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Hot Money vs. Cool Money – A Probability Model for the Trend of Investment Cash Flow

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Abstract. This paper presents a generalized Markov chain probability model for describing and estimating the trends of investment cash flow with changing transition matrices, and simulates the interactions between the market, the hot money flow, and the cool money investments. Numerical procedures for establishing such models and estimating market status are also presented.

Keywords: hot money, Markov chain, equilibrium, trend

1. Introduction

Investment trends in financial market form an important factor in the investment decision making procedure of financial managements. Decisions on whether and how to invest during a particular time period and in a particular market are often made based on the estimates of the trends of investment in that market. In this paper, we concentrate on the trends of hot money versus cool money and we consider Markov chain equilibrium models in estimating the trends.

As a much simplified example, let us collectively call all the hot money cash flow investments in a certain market "sector A". The proportion of the total market investment value that is invested in sector A is recorded at an initial time moment, and then at the end of each month afterwards for several months. Each record shows a market status vector $S = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ where x_1 stands for the percent of the total investment in sector A (hot money) and x_2 for the percent of the total investment not in sector A (cool money). Clearly $0 \le x_1, x_2 \le 1$ and $x_1 + x_2 = 1$. To be more precise, let S_0 be the initial status vector, that is, the status vector at the initial time moment, S_1 be the status vector at the end of the first month, and so on so forth.

Suppose the initial status $S_0 = \begin{bmatrix} 0.05 \\ 0.95 \end{bmatrix}$, that is, at that time 5% of the total market investment is invested in sector A (as hot money) while 95% is not in A (cool money). Suppose the succeeding records show that in each month, 40% of money in sector A stays in it and 60% switch to long term investment (cool money). Meanwhile, 20% of cool money switches to sector A while 80% of cool money stays as cool money. Based on these assumptions regarding the investment flows, we can see that

$$S_1 = \begin{bmatrix} 0.4 & 0.2 \\ 0.6 & 0.8 \end{bmatrix} S_0 = \begin{bmatrix} 0.4 & 0.2 \\ 0.6 & 0.8 \end{bmatrix} \begin{bmatrix} 0.05 \\ 0.95 \end{bmatrix} = \begin{bmatrix} 0.21 \\ 0.79 \end{bmatrix} \quad (1)$$

That is, after one month 21% of the total market investment gets in sector A while 79% is not in A. Similarly we have

$$S_2 = \begin{bmatrix} 0.4 & 0.2 \\ 0.6 & 0.8 \end{bmatrix} S_1 = \begin{bmatrix} 0.4 & 0.2 \\ 0.6 & 0.8 \end{bmatrix} \begin{bmatrix} 0.21 \\ 0.79 \end{bmatrix} = \begin{bmatrix} 0.242 \\ 0.758 \end{bmatrix} \quad (2)$$

So after two months 24.2% is in sector A while 75.8% is not.

If a simple estimate is made based on the records of those few months and we believe that the same changes will happen in every month in long run, then a Markov chain $S_0, S_1, S_2 \ldots$ is formed, with the transition matrix

$$P = \begin{bmatrix} 0.4 & 0.2 \\ 0.6 & 0.8 \end{bmatrix} \quad (3)$$

It is easy to see that this is a regular Markov chain with the equilibrium status vector

$$S = \lim_{k\to\infty} S_k = \begin{bmatrix} 0.25 \\ 0.75 \end{bmatrix} \quad (4)$$

In other words, if the investment flow keeps the same pattern, then eventually the proportion of the total market investment that is in sector A (hot money) will approach 25%, and the market status will stay that way for long run. Note that the equilibrium status S in equation (4) satisfies

$$PS = S \quad (5)$$

Also note that in this case, the proportion in sector A made the largest grow in the first month.

It is also well known that for a regular Markov chain like this, the equilibrium status S does not depend on the initial status vector S_0. That is, for any initial status S_0, under the constant transition matrix $P = \begin{bmatrix} 0.4 & 0.2 \\ 0.6 & 0.8 \end{bmatrix}$, the equilibrium status will always equal $\begin{bmatrix} 0.25 \\ 0.75 \end{bmatrix}$.

In real life investment market, however, situations are not that simple. The transition matrix P usually changes from time period to time period. Therefore, the above simple model might not work well. In this paper, we consider investment trends with changing transition matrices, and try to mathematically model the transition matrices and numerically estimate the equilibrium status.

