good quality range; if greater than 100 nm, and then it is not qualified. The process can move on to fully automated production after the parameter selection is complete and the machinery is stabilized. In such a manufacturing environment, the process yield is very high, and the percentage of nonconforming items is usually at the level of 5×10^{-4} (p_0=500 ppm). For most of time the process remains in-control, but occasionally the nonconformance percentage shifts from the original level to 5×10^{-2} (p_1=0.05). Therefore, a CCC-chart is employed to monitor the cumulative count of conforming items between two consecutive defects for detecting the occasional shift.

According to the previous runs, we have the following information related to the process parameters and cost parameters.

Process parameters: $\lambda = 0.05$, t_0=0.1, t_1=0.3; Cost parameters: s=5, V_0=150, V_1=50, C_0=10, C_1=30.

At present, the CCC-chart takes one item for every 10 items to inspect. As the cycle time for each item is about 36 seconds, the sampling interval length can be set as 0.1 hours (h_0=0.100). For the FSI case, the optimal lower control limit (LCL = 177) that minimizes the hourly expected loss $E(L)$ can be obtained from the economic model by setting $h_1 = h_2 = h_0$ as well as WL=LCL. This scheme has $E(L)$=17.078, out-of-control $AATS$ = 1.950, and $E(FA)$=0.010.

TABLE I
DESIGN PARAMETERS IN THE INDUSTRIAL EXAMPLE

	n	h_1	h_2	WL	LCL
FSI-chart	1	0.10			177
VSI-chart	1	0.12	0.08	1571	186

TABLE II
PERFORMANCE COMPARISONS IN THE INDUSTRIAL EXAMPLE

	$AATS$	$E(FA)$	$E(L)$	%
FSI-chart	1.950	0.010	17.078	
VSI-chart	1.599	0.011	16.799	1.63

5.2. Improvements

Due to the low speed with which the standard CCC chart detects a process change, the process engineer now monitors the process by adding a VSI feature to the CCC chart. In the economic design of the VSI CCC-chart, the minimum value of sampling intervals are considered as the possible minimum time between successive samples available in a work shift, i.e., $h_2 \geq 0.01$. Furthermore, as Liu et al. [4] recommended, the performance of the VSI CCC-chart can be improved significantly when adopting equal probability allocation for the safety and warning region, i.e., $\Pr(\text{LCL} < X \leq \text{WL} \mid X > \text{LCL}) = \Pr(X > \text{WL} \mid X > \text{LCL})$.

Consequently, we consider only the above situation in this study for the sake of convenience.

Tables I and II show the optimal design parameters and corresponding performances for the VSI CCC-chart as well as the FSI CCC-chart. The cost function $E(L)$ for the VSI scheme arrives at minimum of 16.799 if the optimal parameters for the VSI scheme are set by $n=1$, h_1=0.120, h_2=0.080, WL =1571, and LCL =186. If the percent reduction (%) in $E(L)$ is defined as the

$$(E(L)_{FSI} - E(L)_{VSI})/E(L)_{FSI} \times 100 \tag{10}$$

The saving of per hour operation is about 1.63%. Although economic property is slightly better, the VSI scheme has substantially better statistical properties (out-of-control $AATS$=1.599 and $E(FA)$=0.011) than the FSI scheme.

6. Conclusions

This paper has presented an economic design for the VSI CCC-chart used to monitor the defect rate of a high-quality process. The cost function developed here can be used to evaluate the economic performance of the VSI CCC-chart. The assumption that the assignable cause occurs according to an exponential distribution allows the cost function to be obtained by means of the Markov chain approach. An industrial example for illustrating the application of the economic model is provided, and the genetic algorithm is employed to search for the solution of economic design.

7. References

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