2. Models for Changing Transition Matrices

All Let $P(k) = \begin{bmatrix} p_{11}(k) & p_{12}(k) \\ p_{21}(k) & p_{22}(k) \end{bmatrix}$ be the transition matrix for the *k*-th time period. That is, $p_{11}(k)$ stands for the percentage of investment in sector A in the *k*-the period that continues to stay in it, $p_{21}(k)$ for the percentage of investment in A that switches out. Similarly, $p_{12}(k)$ stands for the percentage of money not in A in the *k*-the period that switches to A, while $p_{22}(k)$ for the percentage of money not in A that continues to stay as cool money. Values of $p_{11}(k)$, $p_{21}(k)$, $p_{12}(k)$ and $p_{22}(k)$ change from time period to time period, and hence they are functions of *k*. However, it is always true that

$$0 \le p_{11}(k), p_{21}(k), p_{12}(k),\ p_{22}(k) \le 1 \quad (6)$$

and

$$p_{11}(k) + p_{21}(k) = 1, \ \text{and} \ \ p_{12}(k) + p_{22}(k) = 1 \quad (7)$$

Often, hot money investments are likely to attract new cash flow into them. This is particularly true when hot money starts to rush into a certain market. In these early time periods, a growth of the value of $p_{12}(k)$ may be observed, reflecting investment flowing into the hot money sector. The value of $p_{11}(k)$ may stay quite stable and high, even close to 1, during early time periods because those who start to get in may very much likely to stay in it for some time before they decide whether to stick to it or not. After a while, however, the values of $p_{11}(k)$ and $p_{12}(k)$ may start to decrease, and now the long term on-going investment trend starts to show. In long run, the transition matrix $P(k)$ may eventually get closer and closer to the identity matrix $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, meaning that a certain percentage of the total investment market will stay as hot money, and the rest of the market will be stable long term cool money investments. The market status vector S_k reached in those time periods will then approach the equilibrium status for the hot money sector A.

Therefore, the value of $p_{11}(k)$ may stay close to 1 for a while, then starts to decrease, and eventually comes back up and stays close to 1. The values of $p_{12}(k)$ would grow up for a while in the early stages, then gradually decreases and eventually stays close to 0. The values of $p_{21}(k)$ and $p_{22}(k)$ are determined by $p_{11}(k)$ and $p_{12}(k)$, respectively, due to the equation (7).

Based on above assumptions, many mathematical functions may be used to establish models for $p_{11}(k)$ and $p_{12}(k)$. Below we propose two such models.

1. Upside-down normal distribution model for $p_{11}(k)$

$$M_{11}(k) = 1 - \frac{\alpha_1}{\sqrt{2\pi}\sigma_1} e^{-\frac{1}{2}(\frac{k-\mu_1}{\sigma_1})^2}.$$

Three parameters α_1, μ_1, and σ_1 are involved in the formula of this model. The model is formed by subtracting from 1 a portion of the density function of the normal distribution $N(\mu_1, \sigma_1)$. The curve of $M_{11}(k)$, for example, when α_1= 10, μ_1=20, and σ_1= 10, is shown in Figure 2.1. Note that the value of α_1 determines how low the value of $M_{11}(k)$ can get, μ_1 determines when it will get to that low value, and σ_1 determines how long it will take for the value of $M_{11}(k)$ to approach 1.

2. The normal distribution model for $p_{12}(k)$

$$M_{12}(k) = \frac{\alpha_2}{\sqrt{2\pi}\sigma_2} e^{-\frac{1}{2}(\frac{k-\mu_2}{\sigma_2})^2}.$$

This model is formed by using a portion of the density function of the normal distribution $N(\mu_2, \sigma_2)$. Again, three parameters α_2, μ_2, and σ_2 are involved. In Figure 2.2, we display the curve of $M_{12}(k)$ when α_2= 5, μ_2= 18, and σ_2= 8.

Since models $M_{11}(k)$ and $M_{12}(k)$ each involves three parameters, in order to numerically estimate these parameters, market records are needed for at least three time periods. For example, suppose a record is made at the beginning of the first time period and then at the end of each time period for the first n periods (with $n \geq 3$) and the values of $p_{11}(k)$ and $p_{12}(k)$, for k = 1, 2, … n, are found from these records, then by numerically solving the nonlinear system

$$p_{11}(k) = 1 - \frac{\alpha_1}{\sqrt{2\pi}\sigma_1} e^{-\frac{1}{2}(\frac{k-\mu_1}{\sigma_1})^2} \quad \text{for } k = 1, 2, \dots n \qquad (8)$$

we can determine the parameters α_1, μ_1, σ_1 in $M_{11}(k)$, and thus establish a model for $p_{11}(k)$. Similarly, by solving the nonlinear system

$$p_{12}(k) = \frac{\alpha_2}{\sqrt{2\pi}\sigma_2} e^{-\frac{1}{2}(\frac{k-\mu_2}{\sigma_2})^2} \quad \text{for } k = 1, 2, \dots n \qquad (9)$$

we can establish the model $M_{12}(k)$ for $p_{12}(k)$. Many numerical methods are available for solving nonlinear systems (8) and (9). See, for example, [1], [2] and [3].

3. Numerical Estimates of Investment Trends and Equilibrium Status

In this section, let us use an example to describe the numerical procedure for estimating the investment trends and equilibrium market status. Market records are made at the initial time moment, and then at the end of each month for four months. Suppose the first record shows that the initial status $S_0 = \begin{bmatrix} 0.01 \\ 0.99 \end{bmatrix}$, that is, at that time 1% of the total market is in hot money sector A while 99% is long term cool money investments. Suppose the succeeding records show that the transition matrix for these four months are:

$$P(1) = \begin{bmatrix} 0.998 & 0.231 \\ 0.002 & 0.769 \end{bmatrix}, \qquad P(2) = \begin{bmatrix} 0.994 & 0.289 \\ 0.006 & 0.711 \end{bmatrix},$$

$$P(3)=\begin{bmatrix}0.984 & 0.350\\ 0.016 & 0.650\end{bmatrix},\qquad P(4)=\begin{bmatrix}0.970 & 0.399\\ 0.030 & 0.601\end{bmatrix}.$$

Based on these records, we have that $p_{11}(1) = 0.994$, $p_{11}(2) = 0.984$, $p_{11}(3) = 0.967$, and $p_{11}(4) = 0.926$. By solving the nonlinear system (8) with $n = 4$, we get $\alpha_1 = 4$, $\mu_1 = 10$, and $\sigma_1 = 3$. Therefore the model for $p_{11}(k)$ in this case is

$$M_{11}(k) = 1 - \frac{4}{\sqrt{2\pi}\cdot 3} e^{-\frac{1}{2}(\frac{k-10}{3})^2}.$$

Similarly, the records show that $p_{12}(1) = 0.289$, $p_{12}(2) = 0.350$, $p_{12}(3) = 0.399$, $p_{12}(4) = 0.442$. Solving the nonlinear system (9) with these values we see that $\alpha_2 = 6$, $\mu_2 = 6$, and $\sigma_2 = 5$. Thus the model for $p_{12}(k)$ will be

$$M_{12}(k) = \frac{6}{\sqrt{2\pi}\cdot 5} e^{-\frac{1}{2}(\frac{k-6}{5})^2}.$$

With these two models we may estimate the monthly transition matrix which represents the investment flow in each month, using the formula

$$P(k) \approx \begin{bmatrix} M_{11}(k) & M_{12}(k) \\ 1-M_{11}(k) & 1-M_{12}(k)\end{bmatrix} \qquad (10)$$

Table 3.1 shows the estimates of entries in the transition matrices in the 5-th, 10-th, 15-th, and 20-th months. Note that $P(k)$ approached $I = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ when k gets bigger and bigger.

With the established model for the transition matrix $P(k)$ and the value of the initial status $S_0 = \begin{bmatrix}0.01\\ 0.99\end{bmatrix}$, we may easily estimate the market status S_k for all the succeeding months as well as the equilibrium market status. For example, $S_1 = P(1)S_0 = \begin{bmatrix}0.24\\ 0.76\end{bmatrix}$, $S_2 = P(2)S_1 = \begin{bmatrix}0.46\\ 0.54\end{bmatrix}$, and so on so forth. Table 3.2 shows the estimates of S_k for k from 1 to 30. For convenience, in the table the status vector S_k is displayed as a row vector instead of a column vector. From the table we can see that for hot money sector A the equilibrium market status will be about $\begin{bmatrix}0.43\\ 0.57\end{bmatrix}$. That is, when time goes by the proportion of the total market investment in sector A will approach 43%, and the market status will stay that way in long run.

It is also interesting to mention that, like in the regular Markov chain with constant transition matrix, the equilibrium status in this model does not depend on the initial status vector. For example, if $S_0 = \begin{bmatrix}0.1\\ 0.9\end{bmatrix}$ instead of $\begin{bmatrix}0.01\\ 0.99\end{bmatrix}$, the equilibrium market status will still be $\begin{bmatrix}0.43\\ 0.57\end{bmatrix}$, and this remains true for any value of initial status vector.

4. Further Comments

The example used in Section 3 is a simulated case intended to describe the numerical produce to establish models for changing transition matrices and thus to estimate the investment trends and equilibrium status. In real investment applications, market records need to be examined with care and models should be selected carefully. There is a variety of models that could be applied in the estimating procedure, including exponential models, polynomial models, and so on. Moreover, more market records would become available when time goes by. Therefore, real-time on-going modification of models may be conducted based on the additional information obtained from the newly received market records. Furthermore, many other factors such as developments in other sectors, changes in the economic and political environments, growth of the investment population, etc, may also have impacts on the investment trends. Therefore, more generalized models such as piecewisely defined models or compositions of models may also be needed to address the effects of those factors.

5. References

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TABLE 3.1 Estimates of Transition Matrices

k	$p_{11}(k)$	$p_{21}(k)$	$p_{12}(k)$	$p_{22}(k)$
5	0.867	0.133	0.469	0.531
10	0.468	0.532	0.348	0.652
15	0.867	0.133	0.095	0.905
20	0.998	0.002	0.009	0.991

TABLE 3.2 Estimates of S_k

k	S_k	k	S_k	k	S_k
1	[0.24, 0.76]	11	[0.40, 0.60]	21	[0.42, 0.58]
2	[0.46, 0.54]	12	[0.37, 0.63]	22	[0.43, 0.57]
3	[0.64, 0.36]	13	[0.36, 0.64]	23	[0.43, 0.57]
4	[0.76, 0.24]	14	[0.36, 0.64]	24	[0.43, 0.57]
5	[0.81, 0.19]	15	[0.37, 0.63]	25	[0.43, 0.57]
6	[0.79, 0.21]	16	[0.38, 0.62]	26	[0.43, 0.57]
7	[0.72, 0.28]	17	[0.39, 0.61]	27	[0.43, 0.57]
8	[0.62, 0.38]	18	[0.40, 0.60]	28	[0.43, 0.57]
9	[0.52, 0.48]	19	[0.41, 0.59]	29	[0.43, 0.57]
10	[0.45, 0.55]	20	[0.42, 0.58]	30	[0.43, 0.57]

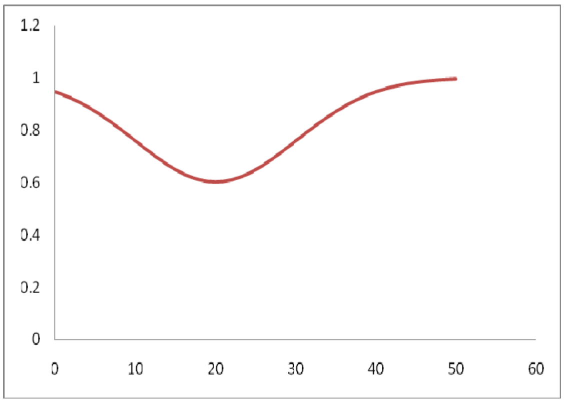

Figure 2.1 The curve of $M_{11}(k)$ when α_1= 10, μ_1=20, and σ_1= 10,

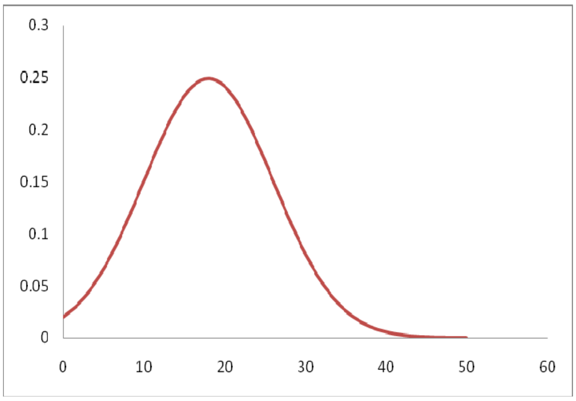

Figure 2.2 The curve of M12(k) when $\alpha 2$= 5, $\mu 2$= 18, and $\sigma 2$= 8.